Nuclear structure input for rp-process rate calculations in the sd shell

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OUTLINE

Introduction

Application of the Nuclear Shell Model

Interactions used

Application of the Isobaric Mass Multiplet Equation

Calculation of (p, gamma) reaction rates

Conclusions
INTRODUCTION

Many states in the final nuclei of the rp process, such as $^{26}$Si, $^{30}$S and $^{36}$K, are not well known, thus requiring input from shell-model calculations.

The total rp-process reaction rate depends on the partial gamma decay widths and the proton decay widths of states in the final nucleus (above the proton-emission threshold) and their Q values.
Reaction rate equation

\[ N_A < \sigma v > = 1.54 \cdot 10^{11} (AT)_9^{-3/2} \omega \gamma [\text{MeV}] e^{\frac{-11.605E_r [\text{MeV}]}{T_9}} \frac{\text{cm}^3}{\text{s mole}} \]

\[ \omega \gamma = \frac{2J_r + 1}{(2J_1 + 1)(2J_T + 1)} \Gamma \]

\(^{32}\text{Cl}(p,g)^{33}\text{Ar}\) Reaction rate for \(T=0.4 \times 10^9\) K

Example: contribution from one individual resonance (5/2+)
APPLICATIONS OF THE SHELL MODEL

1) To calculate the proton decay widths i.e. the spectroscopic factors for proton capture by the target nucleus

2) The partial gamma decay widths of states in the final nucleus

3) The energies of the final nucleus in conjunction with the Isobaric Mass Multiplet Equation (IMME)
Brief review of Interactions USDA and USDB

1983: Hobson Wildenthal obtains USD interaction by fitting SPE and TBME (3+63=66) to ~450 energies in sd shell. USD used in several hundred papers for interpretation of spectroscopic properties of nuclei.

Reasons for deriving new interactions:

- Much more data, especially neutron–rich nuclei
- Many nuclei near middle of sd shell omitted
- Problems with alternative interactions esp. G-matrix

Brief review of Interactions USDA and USDB.
EXPERIMENTAL DATA

• With neutron-rich nuclei and previously omitted nuclei, we used 608 levels in 77 nuclei.

FITTING PROCEDURE

• Minimize deviations (chi-squared) between theor. and exp. energies in several iterations.

For USDA, 30 well-determined LC’s (170 keV rms)

For USDB, 56 well-determined LC’s (130 keV rms)
Generally good agreement with experiment for all sd-shell observables calculated with the effective interactions USDA and USDB [Richter, Mkhize, Brown, Phys. Rev. C 78, 064302 (2008)]

Optimal g factors and effective charges were determined from least-square fits to 48 magnetic moments, 26 quadrupole moments, 111 M1 transitions and 144 E2 transitions.
For **level energies** USDB provided a superior agreement (130 keV rms fit deviations). USD overbinds both the n-rich F and O isotopes. Both USDB and USDA gave improved **binding energies** for neutron-rich nuclei compared to USD.

<table>
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<th>$e_p$</th>
<th>$e_n$</th>
<th>$g_{lp}$</th>
<th>$g_{sp}$</th>
<th>$g_{tp}$</th>
<th>$g_{ln}$</th>
<th>$g_{sn}$</th>
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<td>1</td>
<td>5.586</td>
<td>0</td>
<td>0</td>
<td>-3.86</td>
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(free-nucleon)
Application of the Isobaric Mass Multiplet Equation (IMME)

Example: To determine levels in $^{30}$S

According to the IMME $B = a + bT_z + cT_z^2$
where $B =$ binding energy of a state.

It follows that
$B(^{30}\text{S}) = 2B(^{30}\text{P})_{\text{exp}} - B(^{30}\text{Si})_{\text{exp}} + 2c_{\text{th}}$

If the analogue states are known the only theoretical input is the value of $c$. 
In 1989 W E Ormand and B A Brown (NP A 491, 1) reproduced 42 b coefficients with an rms deviation of 27 keV and 26 c coefficients with an rms dev of 9 keV using a charge-dependent Hamiltonian for A=18-22 and A=34-39. We use USDA/B for the charge-independent part. The full composite Hamiltonians will be referred to as usda-cdpn and usdb-cdpn.
Figure 1: $c$ coefficients from the isobaric mass multiplet equation (IMME: $E = a + bT_z + cT_z^2$) versus state number (in order of increasing energy) in $^{26}$Si based on experimental energies (closed circles) and energies calculated from usdb-cd-fn (open circles). Good general agreement can be seen.

Figure 2: Predicted energies based on the IMME in $^{26}$Si versus experimental excitation energies.
Figure 3: Experimental energies of the $T=1$ states in $A=36$. Negative parity states are connected by dashed lines. The energies above $S_p$ for $^{36}K$ are those measured by Wrede et al., PRC 82, 035805 (2010).
calculation of the reaction rates 

\[ ^{35}\text{Ar} \rightarrow ^{36}\text{K} \]

\[ Q = 1.658 \text{ MeV} \]

with USDB.

\[ \Gamma_\gamma \] has been calculated for 36 K levels.

Figure 4: The total reaction rate versus temperature $T_9$ (GigaK) (top panel) and the contribution of each of the final states (lower panel) with USDB.
The $^30P(p,\gamma)^{30}S$ reaction applying the IMME method to $^{30}S$ was difficult because many energies in the intermediate nucleus $^{30}P$ were ambiguous and uncertain. Fortunately, the experimental energy spectrum was extended to well above the proton emission threshold (4.339 MeV) by recent measurements as far as 7 MeV. So we reversed the IMME procedure, by checking and changing the $^{30}P$ assignments to be consistent with the IMME. This led to the following figure for the $c$ coefficients:

![Graph showing $c$ (keV) as a function of $n$. The graph has data points at $0^+$, $2^+$, $2^+$, $0^+$, $1^+$, $3^+$, $2^+$, $4^+$, $0^+$, $3^+$, $2^+$, $4^+$, and $0^+$. The $y$-axis is labeled $c$ (keV) and ranges from 150 to 300. The $x$-axis is labeled $n$ and ranges from 1 to 17.]
$^{29}$P($1/2^+$) → $^{30}$S

Q = 4.400 MeV

Contribution (%)

log(rate)

log(T9)
Uncertainties in the resonant capture reaction rate

A detailed analysis of the various possible sources of error have been given in Phys Rev C 83, 066803 (2011) for $^{25}\text{Al} (p,\gamma) ^{26}\text{Si}$.

The effect of using different interactions can be estimated from the ratios of the reaction rates, e.g. for $^{30}\text{P} (p,\gamma) ^{30}\text{S}$.
The usdb-cdpn rate for $P_{29}(p,\gamma)S_{30}$ is compared to the rate in the 2010 Evaluation (Iliades et al. NP 841, 31). The solid line is the median rate ratio and the dashed lines are the low high and low ratios.
CONCLUSIONS

• We have considered (p,\(\gamma\)) reactions leading to final nuclei in \(^{26}\text{Si}\), \(^{36}\text{K}\) and \(^{30}\text{S}\) (all published) and \(^{31}\text{S}\) (preliminary).

• Our method for determining energies of states in the final nuclei, based on the IMME with experimental energies for the \(T = 1\) analogue states and the theoretical c-coefficients, should be extended to other cases in the sd shell.
• The use of different interactions and approximations gives an indication of the theoretical error in the rates

• The contribution of negative parity states should be taken into account from measurements, if possible, or using S and $T_{1/2}$ values from the mirror nuclei as an estimate
THANK YOU!
Figure 6. Relative contributions to the reaction rates for $x = -E_{\text{res}} / (kT)$ with $T9 = 10$. Resonant reaction rate $\alpha \Sigma_f \omega \gamma_{if} e^{-E_{\text{res}} / (kT)}$. 
Why measure excitation energies?

\[ N_A < \sigma v > = 1.54 \cdot 10^{11} (AT_9)^{-3/2} \omega \gamma \ [\text{MeV}] e^{\frac{-11.605E_r [\text{MeV}]}{T_9}} \frac{\text{cm}^3}{\text{s mole}} \]

\[ \omega \gamma = \frac{2J_r + 1}{(2J_1 + 1)(2J_T + 1)} \frac{\Gamma_p \Gamma_\gamma}{\Gamma} \]

**32Cl(p,g)33Ar** Reaction rate for T=0.4 x 10^9 K

Example: contribution from one individual resonance (5/2+)

![Graph showing reaction rate vs. resonance energy](Graph.png)