Cosmology with Minkowski Functionals and Moments of Weak Lensing Fields

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with Z.Haiman, L.Hui, M.May and J.M.Kratochvil
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Cosmological information in non weak lensing maps

- CMB temperature $\rightarrow$ gaussian $\rightarrow$ two point function
  \[ \xi(x_1, x_2) = \langle \delta T(x_1) \delta T(x_2) \rangle \]
- Fourier equivalent: Power spectrum
  \[ \langle \delta \hat{T}(k) \delta \hat{T}(k') \rangle = (2\pi)^3 P(k) \delta(k + k') \]
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\[ \frac{d\theta_i'}{d\theta_j} = (1 - \kappa)\delta_{ij} - \gamma_1 \sigma^3_{ij} - \gamma_2 \sigma^1_{ij} \]

• Convergence \( \kappa \) (galaxy magnification due to lensing) \( \rightarrow \) highly non gaussian \( \rightarrow \) two point statistics \( \xi(x_1, x_2) = \langle \kappa(x_1)\kappa(x_2) \rangle + \cdots \)
Cosmological information in non weak lensing maps

- \( \frac{d\theta'_i}{d\theta_j} = (1 - \kappa)\delta_{ij} - \gamma_1 \sigma_{ij}^3 - \gamma_2 \sigma_{ij}^1 \)

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Our framework

1000 realizations of simulated $\kappa$ maps for a fiducial $\Lambda$CDM model with $(\Omega_m, w, \sigma_8) = (0.26, -1.0, 0.798)$

- 1000 realizations for each parameter variation
- Single redshift plane at $z_s = 2$
- Galaxy shape noise added assuming $n_{gal} = 15\text{arcmin}^{-2}$
- Gaussian smoothing with a variable window size $\theta_G = 1 \div 15 \text{arcmin}$
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Topological descriptors: beyond gaussian statistics

Consider the excursion sets $\Sigma(\nu) = \{ \kappa > \nu \sigma_0 \}$
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**Figure**: Full simulated convergence map
Figure: Excursion set with threshold $\nu = \kappa_T / \sigma_0 = 0.01 / \sigma_0$
Minkowski Functionals are...

- $V_0(\nu)$: area of the black regions
- $V_1(\nu)$: length of the boundaries of the black regions
- $V_2(\nu)$: genus of the black regions (number of connected regions - number of holes in them)
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Analytical study of Minkowski Functionals

- If the underlying random field is gaussian they are completely determined by $\sigma_0^2 = \langle \kappa^2 \rangle$ and $\sigma_1^2 = \langle |\nabla \kappa|^2 \rangle$
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\[ V_1(\nu) = \frac{1}{8\sqrt{2}} \frac{\sigma_1}{\sigma_0} \exp \left( -\frac{\nu^2}{2} \right) \]
The real $\kappa$ field is non gaussian...

- MFs $\leftrightarrow$ moments $\langle \kappa^n | \nabla \kappa |^{2m} \rangle$ via series expansion
- $N$-th order term is proportional $\sigma_0^N H_k(N) \langle \nu \rangle e^{-\nu^2/2}$
- The proportionality coefficient is a real space moment of order $2 + N$
- We call \textit{one point moments} the ones with $m = 0$, and \textit{moments with gradients} the ones with $m \neq 0$

Does the series converge?
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Measured

Gaussian: $O(1)$

$O(\sigma_0)$

$O(\sigma_0^2)$
\[ \Delta \chi^2 = (V_{\text{pert}} - V_{\text{meas}})_i (C_V^{-1})_{ij} (V_{\text{pert}} - V_{\text{meas}})_j \]

- \[ \Delta \chi^2(\theta_G = 1') \approx 2000 \rightarrow \text{doesn't converge!} \]
- \[ \Delta \chi^2(\theta_G = 15') \approx 0.1 \rightarrow \text{converges!} \]
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Downside: distinguishing power

- $\Delta \chi^2_{cosmo}(\theta_G = 1') \approx 5 \rightarrow \text{can distinguish!}$
- $\Delta \chi^2_{cosmo}(\theta_G = 15') \approx 0.5 \rightarrow \text{cannot distinguish!}$
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Fisher Constraints on Cosmology

- Power spectrum observable probes:

\[ C = (C_1, C_2, ...) \]

\[ C \] is a \( l_{\text{max}} \) sized vector.

- We use instead:

\[ D = \left( V_0^{\nu_1}, V_0^{\nu_2}, ..., V_1^{\nu_1}, V_1^{\nu_2}, ..., V_2^{\nu_1}, V_2^{\nu_2}, ..., \langle \kappa^2 \rangle, \langle \kappa^3 \rangle, \langle \kappa^4 \rangle \right) \]

\( D \) is a \( 3N_{\text{bins}} + 9 \) sized vector.

- Measure the covariances \( C_{ij} = \langle D_i D_j \rangle \) and compute the parameters \( (p_\alpha = (\Omega_m, w, \sigma_8)) \) Fisher matrix

\[ F_{\alpha\beta} = \frac{\partial D_i}{\partial p_\alpha} C^{-1}_{ij} \frac{\partial D_j}{\partial p_\beta} \]

- Marginalized errors on the parameters

\[ \Delta p_\alpha = \sqrt{(F^{-1})_{\alpha\alpha}} \]
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Robustness checks

- We are trying to estimate a $\sim 3N_{\text{bins}} \times 3N_{\text{bins}} \sim 300 \times 300$ covariance matrix using 1000 realizations...
- Accuracy is not guaranteed
- Use of modified Fisher matrix formalism: use of auxiliary, independent, map set to measure $\partial D_i / \partial p_{\alpha}$ and $C_{ij}$
- Gets rid of statistical outliers
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Constraints vs $N_{\text{bins}}$ used

$\Omega_m$, $w$, $\sigma_8$
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• MF not equivalent to moments for smoothing scales of $\sim 1'$ (series doesn’t converge)
• For $\theta_G = 1'$ the MF give a factor of $1.5 \div 2$ better constraints than moments alone
• Most of the information that moments carry is stored in low order moments of gradients

Future prospects
• Study the accuracy of MFs covariance matrices
• Impact of systematic errors, currently under investigations
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Thank you for your attention!