Geometry as a Semiclassical Effect in a Quantum World - Emergent gravity from matrix models

Talk presented by Daniel N. Blaschke

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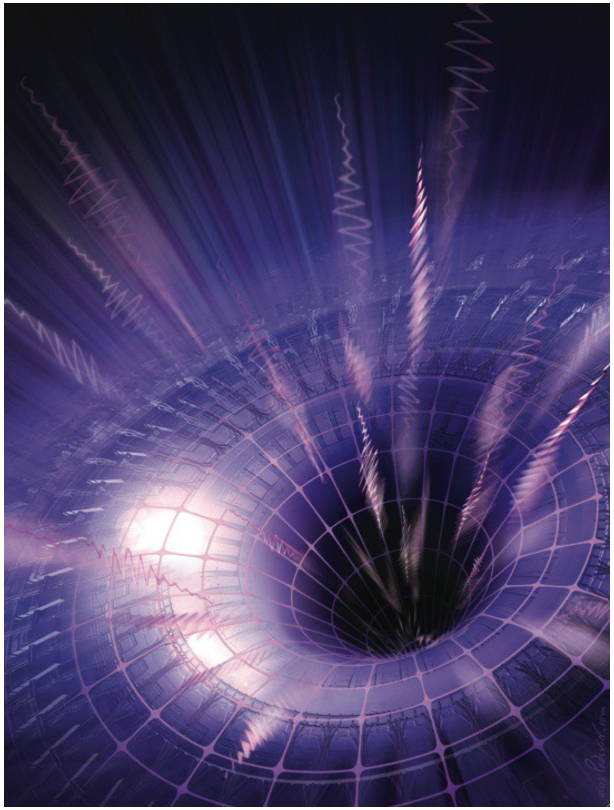
Motivation

- incompatibility between GR and QFT

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \langle T_{\mu\nu} \rangle$$

lhs: classical Einstein tensor, rhs: ev of an operator



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http://web.physics.ucsb.edu/~giddings/sbgw/physics.html

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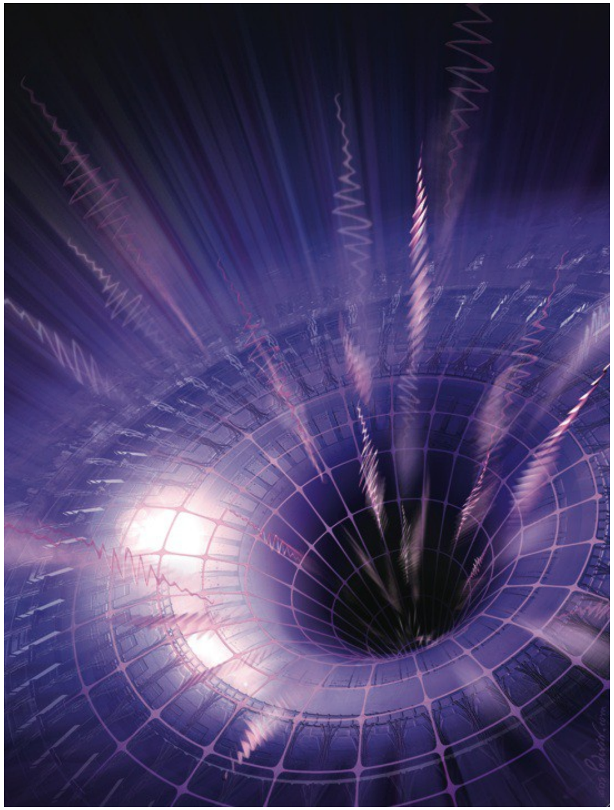
- natural limit in experimental length resolution: better length resolution requires higher energy, energy required for resolution of the Planck length has a Schwarzschild radius of the Planck length

$$\Delta x_\mu \simeq \lambda_p = \sqrt{\frac{G\hbar}{c^3}} \simeq 10^{-33}\mathrm{cm}$$



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What is a "non-commutative" space?

Geometric space	⟷	Commutative C*-algebra (according to Gel'fand-Naimark theorem)
Non-commutative space	⟷	Non-commutative C*-algebra (A. Connes 1990s)

Commutator of the coordinates has the general form:

$$[\hat{x}^i, \hat{x}^j] := \hat{x}^i \hat{x}^j - \hat{x}^j \hat{x}^i = i\theta^{ij}(\hat{x}) \,,$$

$$\theta^{ij}(x) = \text{const}, \quad \theta^{ij}(x) = \lambda^{ij}_k \hat{x}^k, \,\ldots$$

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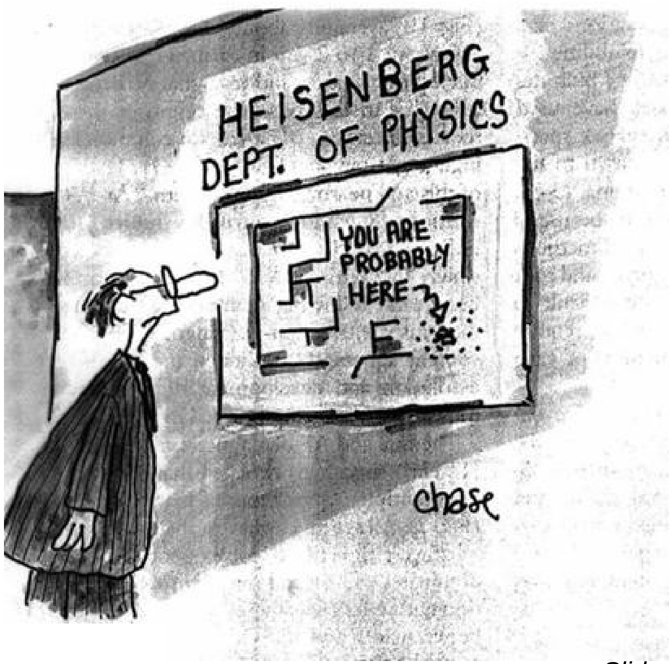

e.g. constant case leads to uncertainty relation

$$\Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\theta^{\mu\nu}| \sim (\lambda_p)^2$$

QFT on deformed space-time

Isomorphism mapping between NC algebra and commutative one, e.g. Weyl map:

$$\mathcal{W} : \mathcal{A} \to \widehat{\mathcal{A}}, \qquad x^i \mapsto \hat{x}^i \,, \qquad \hat{\mathcal{W}}[f]\hat{\mathcal{W}}[g] = \hat{\mathcal{W}}[f \star g]$$

For a field theory in Euclidean space this means:

interaction vertices gain phases, whereas propagators remain unchanged, e.g.:

$$S = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \star \partial^\mu \phi + \frac{m^2}{2} \phi \star \phi + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right)$$

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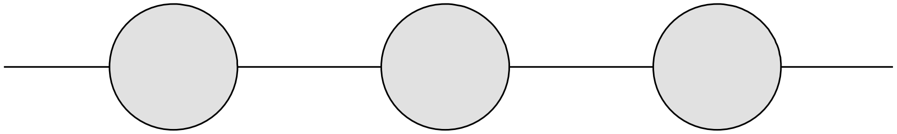
and some Feynman integrals ("non-planar diagrams") have phases which act as UV-regulators

$$\frac{1}{4} \int d^4k \frac{e^{ik\theta p}}{k^2 + m^2} \approx \frac{1}{(\theta p)^2} + c.m^2 \ln(\theta p)^2$$

➡ origin of the UV/IR mixing problem



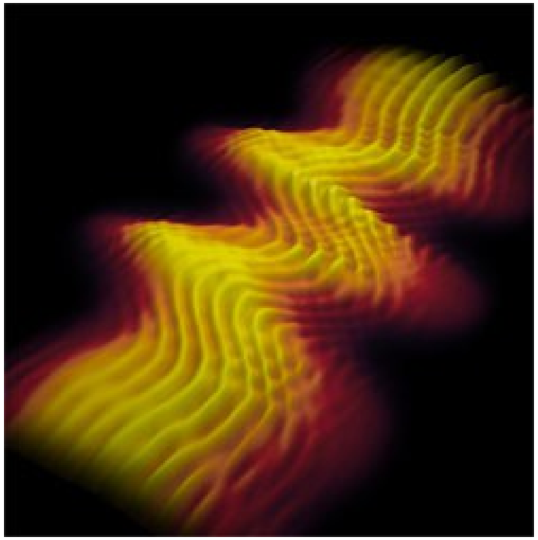
(solved by adding $\tilde{x}^2 \phi^2$ to action)

Gauge fields on theta-deformed spaces

Non-commutative Yang-Mills action:

$$S = \frac{1}{4} \int d^D x \, \mathrm{tr}_N \left(F_{\mu\nu}(x) \star F^{\mu\nu}(x) \right) \; ,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \underline{- ig[A_\mu \stackrel{\star}{,} A_\nu]}$$



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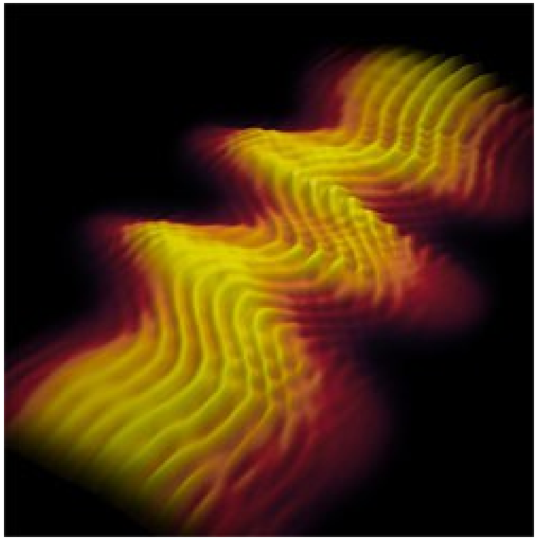

It is invariant under the infinitesimal gauge transformations

$$\delta_\alpha A_\mu(x) = \partial_\mu A(x) - ig[A_\mu(x) \overset{\star}{,} \alpha(x)] \, ,$$

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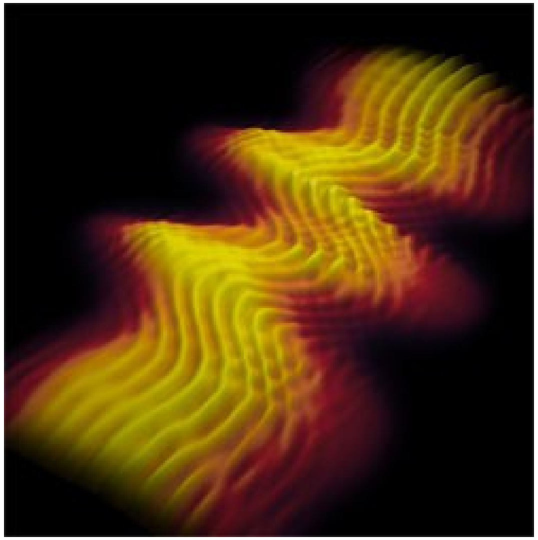

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IR divergent terms: $\Pi^{\mathrm{IR}}_{\mu\nu}(p) \propto \frac{\tilde{p}_\mu \tilde{p}_\nu}{(\tilde{p}^2)^2} ,$ $\qquad \tilde{p}^\mu := \theta^{\mu\nu} p_\nu ,$

Covariant coordinates & matrix models

define "covariant" coordinates: $\widetilde{X}_\mu := \tilde{x}_\mu + gA_\mu, \qquad \tilde{x}_\mu := \theta^{-1}_{\mu\nu} x^\nu$

$$\widetilde{X}_\mu \to u(x) \star \widetilde{X}_\mu \star u(x)^\dagger\,,$$

$$\Rightarrow \quad i\left[\tilde{X}_\mu \stackrel{\star}{,} \tilde{X}_\nu\right] = \theta^{-1}_{\mu\nu} - gF_{\mu\nu}$$

NC Yang-Mills action: $\int d^4x F_{\mu\nu} \star F^{\mu\nu}$

$$\to \quad -\frac{1}{g^2} \int d^4x \left[\tilde{X}_\mu \stackrel{\star}{,} \tilde{X}_\nu\right] \star \left[\tilde{X}^\mu \stackrel{\star}{,} \tilde{X}^\nu\right]$$

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Yang-Mills matrix model: $S_{YM} = -\mathrm{Tr}[X^a, X^b][X^c, X^d]\eta_{ac}\eta_{bd}$

Emergent gravity from matrix models

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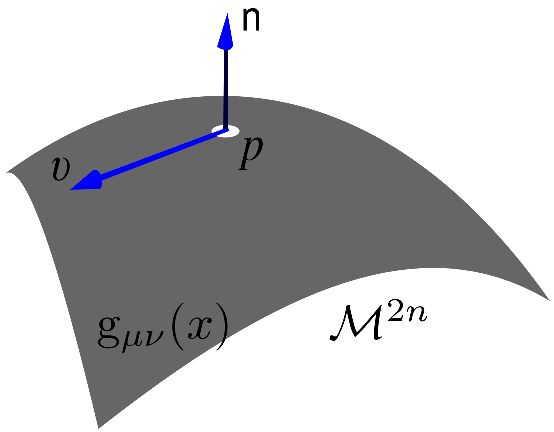
$X^a = \left(X^\mu, \Phi^i\right), \ \mu = 1, \ldots, 2n, \ i = 1, \ldots, D - 2n,$
so that $\Phi^i(X) \sim \phi^i(x)$ define embedding $\mathcal{M}^{2n} \hookrightarrow \mathbf{R}^D$
$g_{\mu\nu}(x) = \partial_\mu x^a \partial_\nu x^b \eta_{ab}$ (in semi-classical limit)



$$G^{\mu\nu} = e^{-\sigma}\theta^{\mu\rho}\theta^{\nu\sigma}g_{\rho\sigma}, \qquad e^{-\sigma} \equiv \frac{\sqrt{\det \theta^{-1}_{\mu\nu}}}{\sqrt{\det G_{\rho\sigma}}}$$

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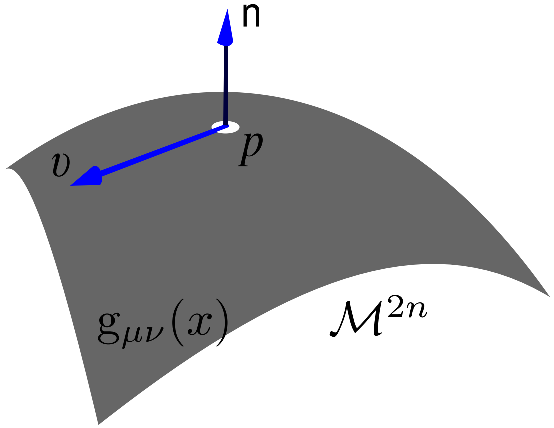

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$$S[\phi] = -\mathrm{Tr}\left[X^a, \Phi\right]\left[X^c, \Phi\right]\eta_{ac} \sim \int d^4x \sqrt{\det \theta^{-1}_{\mu\nu}}\ \{x^a, \varphi\}_{\mathrm{PB}}\{x^c, \varphi\}_{\mathrm{PB}}\,\eta_{ac}$$
$$= \int d^4x \sqrt{\det \theta^{-1}_{\mu\nu}}\ \theta^{\mu\nu}\partial_\mu x^a \partial_\nu\varphi\, \theta^{\rho\sigma}\partial_\rho x^c \partial_\sigma\varphi\, \eta_{ac}$$
$$= \int d^4x \sqrt{\det G_{\mu\nu}}\, G^{\nu\sigma}\partial_\nu\varphi\partial_\sigma\varphi$$

Introducing the IKKT model

$$S_{\rm IKKT} = {\rm Tr}\left(\left[X^a, X^b\right]\left[X_a, X_b\right] + \bar{\Psi} \not{D} \Psi\right) \ ,$$

$$\not{D}\Psi := \gamma_a \left[X^a, \Psi\right] \ , \qquad \{\gamma_a, \gamma_b\} = 2\eta_{ab}$$

IKKT matrix model is supersymmetric and expected to be renormalizable

- cf. *Nucl.Phys.* **B498** (1997) 467.

Majorana-Weyl spinor $\Psi = \mathcal{C}\bar{\Psi}^T$, is invariant under SUSY:

$$\delta^1\Psi = \frac{i}{4}[X^a, X^b][\gamma_a, \gamma_b]\epsilon \, , \qquad \delta^1 X^a = i\bar{\epsilon}\gamma^a\Psi$$

$$\delta^2\Psi = \xi \, , \qquad \delta^2 X^a = 0$$

Introducing the IKKT model

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Further symmetries:

$$X^a \to U^{-1}X^aU\,, \qquad \Psi \to U^{-1}\Psi U\,, \qquad U \in U(\mathcal{H})\,, \quad \text{gauge inv.}$$

$$X^a \to \Lambda(g)^a_b X^b\,, \qquad \Psi_\alpha \to \tilde{\pi}(g)^\beta_\alpha \Psi_\beta\,, \qquad g \in \widetilde{SO}(D)\,, \quad \text{rotations,}$$

$$X^a \to X^a + c^a\mathbb{1}\,, \qquad c^a \in \mathbb{R}\,, \quad \text{translations}$$

The IKKT model

$$S_{\mathrm{IKKT}} = \mathrm{Tr}\left(\left[X^a, X^b\right]\left[X_a, X_b\right] + \bar{\Psi}\gamma_a\left[X^a, \Psi\right]\right)$$

- Originally proposed as non-perturbative definition of type IIB string theory,

- Seems to provide a good candidate for quantum gravity and other fundamental interactions, i.e.

$$X^a = \begin{pmatrix} \bar{X}^\mu - \theta^{\mu\nu} A_\nu(\bar{X}^\mu) \\ \Phi^i(\bar{X}^\mu) \end{pmatrix}$$

cf. e.g. *JHEP* **09** (2011) 115, *Prog. Theor. Phys.* **125** (2011) 521

- Assume soft breaking of SUSY below some scale Λ, e.g. by compactified brane solutions of type

$$\mathcal{M} = \mathcal{M}^4 \times \mathcal{K}$$

Conclusion and Outlook

- ✔ Introduced and motivated the concept of NCQFTs

- ✔ Introduced matrix models, especially the IKKT model and its properties, such as emergent gravity

- ✔ Commented on SUSY-breaking and model building

- ✔ Many interesting open questions

References

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Thank you for your attention!