Cosmology with Minkowski Functionals and Moments of Weak Lensing Fields

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27th Texas Symposium of Relativistic Astrophysics

arXiv 1309.4460, PRD accepted

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Outline

Cosmological information in non weak lensing maps

- CMB temperature → gaussian → two point function $\xi(\boldsymbol{x}_1, \boldsymbol{x}_2) = \langle \delta T(\boldsymbol{x}_1) \delta T(\boldsymbol{x}_2) \rangle$
- Fourier equivalent: Power spectrum $\langle \delta \hat{T}(\mathbf{k}) \delta \hat{T}(\mathbf{k}') \rangle = (2\pi)^3 P(k) \delta(\mathbf{k} + \mathbf{k}')$

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Cosmological information in non weak lensing maps

- $\frac{d\theta'_i}{d\theta_j} = (1-\kappa)\delta_{ij} - \gamma_1 \boldsymbol{\sigma}^3_{ij} - \gamma_2 \boldsymbol{\sigma}^1_{ij}$
- Convergence κ (galaxy magnification due to lensing) $\rightarrow$ highly non gaussian $\rightarrow$ two point statistics $\xi(\boldsymbol{x}_1, \boldsymbol{x}_2) = \langle \kappa(\boldsymbol{x}_1)\kappa(\boldsymbol{x}_2)\rangle +$??

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Our framework

- 1000 realizations of simulated κ maps for a fiducial ΛCDM model with $(\Omega_m, w, \sigma_8) = (0.26, -1.0, 0.798)$
- 1000 realizations for each parameter variation
- Single redshift plane at $z_s = 2$
- Galaxy shape noise added assuming $n_{gal} = 15 \mathrm{arcmin}^{-2}$
- Gaussian smoothing with a variable window size $\theta_G = 1 \div 15\,\mathrm{arcmin}$

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Topological descriptors: beyond gaussian statistics

Consider the excursion sets $\Sigma(\nu) = \{\kappa > \nu\sigma_0\}$

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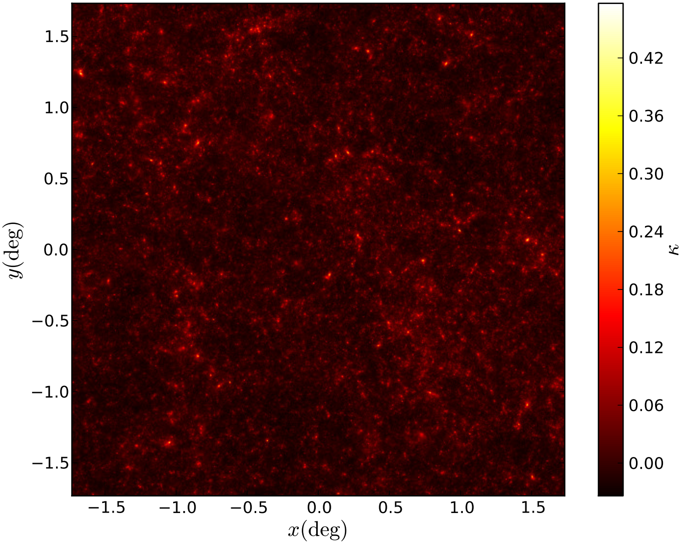

Figure : Full simulated convergence map

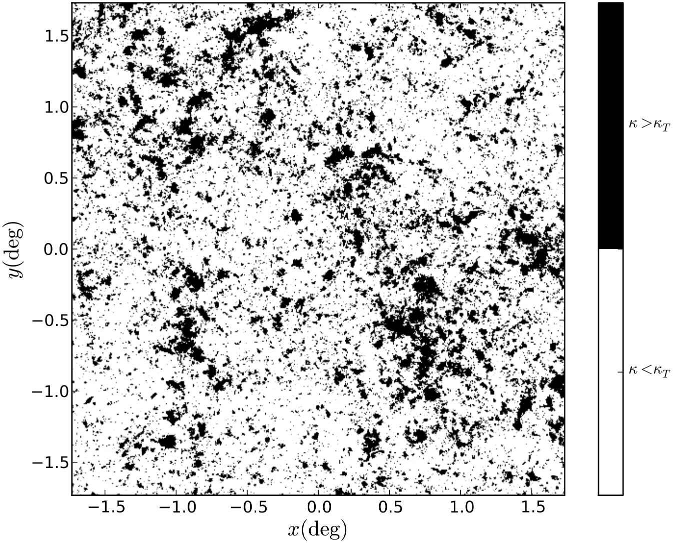

Figure : Excursion set with threshold $\nu = \kappa_T/\sigma_0 = 0.01/\sigma_0$

Minkowski Functionals are...

- $V_0(\nu)$: area of the black regions
- $V_1(\nu)$: length of the boundaries of the black regions
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Analytical study of Minkowski Functionals

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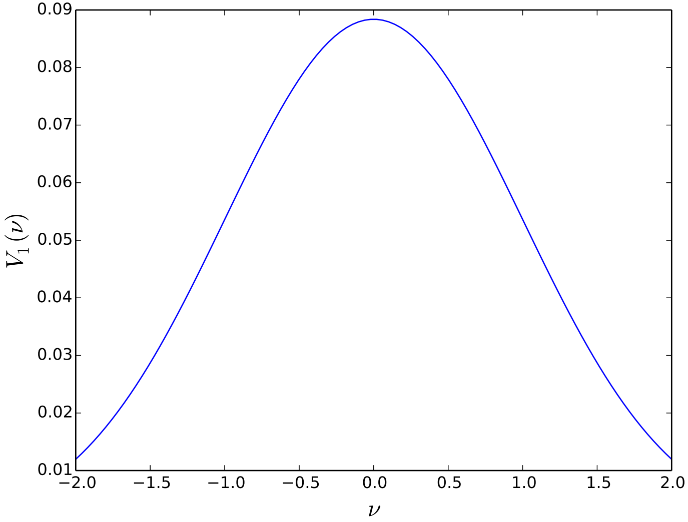
$$V_1(\nu) = \frac{1}{8\sqrt{2}}\frac{\sigma_1}{\sigma_0}\exp\left(-\frac{\nu^2}{2}\right)$$



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The real κ field is non gaussian...

- MFs $\leftrightarrow$ moments $\langle\kappa^n|\nabla\kappa|^{2m}\rangle$ via series expansion
- $N-\mathrm{th}$ order term is proportional $\sigma_0^N H_{k(N)}(\nu)e^{-\nu^2/2}$
- The proportionality coefficient is a real space moment of order $2+N$
- We call *one point moments* the ones with $m=0$, and *moments with gradients* the ones with $m\neq 0$

Does the series converge?

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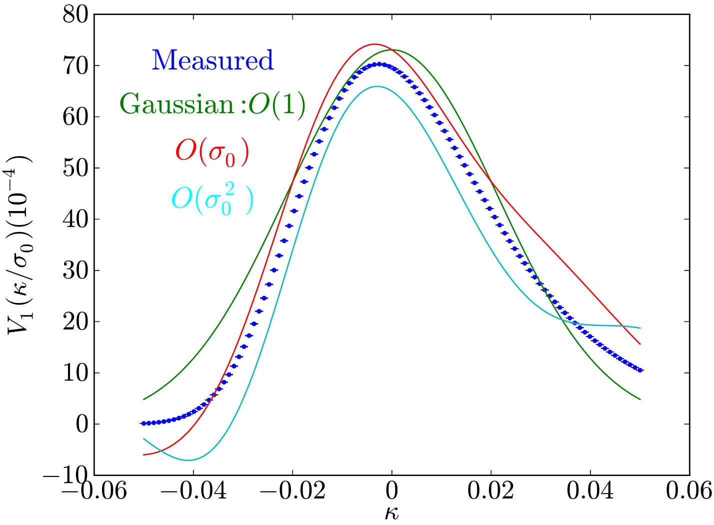
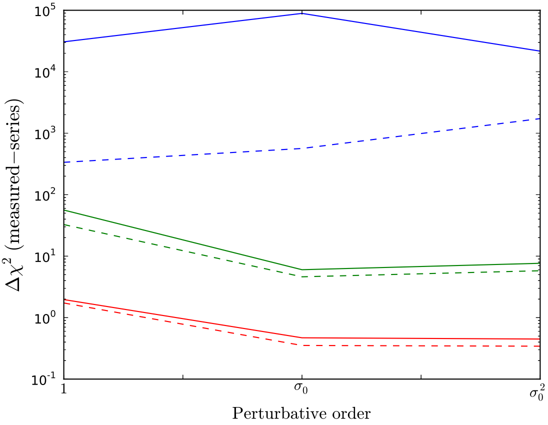


$$\Delta\chi^2 = (V_{pert} - V_{meas})_i (C_V^{-1})_{ij} (V_{pert} - V_{meas})_j$$



- $\Delta\chi^2(\theta_G = 1') \approx 2000 \rightarrow$ doesn't converge!
- $\Delta\chi^2(\theta_G = 15') \approx 0.1 \rightarrow$ converges!

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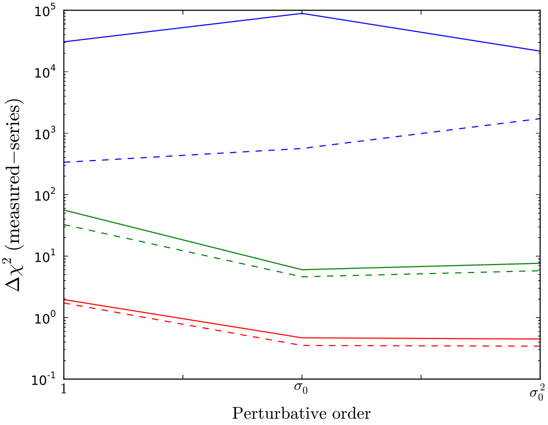

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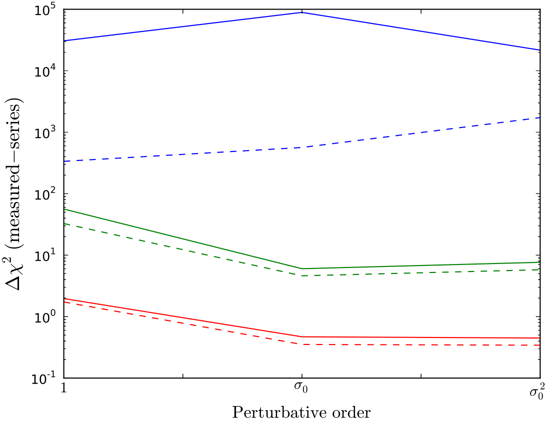

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Downside: distinguishing power

- $\Delta\chi^2_{cosmo}(\theta_G = 1') \approx 5 \rightarrow$ can distinguish!
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Fisher Constraints on Cosmology

- Power spectrum observable probes:

$$\mathbf{C} = \overbrace{(C_1, C_2, ...)}^{l_{max}\text{ sized vector}}$$

- We use instead:

$$\mathbf{D} = \overbrace{(\underbrace{V_0^{\nu_1}, V_0^{\nu_2}, ...,}_{N_{bins}} \underbrace{V_1^{\nu_1}, V_1^{\nu_2}, ...,}_{N_{bins}} \underbrace{V_2^{\nu_1}, V_2^{\nu_2}, ...,}_{N_{bins}} \underbrace{\langle\kappa^2\rangle, \langle\kappa^3\rangle, \langle\kappa^4\rangle}_{\text{9 Moments}})}^{3N_{bins}+9\text{ sized vector}}$$

- Measure the covariances $C_{ij} = \langle D_i D_j \rangle$ and compute the parameters ($p_\alpha = (\Omega_m, w, \sigma_8)$) Fisher matrix

$$F_{\alpha\beta} = \frac{\partial D_i}{\partial p_\alpha} C_{ij}^{-1} \frac{\partial D_j}{\partial p_\beta}$$

- Marginalized errors on the parameters

$$\Delta p_\alpha = \sqrt{(F^{-1})_{\alpha\alpha}}$$

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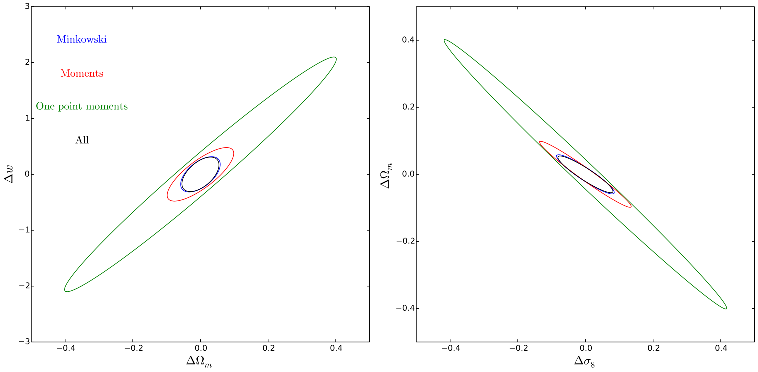
Minkowski
Moments
One point moments
All
Δw
$\Delta\Omega_m$
3
2
1
0
−1
−2
−3
−0.4
−0.2
0.0
0.2
0.4
$\Delta\Omega_m$
$\Delta\sigma_8$
0.4
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Robustness checks

- We are trying to estimate a $\sim 3N_{bins} \times 3N_{bins} \sim 300 \times 300$ covariance matrix using 1000 realizations...
- Accuracy is not guaranteed
- Use of modified Fisher matrix formalism: use of auxiliary, independent, map set to measure $\partial D_i/\partial p_\alpha$ and C_{ij}
- Gets rid of statistical outliers

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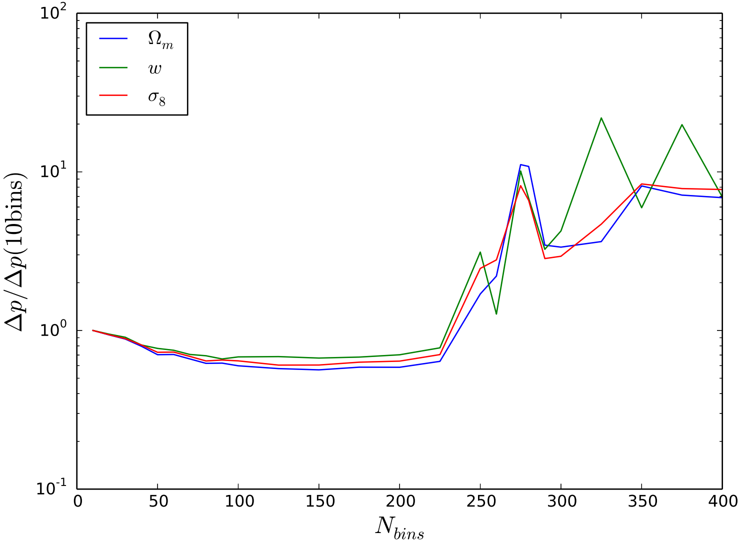
Constraints vs N_{bins} used



- MF perturbation series seems to converge for smoothing scales $\theta_G \sim 15'$ (but distinguishing power is lost)
- MF not equivalent to moments for smoothing scales of $\sim 1'$ (series doesn't converge)
- For $\theta_G = 1'$ the MF give a factor of $1.5 \div 2$ better constraints than moments alone
- Most of the information that moments carry is stored in low order *moments of gradients*

Future prospects

- Study the accuracy of MFs covariance matrices
- Impact of systematic errors, currently under investigations

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