Charged particle motion near magnetized black holes

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Based on:

V.F. & A.Shoom, Phys.Rev.D82: 084034 (2010);
V.F., Phys.Rev. D85: 024020 (2012);
A.M. Al Zahrani, V.F. & A.Shoom, D87: 084043 (2013)

27th Texas Symposium, December 12, 2013

MOTIVATIONS

There are indications that magnetic field plays an important role in astrophysical black holes

We study a simple (toy) model: a charged particle motion near a magnetized black hole

We demonstrate that even a weak regular magnetic field in the vicinity of a black hole dramatically changes such characteristics as ISCO radius and period of motion.

Effect of magnetic field on charged particles

$$\Omega_c / \Omega_K \sim \frac{\text{Cyclotron frequency}}{\text{Keplerian frequency at ISCO}}$$

$$\sim b = \frac{qBMG}{mc^4}$$

<u>For b>1 magnetic field essentially modifies motion of a charged particle</u>

For a proton $b = 1$ for:

$$M = 10 M_{\odot} \text{ if } B = 2G$$

$$M = 10^{9} M_{\odot} \text{ if } B = 2 \times 10^{-8} G$$

$$B \ll \frac{c^{4}}{G^{3/2} M_{\odot}} \left(\frac{M_{\odot}}{M} \right) \times 10^{19} (M_{\odot}/M) \mathrm{G}$$

In BZ mechanism:

$\mathrm{B} \sim 10^{4} G$ is required to produce power $\sim 10^{45} erg/\sec$ seen in jets from supermassive ($M \sim 10^{9} M_{\odot}$) BH;

$\mathrm{B} \sim 10^{15} G$ is required to produce power $\sim 4 \times 10^{52} erg/\sec$ seen in GRB (for BH with $M \sim 10 M_{\odot}$).

The problem set up

- We consider a Schwarzschild black hole immersed into a static and axisymmetric magnetic field which approaches a constant value far away of the black hole.

- The magnetic field is regular between the black hole horizon and accretion disk.

- We study motion of charged particles in the presence of the magnetic and the black hole gravitational fields, neglecting their mutual interaction.

- In the first part: We study motion in the equatorial plane of the black hole, which is orthogonal to the direction of the magnetic field.

Magnetized black hole

The Schwarzschild space-time:

$$ds^2 = -\left(1 - \frac{r_g}{r}\right) dt^2 + \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

Killing vectors: $\xi_{(\phi)} = \frac{\partial}{\partial \phi}, \qquad \xi^{a;b}{}_{;b} = 0, \qquad (R_{ab} = 0)$

The electromagnetic 4-potential: $A^a = \frac{1}{2} B \xi^a_{(\phi)}$

Static, axisymmetric, uniform at infinity magnetic field:

$$B^a \partial_a = B\left(1 - \frac{r_g}{r}\right)^{1/2} \left[\cos\theta \frac{\partial}{\partial r} - \frac{\sin\theta}{r} \frac{\partial}{\partial \theta}\right]$$

Dynamical equations

Motion of a charged particle:

$$m\frac{du^a}{d\tau} = qF^a{}_b u^b, \qquad u^a u_a = -1$$

Generalized momentum: $P_\mu = mu_\mu + qA_\mu$.

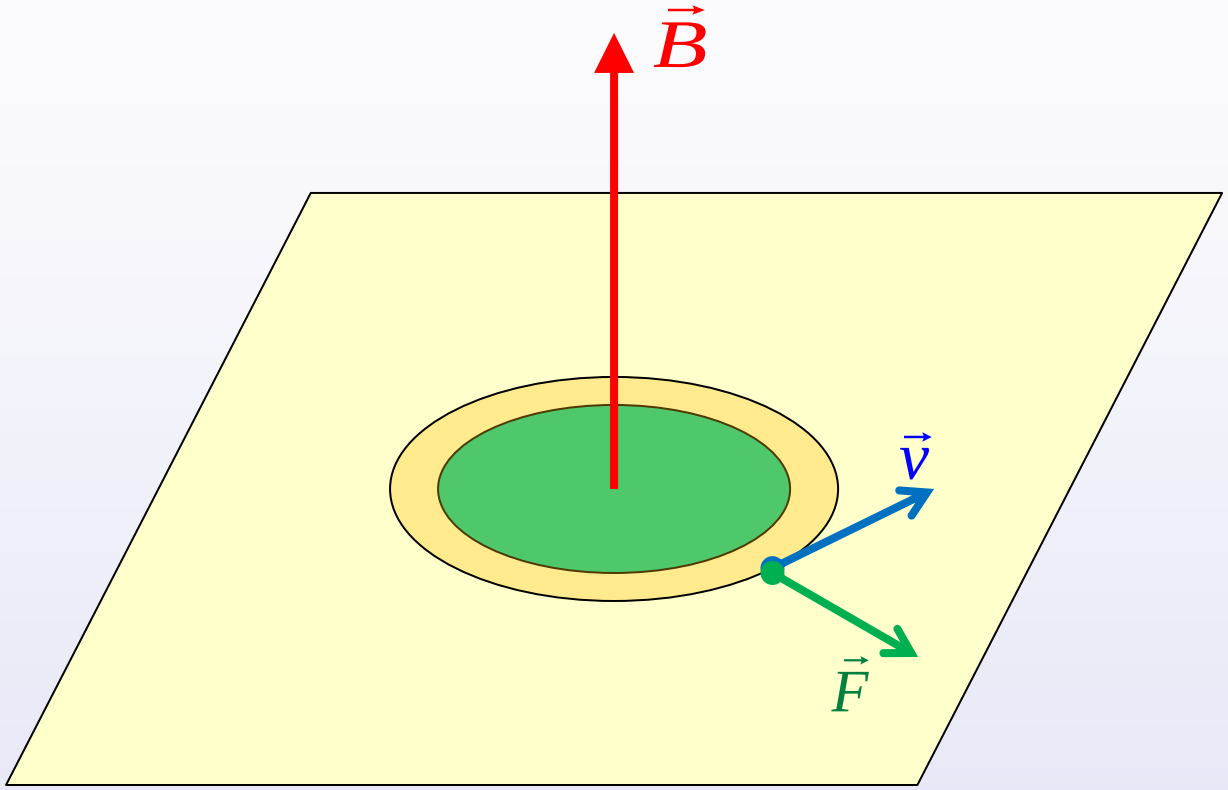
Integrals of motion:

$$E \equiv -\xi^a_{(t)} P_a = m\frac{dt}{d\tau}\left(1 - \frac{r_g}{r}\right),$$

$$L \equiv \xi^a_{(\phi)} P_a = m\frac{d\phi}{d\tau} r^2 \sin^2\theta + \frac{qB}{2} r^2 \sin^2\theta$$

Motion in Equatorial Plane

V.F. & A.Shoom, Phys.Rev.D82:084034 (2010)



$\dot{\varphi} > 0 \rightarrow l > 0$ $\vec{F} = q[\vec{v} \times \vec{B}]$ is repulsive force

Motion of a Charged Particle: Strong Field Case

Dimensionless quantities:

$$T = t / r_g, \quad \rho = r / r_g, \quad \sigma = \tau / r_g,$$

$$\ell = L / (m r_g), \quad b = q B r_g / (2m)$$

Dynamical equations:

$$\left(\frac{d\rho}{d\sigma}\right)^2 = \tilde{E}^2 - U, \qquad \frac{d\phi}{d\sigma} = \frac{\ell}{\rho^2} - b, \quad \frac{dT}{d\sigma} = \frac{\tilde{E}\rho}{\rho - 1}$$

Attractive Lorentz force: $\ell = -|\ell|$

Repulsive Lorentz force: $\ell = +|\ell|$

Effective potential

$$U = \left(1 - \frac{1}{\rho}\right)\left[1 + \frac{(\ell - b\rho^2)^2}{\rho^2}\right]$$

At horizon: $U|_{\rho \to 1} \to 0;$ At infinity: $U|_{\rho \to +\infty} \to b^2 \rho^2$

max = # min $\Rightarrow$ even number of extremal points in $\rho \in (1, \infty)$

Innermost stable circular orbits

$$U_{,\rho} = 0, \qquad U_{,\rho\rho} = 0$$

Lessons

(i) Radius of ISCO $\rho_+ |_{b>>1} \sim 1 + \frac{1}{b\sqrt{3}}$

(ii) Energy release $\Delta E_{ISCO+} \sim mc^2 (1 - \frac{2}{3^{3/4} b^{1/2}})$

(iii) Angular velocity $\Omega_+ |_{b>>1} \sim \frac{3^{3/4}}{6b^{1/2}}$

Magnetized BH as “Particle Accelerator”

V.F., Phys.Rev. D85: 024020 (2012);

Banados, Silk & West PRL 103, 111102 (2009): Center of mass energy for collision of 2 particles near the horizon of a rotating black hole can be arbitrary large for special (fine tuned) choice of their angular momenta and $\alpha \equiv a/M \to 1$. The effect is propotional to $(1-\alpha)^{-1/4}$.

Similar effect occurs in magnetised (even non-rotating) black holes. The effect is proportional to $b^{1/4}$.

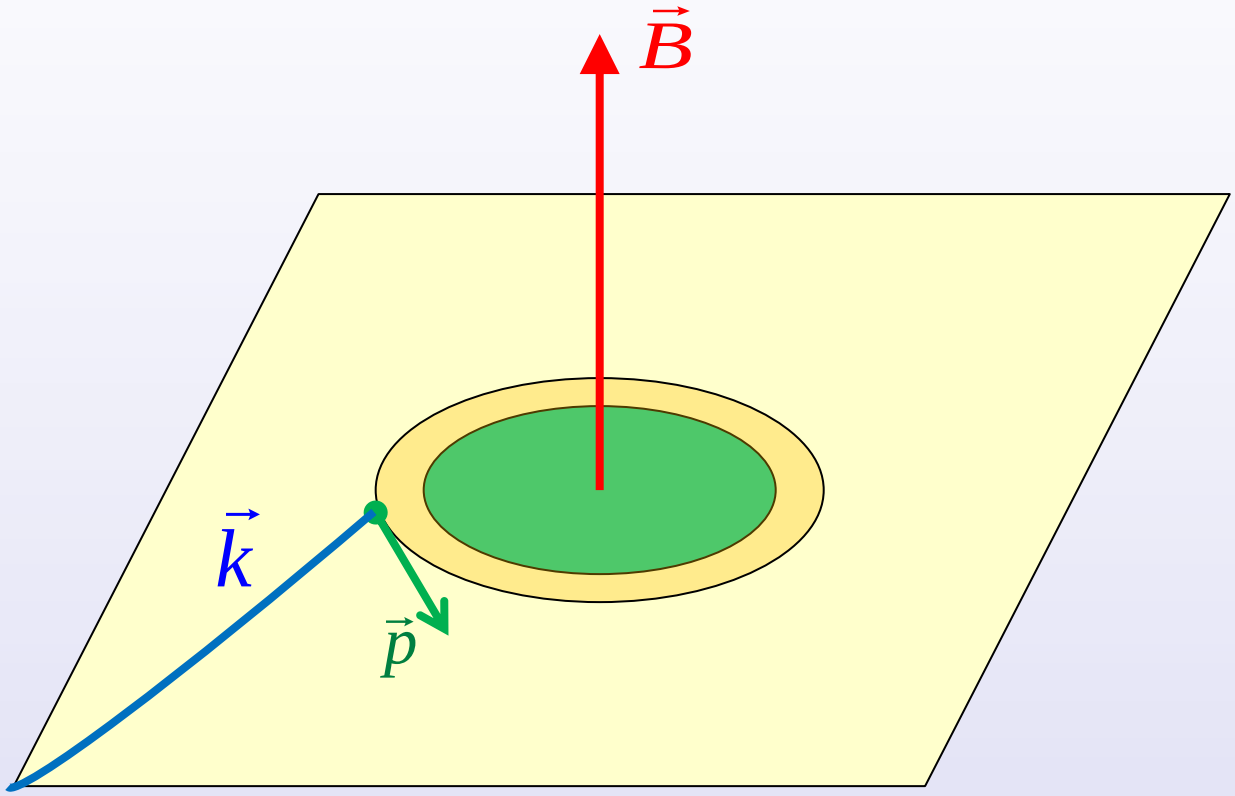
Consider collision of 2 particles in the vicinity of magnetized BH.

(1) First charged particle (with charge q and mass m) is at ISCO.

(2) Second (neutral) particle of mass μ is freely falling.

At the moment of collision the four momentum is $P^{\lambda} = p^{\lambda} + k^{\lambda}$,

and the center-of-mass energy M is: $M^2 = m^2 + \mu^2 - 2(p,k)$


$\vec{B}$
$\vec{k}$
$\vec{p}$

$$\mathrm{M} \sim \frac{(2\mathrm{m}\gamma\mathrm{E})^{1/2}}{(\rho_{\mathrm{ISCO}} - 1)^{1/4}} \quad \Rightarrow \quad M \sim 1.74\, b^{1/4}\sqrt{mE}$$

$$1 - a \Leftrightarrow b^{-1}$$

Generalization to magnetized rotating BHs:
Igata, Harada & Kimura, PRD 85, 104028 (2012)

Motion out of Equatorial Plane

A.M. Al Zahrani, V.F. & A.Shoom, D87: 084043 (2013)

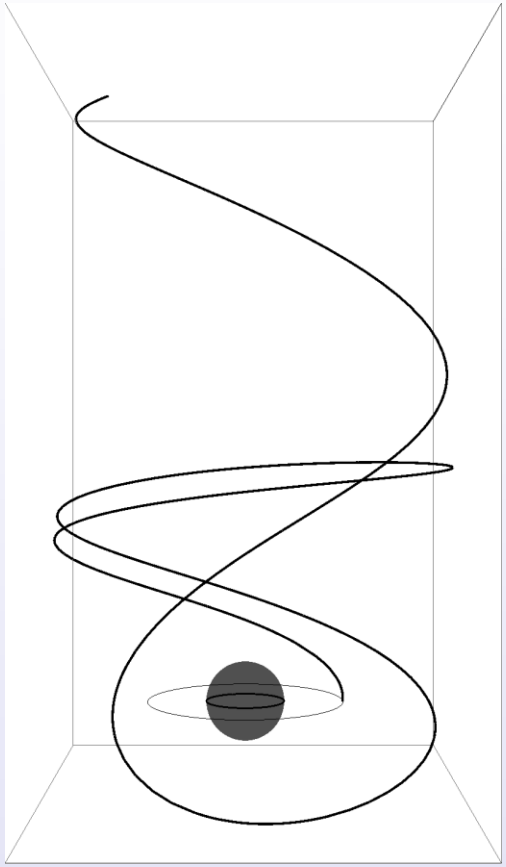
We consider again a charged particle (with charge q and mass m) revolving in the equatorial plane around a magnetized non-rotating black hole at the ISCO. We suppose now that it is `kicked' out of this orbit by some other particle or photon and gets an orthogonal to the plane velocity $v = -r\dot{\theta}$. What is the critical escape velocity v_* and what are properties of the near critical motion?

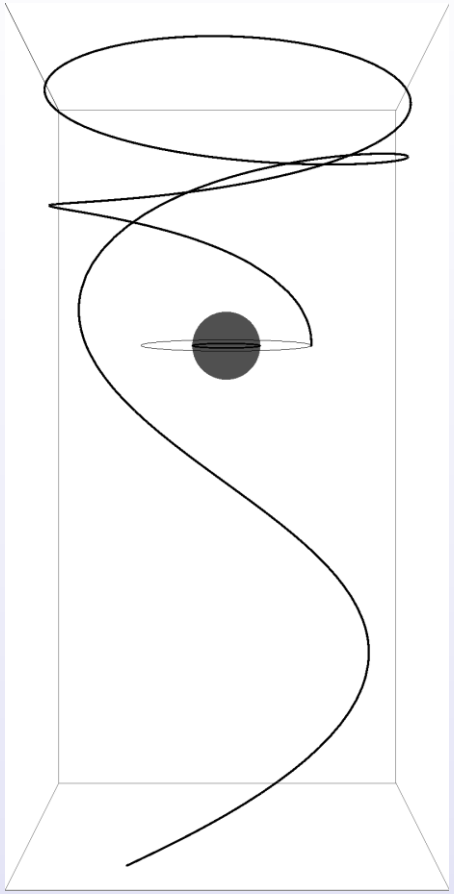
Three possible asymptotic types of motion:

(1) Capture (red);

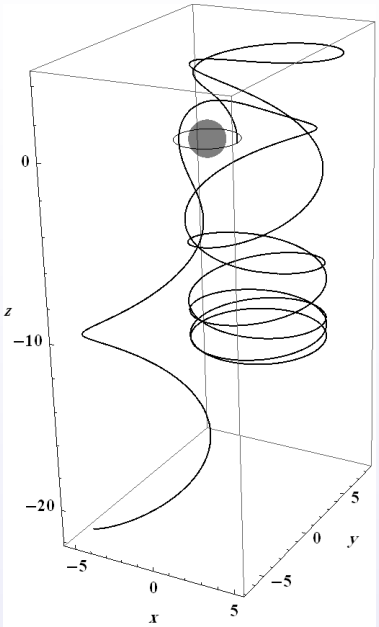
(2) Escape in the direction of **B** (green);

(3) Escape in the direction opposite to $-\mathbf{B}$ (yellow).





Examples of escape trajectories.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y
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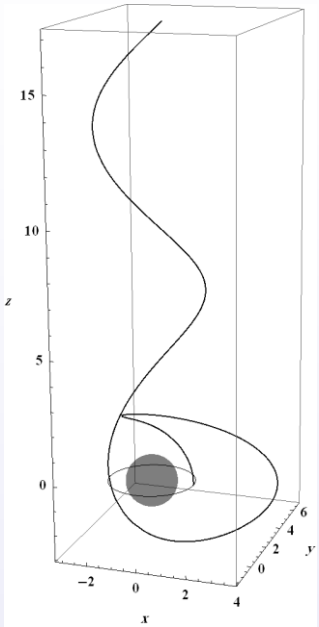
0
−5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y
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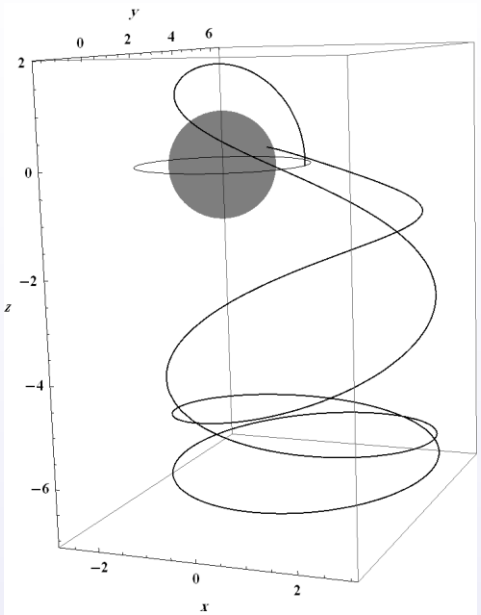
2
0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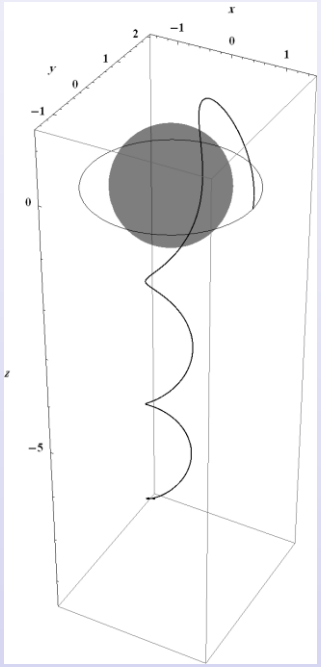
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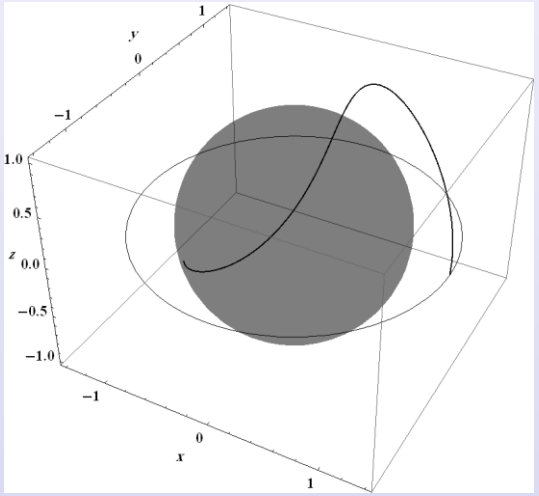
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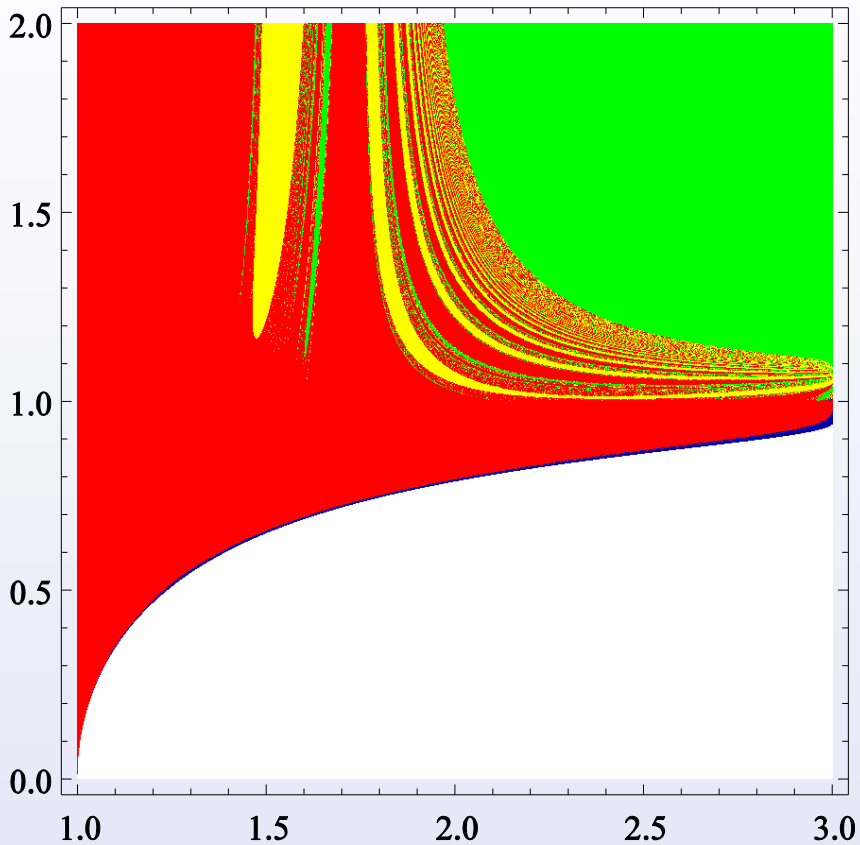
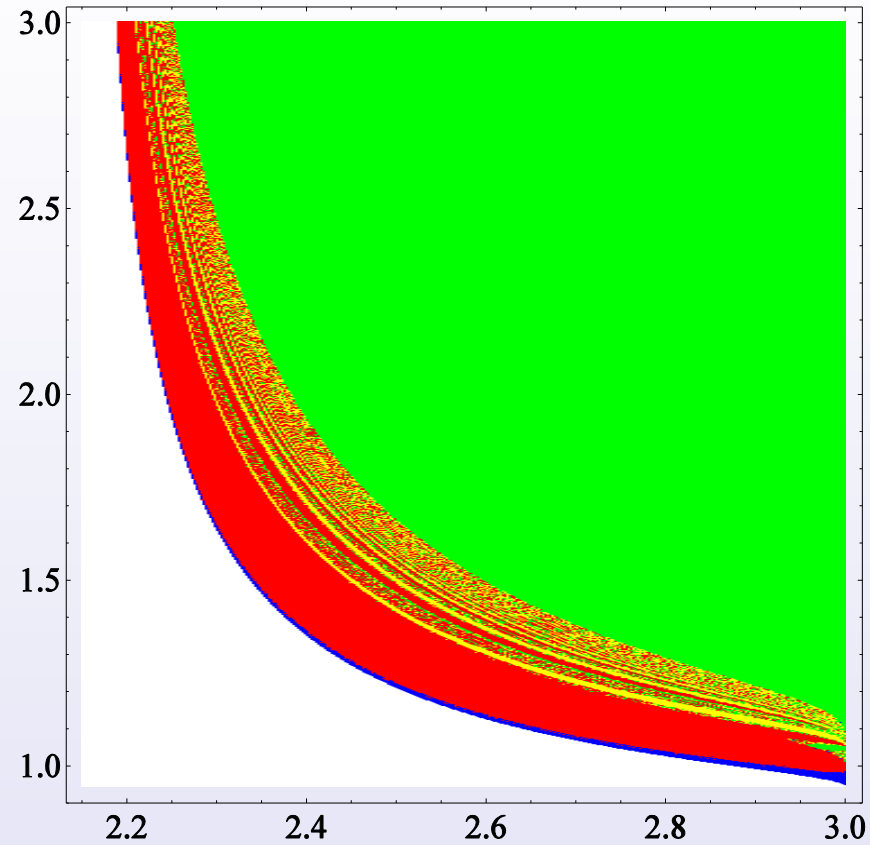
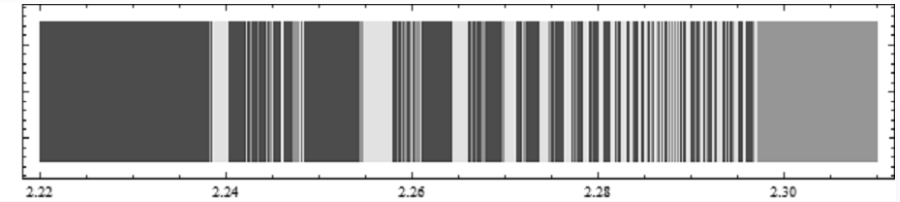
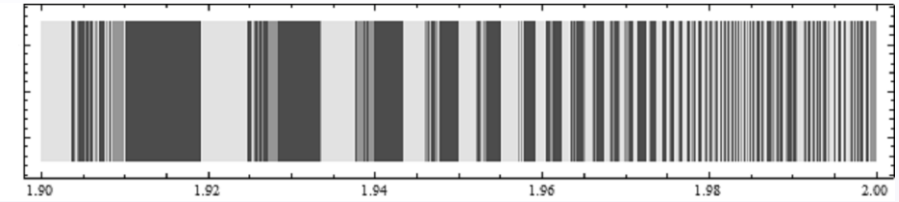




Basin of boundaries plots for a charged particle kicked from the last stable circular orbits at different radii $r_{ISCO}/2M$ defined by the magnetic field b (horizontal axis) with different kicking energies (vertical axis). Left plot for $l > 0$, right plot for $l < 0$.





Stripes from fractal regions in the vicinity of the critical escape.

Left plot for $E \approx 1.9$ for $l > 0$. Right plot for $E \approx 2.5$ for $l < 0$.

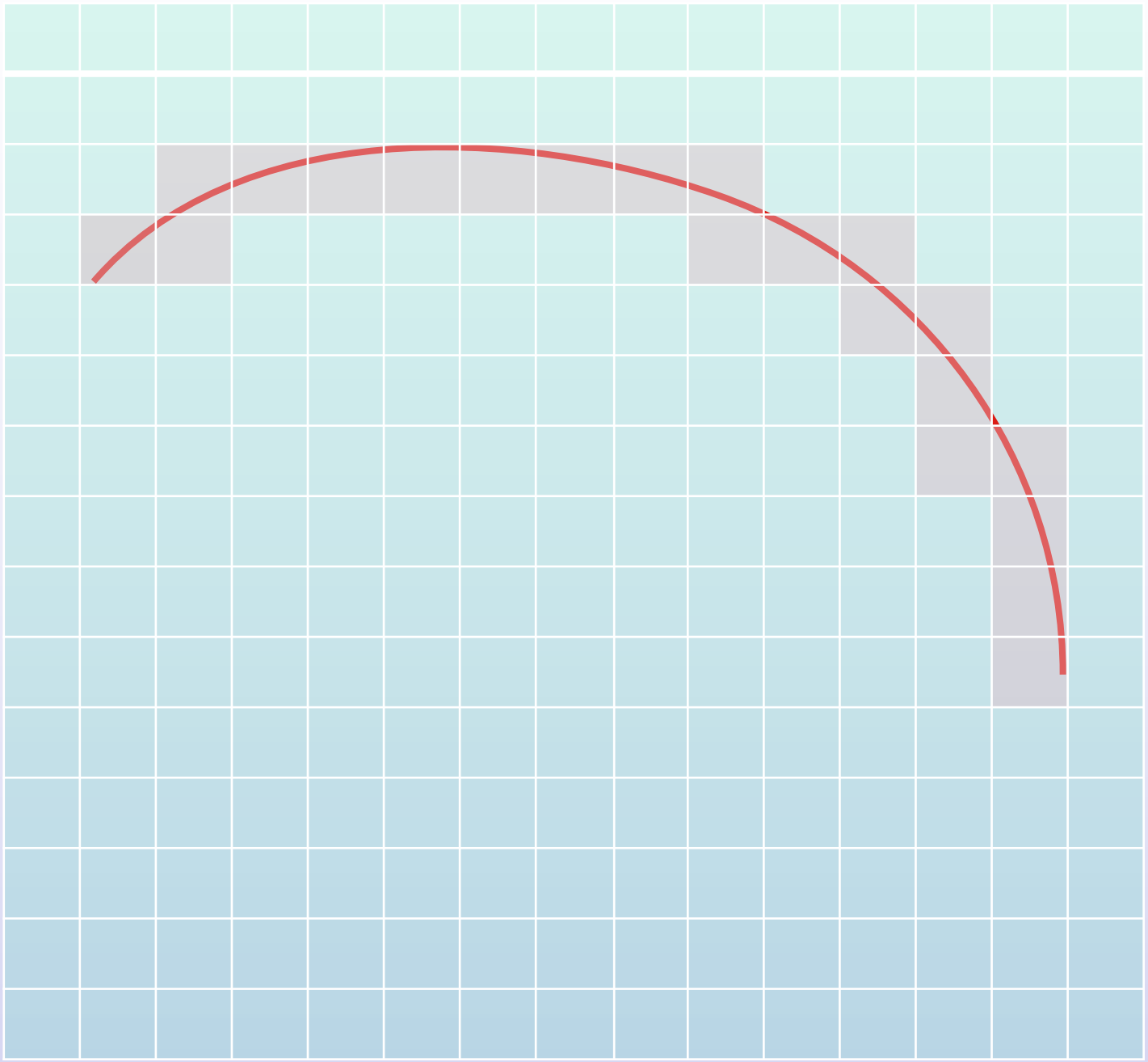
(dark--capture, grey--(+)escape, light grey--(-)escape).

(For discussion of basin of boundaries approach for scattering problems see, e.g. “Chaos in Dynamical Systems” by E.Ott)

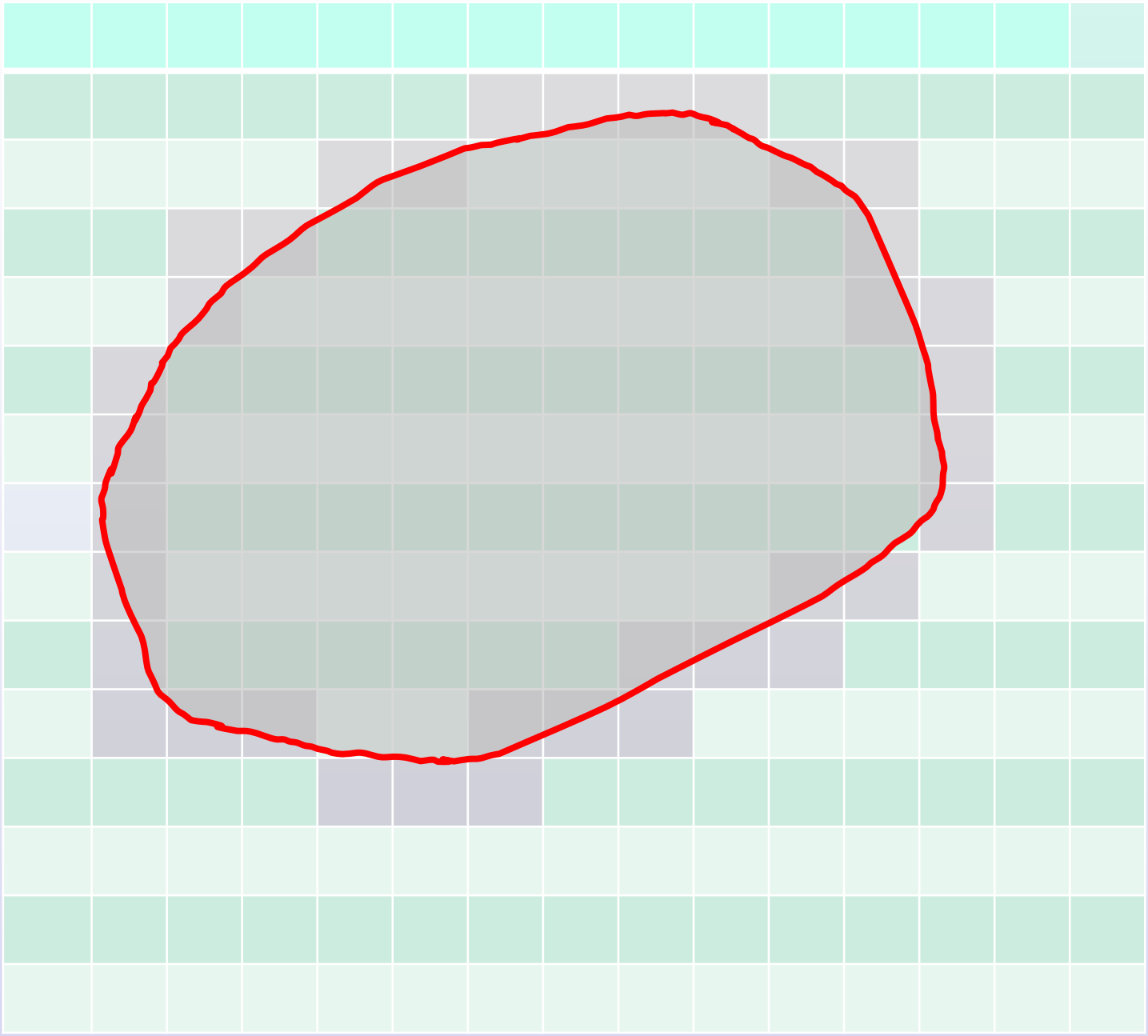
Denote by $N(\varepsilon)$ a number of square stripes of size ε, which is required to cover a basin boundary.

Each of the stripes must contain at least 2 different colors. The box-counting fractal dimension D_f is

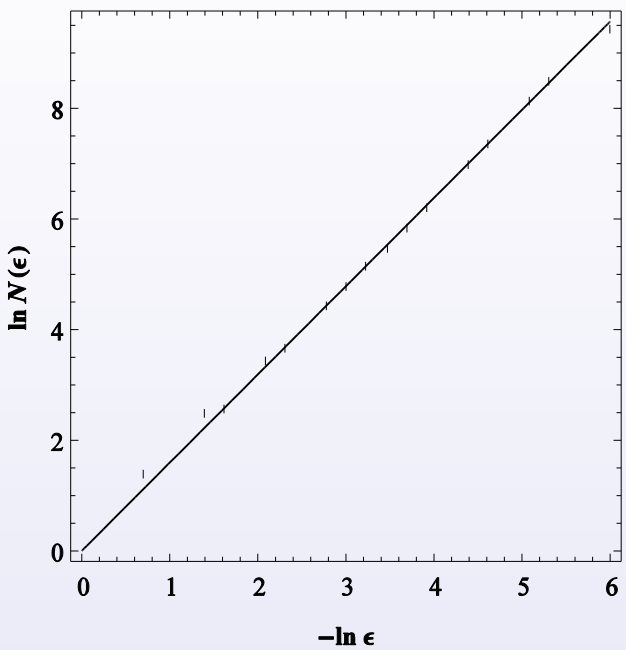
$$D_f = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln \varepsilon^{-1}}, \quad 1 \le D_f < 2$$

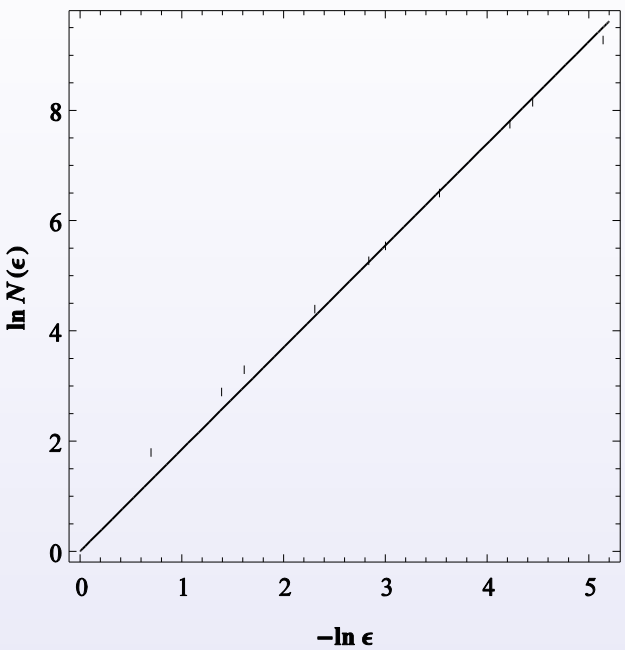


$$N \sim L/\varepsilon, \quad D_f = 1$$



$$N \sim S/\varepsilon^2, \quad D_f = 2$$





The box counting dimension. Plots of $\ln \varepsilon$ vs. $\ln(1/\varepsilon)$.

Left plot for $l > 0$, right one for $l < 0$.

$$D_f \approx 1.60, l > 0; \quad D_f \approx 1.85, l < 0$$

Main result: near-critical-escape motion is chaotic.

Summary

(1) Magnetic field effect on charged particle motion is strong when $b > 1$. This seams to be the case for realistic astrophysical black holes.

(2) Magnetic field makes position of the ISCO closer to the gravitational radius. Efficiency of energy release and period for ISCO particles strongly depend of the parameter b.

(3) Center-of-mass energy for collision of a free falling particle (photon) and a charged particle revolving near a magnetized black hole can be (at least formally) large ($\sim b^{1/4}$).

(4) Near-critical-escape motion out of the equatorial plane is chaotic and basin of boundaries plots have fractal structure.

(5) Possible applications: Broadening of Kα lines of Iron ions as probes of the magnetic field in the black hole infinity

(6) Rotating magnetized black holes

Charged particle: ISCO motion in magnetic field:

$$p^{\mu} = m\gamma(e^{\mu}_{(t)} + v e^{\mu}_{(\varphi)})$$

Freely falling particle: $k^{\mu} = (E/f, \dot{r}, \dot{\theta}, L_z / r^2)$

$$\rho_+ \big|_{b>>1} = 1 + \frac{1}{b\sqrt{3}} + O(b^{-2}),$$

$$\rho_- \big|_{b>>1} = \frac{5+\sqrt{13}}{4} + \frac{41-11\sqrt{13}}{36\sqrt{13}b^2} + O(b^{-4})$$

$$\Omega_+ \big|_{b>>1} = \frac{3^{3/4}}{6b^{1/2}} + O(b^{-3/2}),$$

$$\Omega_- \big|_{b>>1} = -0.34 + \frac{0.41}{b^2} + O(b^{-4})$$