Nuclear Matter Incompressibility and Giant Monopole Resonances

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Iron core in core-collapse supernovae

The iron core grows and becomes dynamically unstable as it approaches the critical mass. The main reactions are

$$^{56}Fe \rightarrow 13\,^{4}He + 4n$$ ← Needs energy (124 MeV)

$$p + e^{-} \rightarrow n + \nu_e$$ ← Neutrino escapes

↓

Pressure decreases and the core collapses

The Equation of State (EoS) has to cover densities of $10^9 - 10^{15}$ g/cm^3

Nuclei are bound into nuclei until they merge ($\rho_0 \sim 10^{14}$ g/cm^3)

Pressure is essentially dominated by e^- ($\gamma = \partial \ln P / \partial \ln \rho = 4/3$)

For $\rho > \rho_0$, nuclear matter becomes very hard ($\gamma \sim 2$)

EOS

Below nuclear density:

- Matter is described by nuclei surrounded by a "gas" of nucleons and alpha particles

- Electrons are uniformly distributed in space, inside and outside nuclei.
- In equilibrium

$$\mu_e = \mu_n - \mu_p$$

- At high densities nuclear matter fills the space uniformly
- Techniques for computing EOS: **liquid drop**, **Thomas-Fermi**, **Hartree-Fock**
- EOS is obtained by minimizing the free energy of the system for each entropy and Y_e

EOS

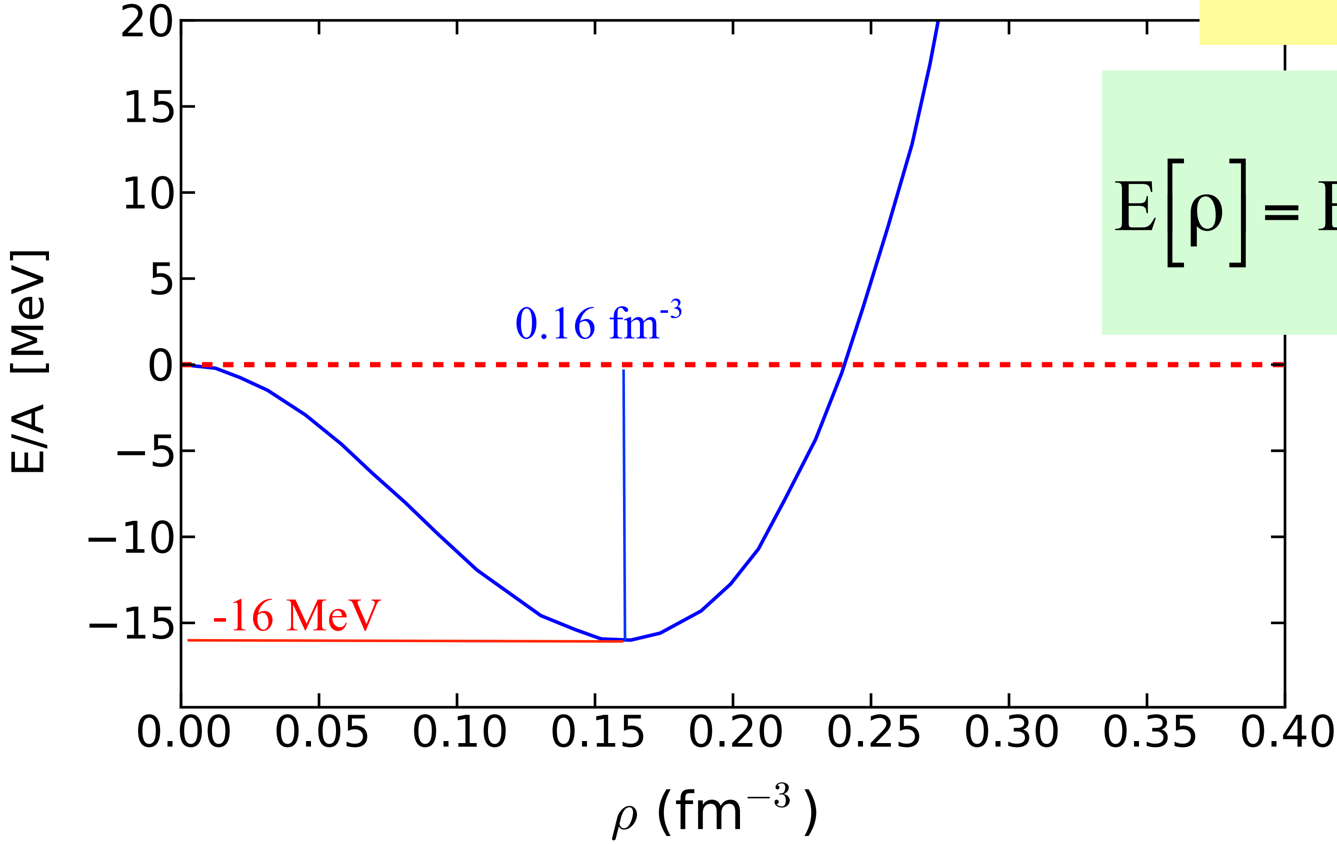
Above nuclear density:

- Maximum density occurs when the inner central core reaches $\rho \sim 5\rho_0$

- the compressibility has to be enough to allow the existence of neutron stars with $M \sim 1.4\ M_\odot$ or larger.

$$K_\infty = 9\rho^2 \left.\frac{d^2\left[E(\rho)/\rho\right]}{d\rho^2}\right|_{\rho_0}$$

$$E[\rho] = E[\rho_0] + \frac{1}{18} K_\infty \left(\frac{\rho - \rho_0}{\rho_0}\right)^2$$

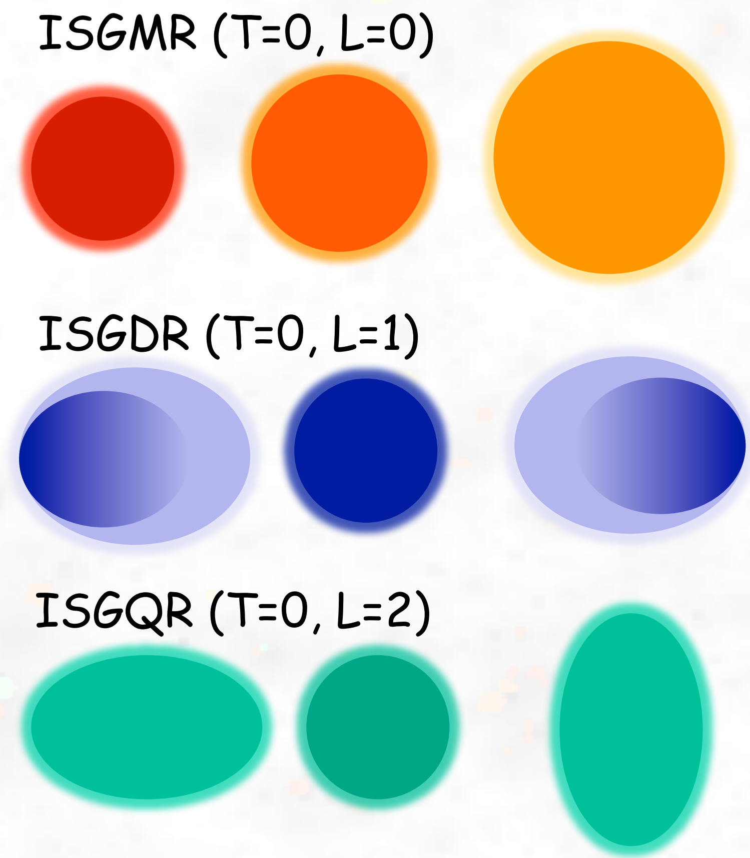


EOS – Experimental Observations

- **Analysis of Giant Isoscalar Resonances**
- Relativistic heavy ion collisions
- Neutron masses

Giant Resonance: Coherent vibration of nucleons in a nucleus

- Resonances related to incompressibility: **ISGMR**, ISGDR, ISGQR



$$E_{ISGMR} \approx \sqrt{\frac{K_A}{m\langle r^2 \rangle}}$$

$$c \approx 1$$

$$K_A = K_\infty \left(1 + cA^{-1/3}\right) + K_\tau \left(\frac{N-Z}{A}\right)^2 + K_{Coul} Z^2 A^{-4/3}$$

- K_{Coul} is basically model independent
- Measurements over several isotopes should give K_τ
- K_τ critical to understand neutron stars

EOS – Theoretical Methods

- Build an energy functional E[ρ] using an mean field calculation

  Each such a functional characterizes a K_∞

- Get excitations such as the ISGMR from a self-consistent QRPA calculation

For the nucleon-nucleon interaction

$$V(\mathbf{r}_i,\mathbf{r}_j) = V_{ij}^{NN} + V_{ij}^{Coul}$$

$$V_{ij}^{Coul} = -\frac{e^2}{4}\sum_{i,j=1}^{A}\frac{\tau_{ij}^2+\tau_{ij}}{\left|\mathbf{r}_i-\mathbf{r}_j\right|},$$

$$\tau_{ij} = \tau_i + \tau_j$$

$$V_{ij}^{NN} = t_0(1+x_0P_{ij}^{\sigma})\delta(\mathbf{r}_i-\mathbf{r}_j)+\frac{1}{2}t_1(1+x_1P_{ij}^{\sigma})[\overleftarrow{\mathbf{k}}_{ij}^{2}\delta(\mathbf{r}_i-\mathbf{r}_j)+\delta(\mathbf{r}_i-\mathbf{r}_j)\overrightarrow{\mathbf{k}}_{ij}^{2}]+$$

$$t_2(1+x_2P_{ij}^{\sigma})\overleftarrow{\mathbf{k}}_{ij}\delta(\mathbf{r}_i-\mathbf{r}_{jj})\overrightarrow{\mathbf{k}}_{ij}+\frac{1}{6}t_3(1+x_3P_{ij}^{\sigma})\rho^{\alpha}\left(\frac{\mathbf{r}_i+\mathbf{r}_j}{2}\right)\delta(\mathbf{r}_i-\mathbf{r}_j)+$$

$$iW_0\overleftarrow{\mathbf{k}}_{ij}\delta(\mathbf{r}_i-\mathbf{r}_j)(\vec{\sigma}_i+\vec{\sigma}_j)\overrightarrow{\mathbf{k}}_{ij},$$

$t_i,\ x_i,\ \alpha,\ W_0$ are 10 **Skyrme** parameters

$$\mathcal{E}[\rho] = \left\langle \Phi \middle| T + V_{ij}^{Coul} + V_{ij}^{NN} \middle| \Phi \right\rangle$$

+ pairing

HF + BCS

$$\Delta_i = \frac{1}{2}\sum_j \frac{G_{ij}\Delta_j}{\sqrt{(\varepsilon_j - \lambda)^2 + \Delta_j^2}}$$

HFB

$$\begin{pmatrix} h_{HF} - \lambda & \Delta \\ -\Delta & -h_{HF} + \lambda \end{pmatrix}\begin{pmatrix} u_k \\ v_k \end{pmatrix} = E_k \begin{pmatrix} u_k \\ v_k \end{pmatrix}$$

v_{NN}^{eff} = Skyrme + pairing force

$$V = V_0\left[1 - \eta\left(\frac{\rho(\mathbf{r})}{\rho_0}\right)^{\alpha}\right]\delta(\mathbf{r}_1 - \mathbf{r}_2), \qquad \rho_0 = 0.16\ \mathrm{fm}, \quad \alpha = 1$$

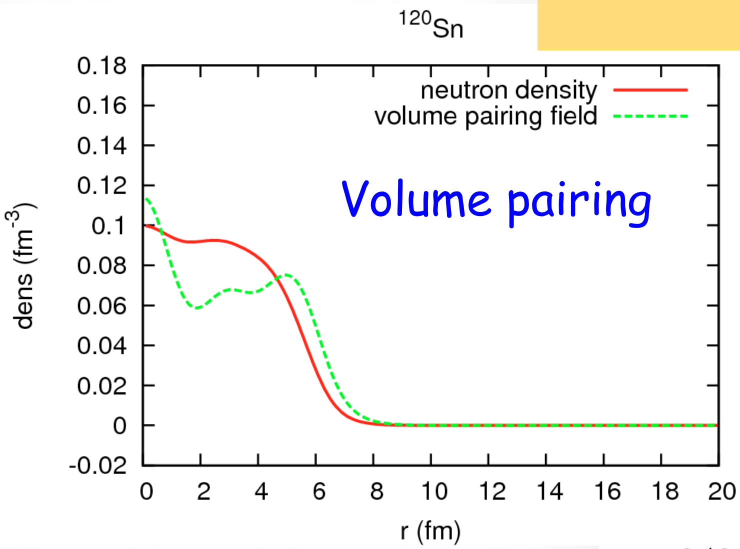
$$\eta = \begin{cases} 0, & \text{"volume" pairing} \\ 1, & \text{"surface" pairing} \\ 1/2, & \text{"mixed" pairing} \end{cases}$$

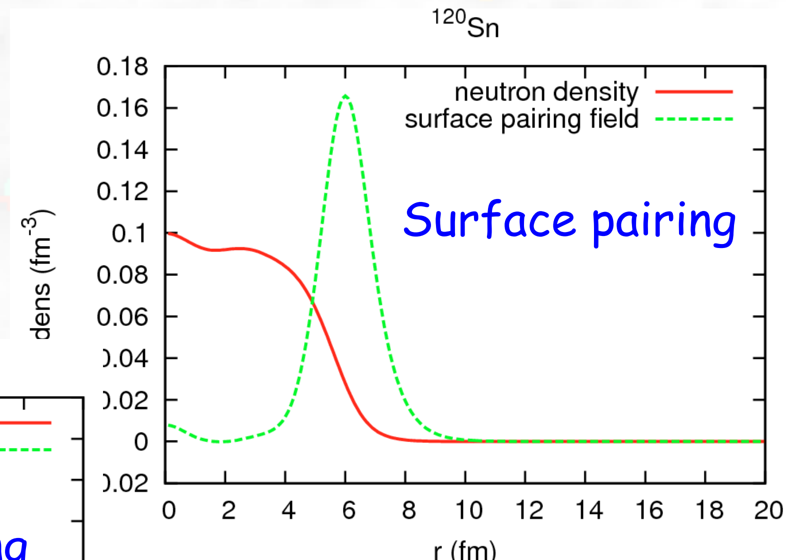
EOS + Pairing

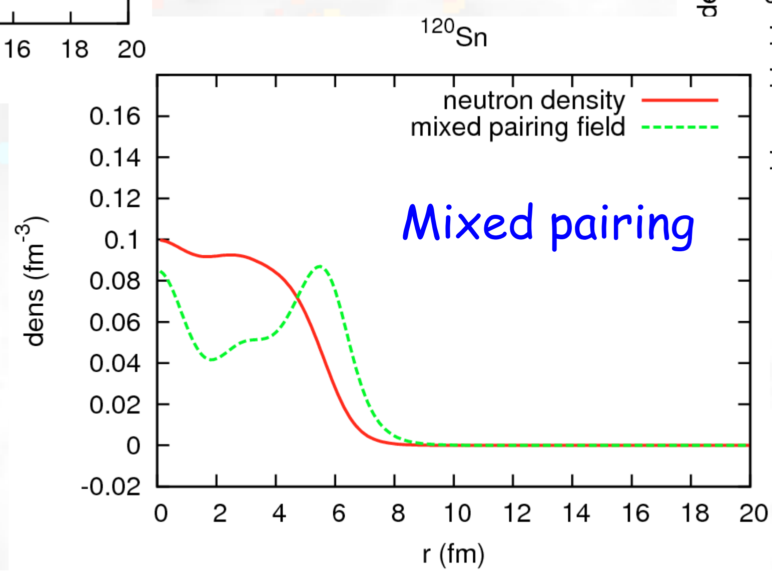
Protons and neutrons tend to pair up, much like Cooper pairs of electrons in superconductors. Pairing is important in nuclei and neutron stars, but a clear understanding of **the microscopic foundation of the pairing functional is still lacking**.

$$v(\mathbf{r},\mathbf{r}') = v_0\left[1-\eta\left(\frac{\rho}{\rho_0}\right)^{\gamma}\right]\delta(\mathbf{r}-\mathbf{r}')$$

Skyrme	K_∞
SLy5	230
SkM*	216
Skxs20	202







Blaizot
Phys. Rep. 64, 171 (1980)

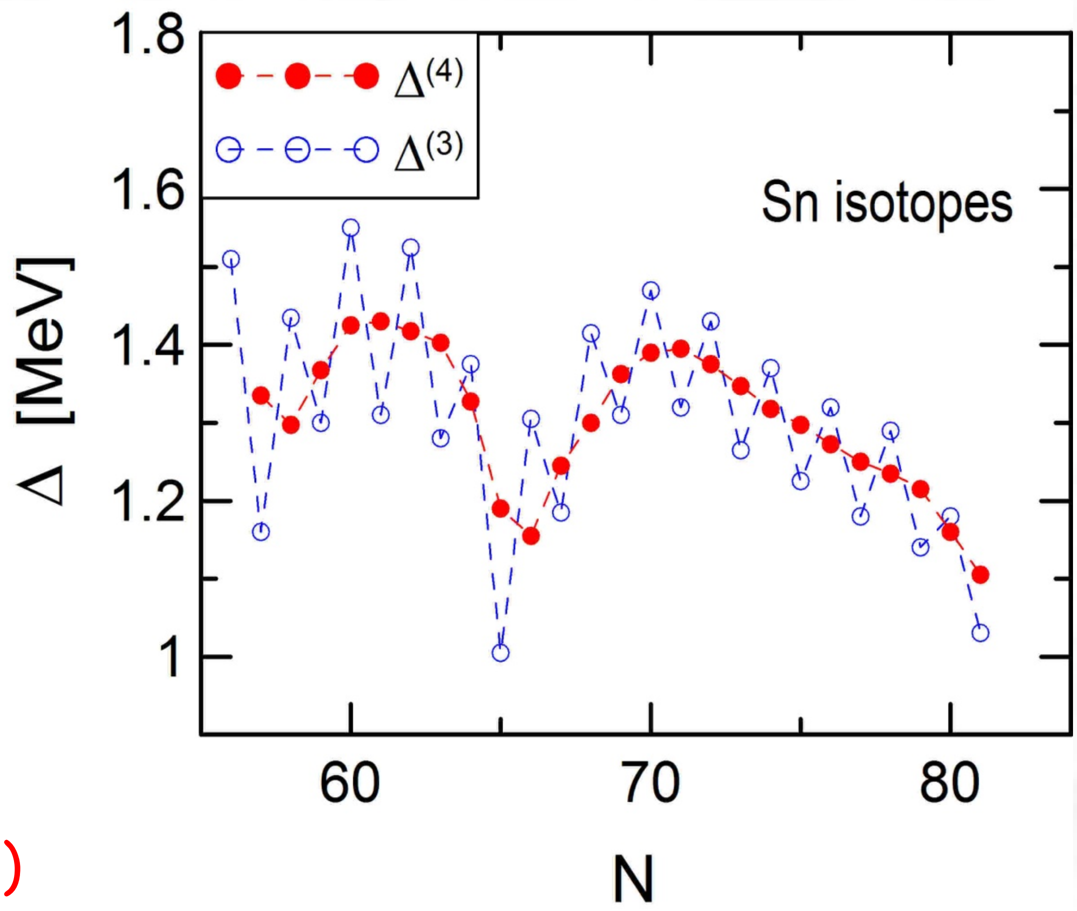
Shlomo, Youngblood
PRC 47, 529 (1993)

Pairing Measure

three-point $$\Delta^{(3)} = \frac{1}{2}(-1)^N\left[B(N-1)+B(N+1)-2B(N)\right]$$

four-point $$\Delta^{(4)} = \frac{1}{4}(-1)^N\left[3B(N-1)-3B(N)-B(N-2)+B(N+1)\right]$$

or higher ?



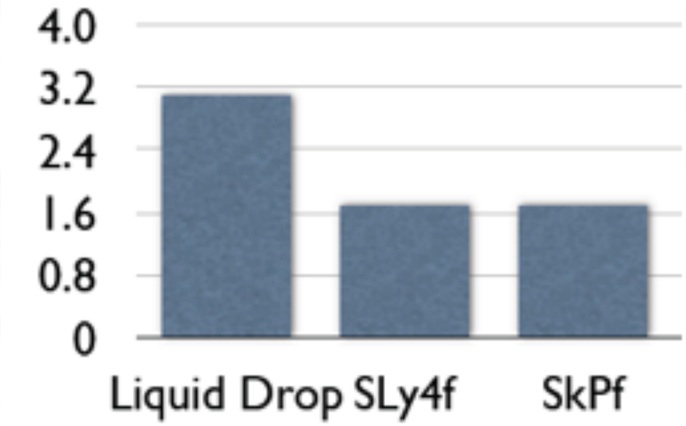
From experiment:

- $\Delta^{(3)}$ larger for $(-1)^N = +1$
- $\Delta^{(3)}$ smaller for $(-1)^N = -1$
- $\Delta^{(4)}$ reflects average of $\Delta^{(3)}$ (N) and $\Delta^{(3)}$ (N-1)

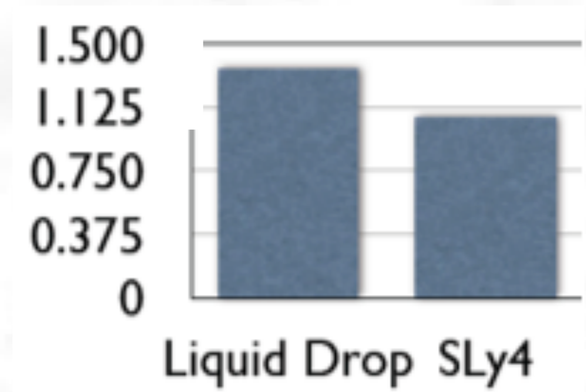
($\Delta^{(4)}$ no additional information)

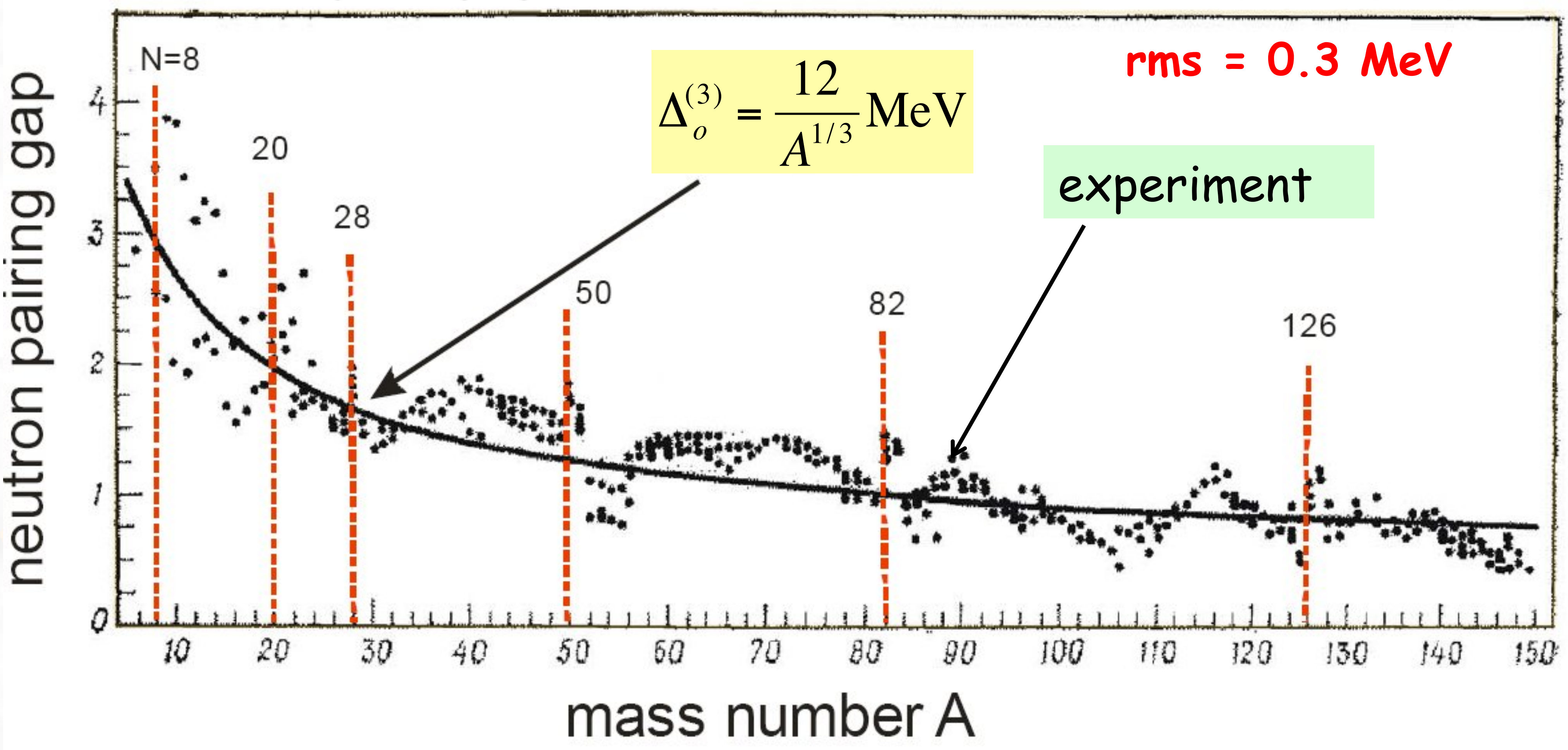
Can Microscopic Models do better than LDM?

rms for **binding** energies

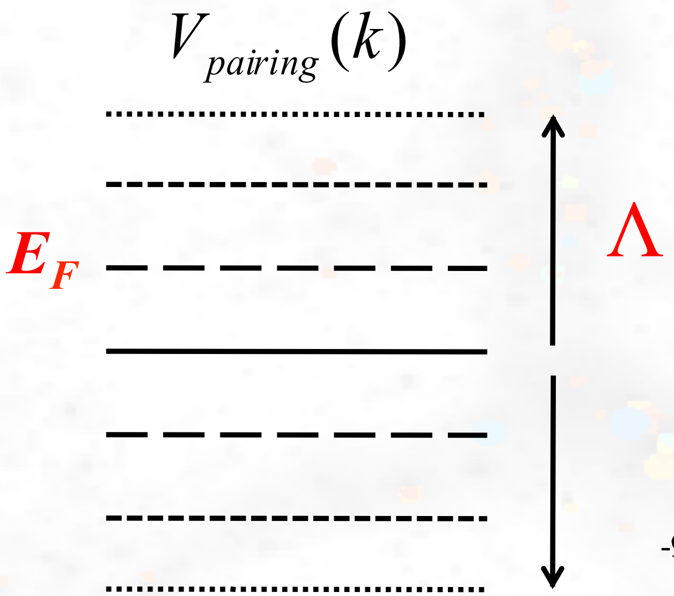


rms for **separation** energies

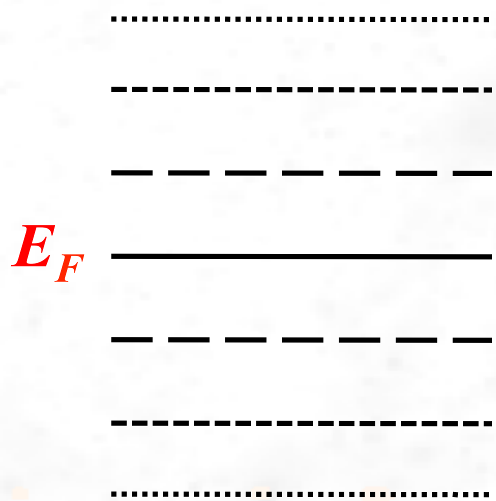




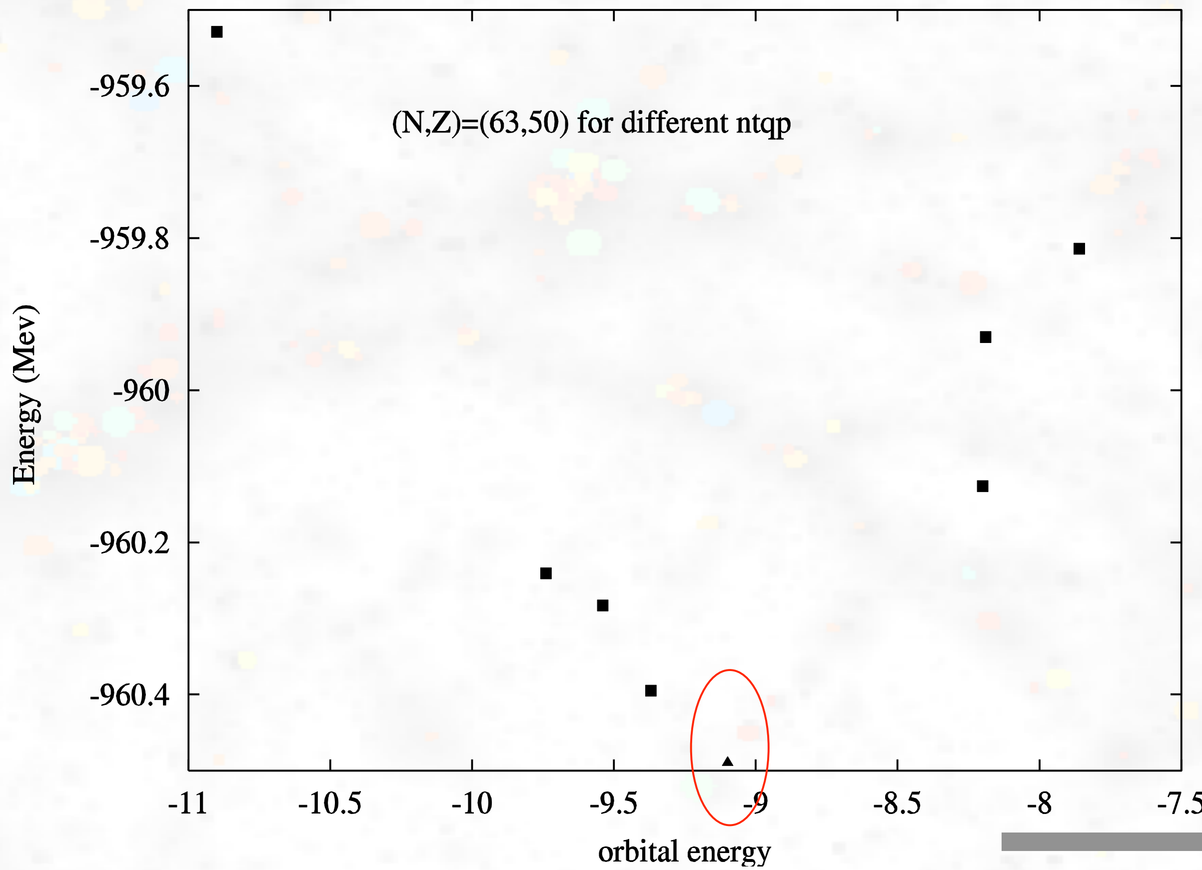
Blocking Procedure for Odd Nucleons



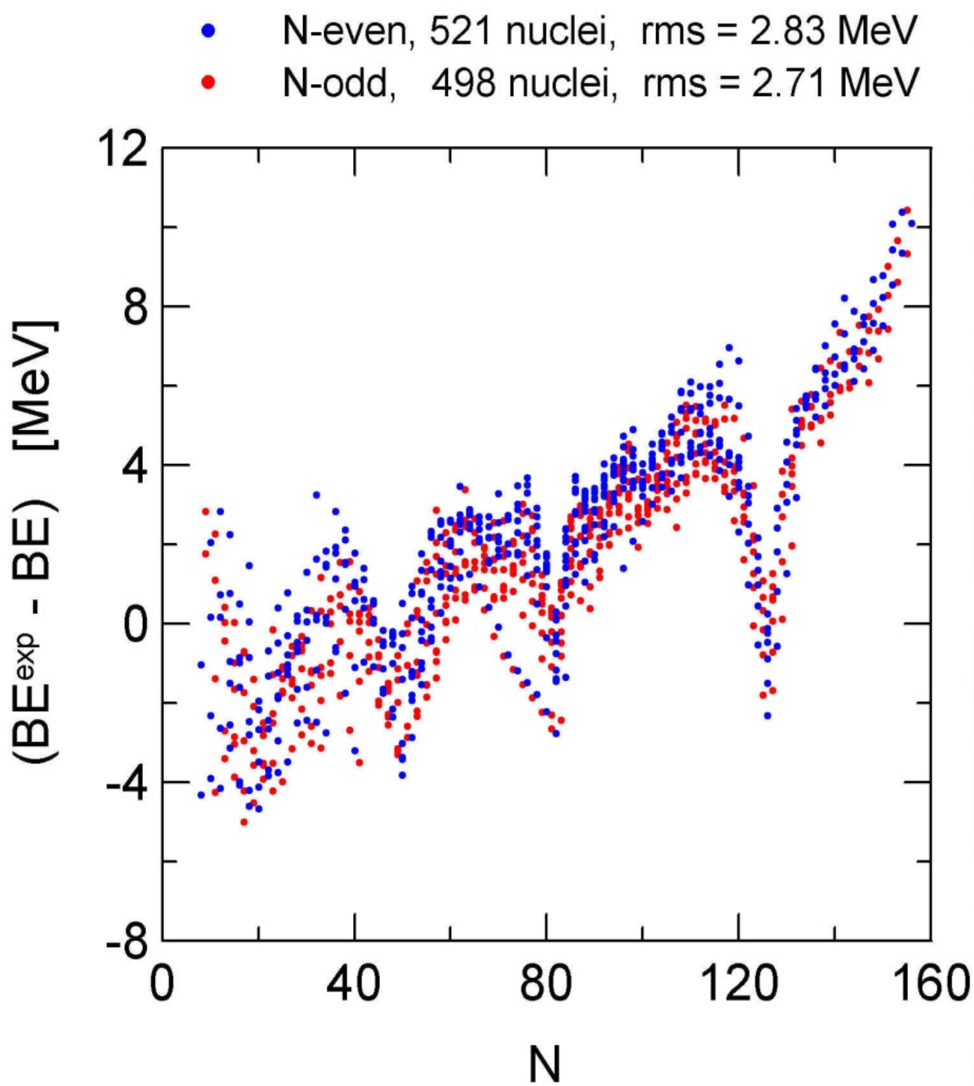
$\Lambda \sim 50$ MeV



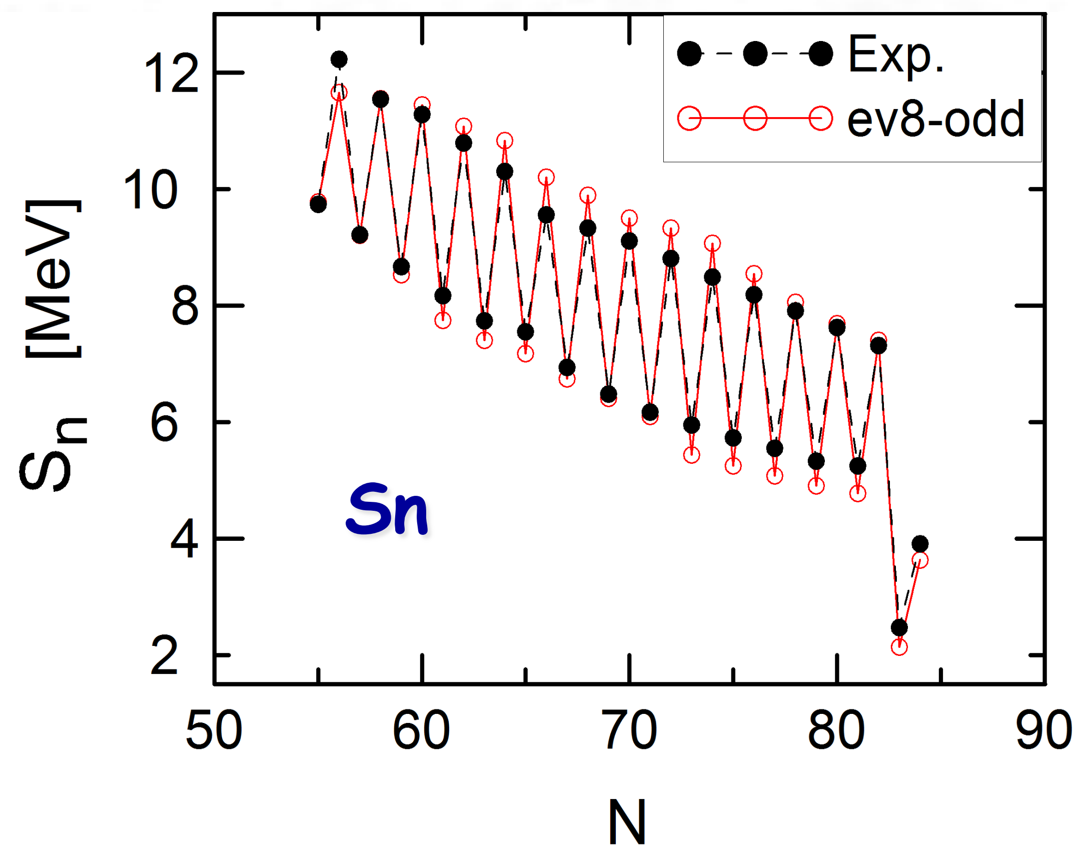
$v^2 \to \frac{1}{2}$, $u \to 0$
for one orbit and its time-conjugate partner



Pairing Improves Nuclear Properties



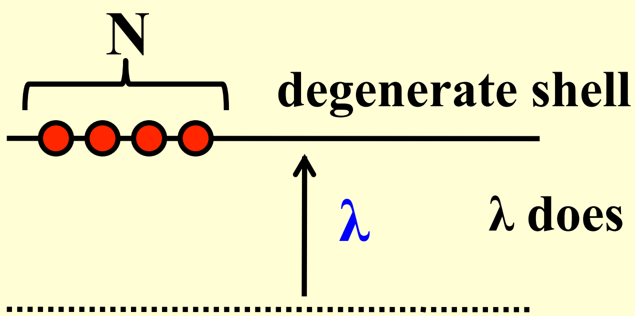
Separation energies staggering



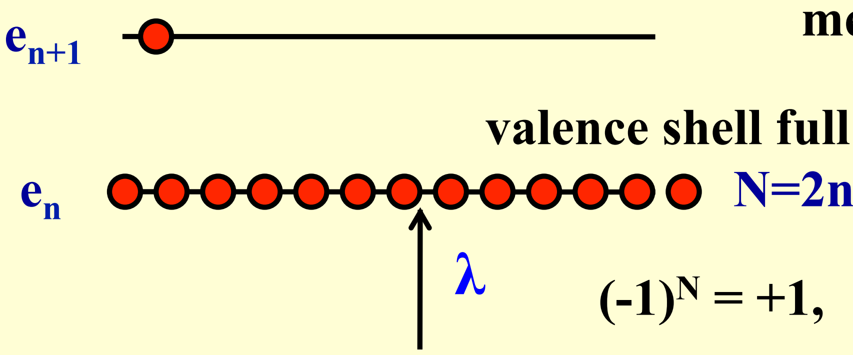
Pairing vs Level Properties

$$\Delta^{(3)} = \frac{1}{2}(-1)^N\left[B(N-1)+B(N+1)-2B(N)\right] \Rightarrow 2(-1)^N\Delta^{(3)} \cong \frac{\partial^2 B}{\partial N^2} = \frac{\partial\lambda}{\partial N} = \frac{1}{g(\lambda)}$$

Fermi energy ($\lambda = \partial B/\partial N$) s.p. level density ($g(e) = dN/\partial e$)



λ does not vary with N → Δ(3) = 0



measure of gap in single-particle spectrum

$(-1)^N = +1$, $d\lambda/dN \approx e_{n+1} - e_n$ → $\Delta(3) = (e_{n+1} - e_n)/2$

Jahn-Teller mechanism: spherical symmetry spontaneously broken

⇒ (2j+1) → double-degenerate orbits ⇒ $\Delta^{(3)}$ alternates for $(-1)^N$ = + and -

Pairing vs Level Properties

Macroscopic-microscopic model:

$$B = E_{sp} - \widetilde{E}_{sp} + E_{macro}, \qquad E_{sp} = \sum_{k=1}^{A} e_k$$

$$E_{macro} = \cdots + a_I \frac{(N-Z)^2}{A}, \qquad a_I = 23\text{ MeV}$$

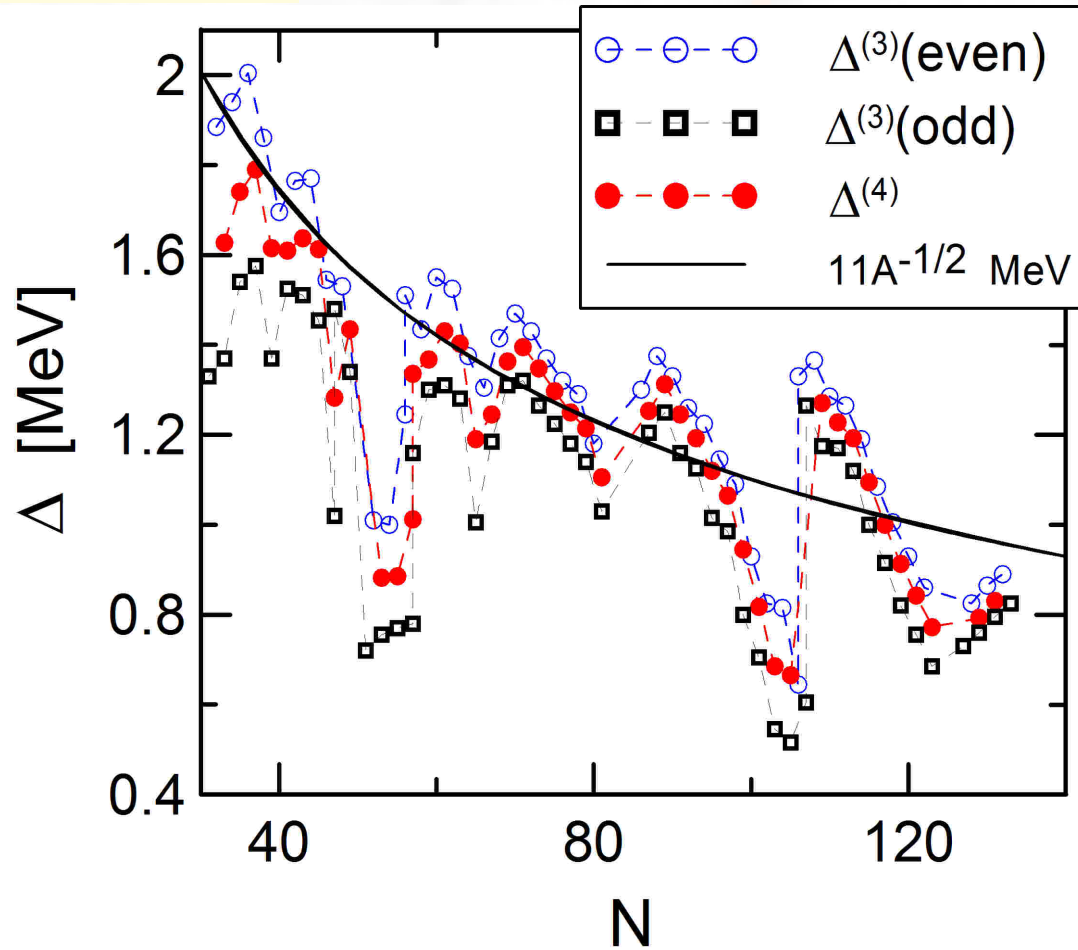
$$\Rightarrow \quad \Delta^{(3)} = \frac{23}{A}\text{ MeV}$$

$$\tilde{E}_{sp}\ \text{contribution}: \quad g(\lambda) = \frac{3a}{\pi^2}, \quad a \cong \frac{A}{8}\text{ MeV}$$

$$\Rightarrow \quad \Delta^{(3)} \cong -\frac{1}{g(\lambda)} \cong -\frac{25}{A}\text{ MeV}$$

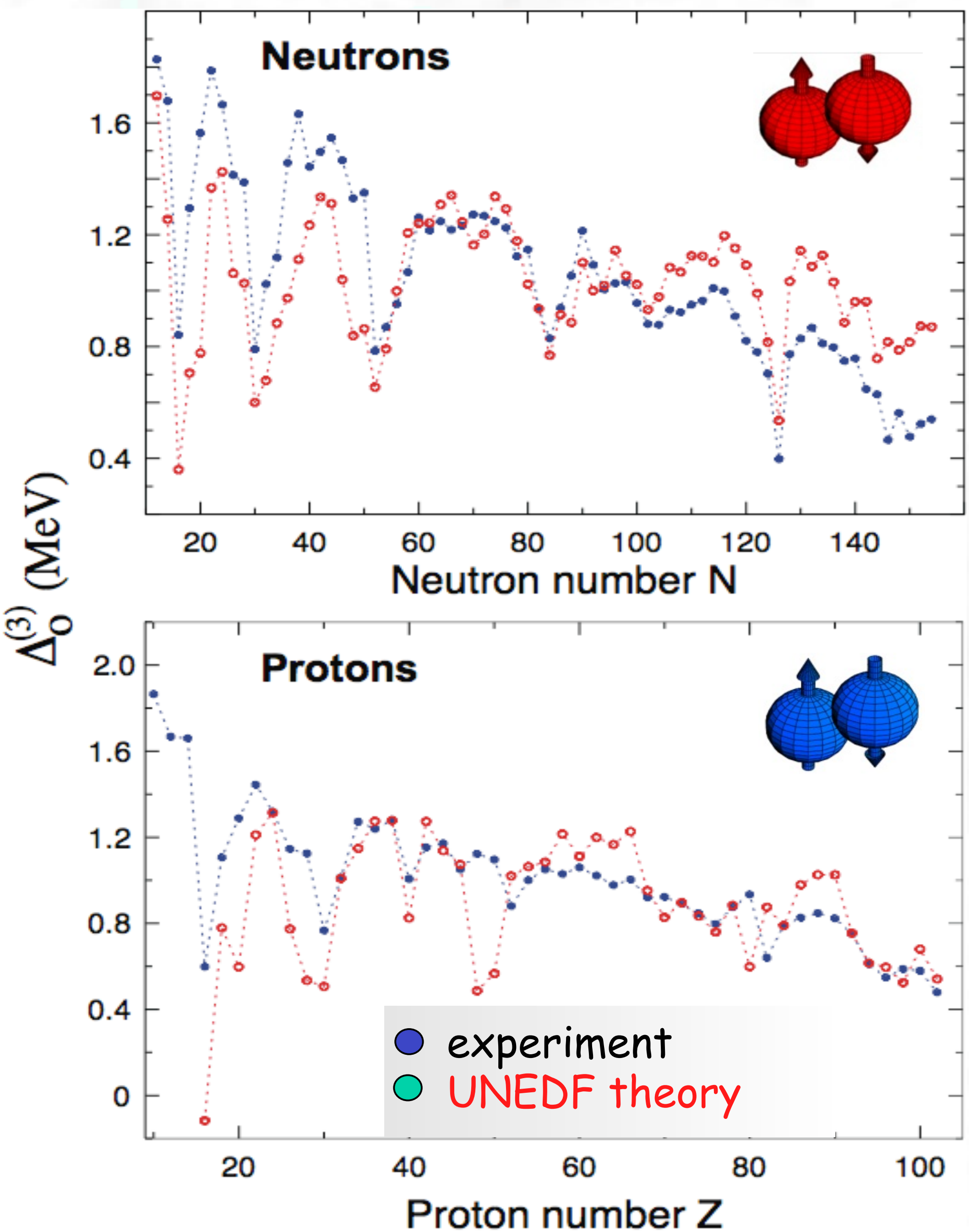
$$\Delta^{(3)}(N) \cong -\frac{1}{2}\delta e$$
$$+\ \text{LDM corrections}$$

$$2\left[\Delta^{(3)}(\text{even}) - \Delta^{(3)}(\text{odd})\right] \cong e_{n+1} - e_n$$



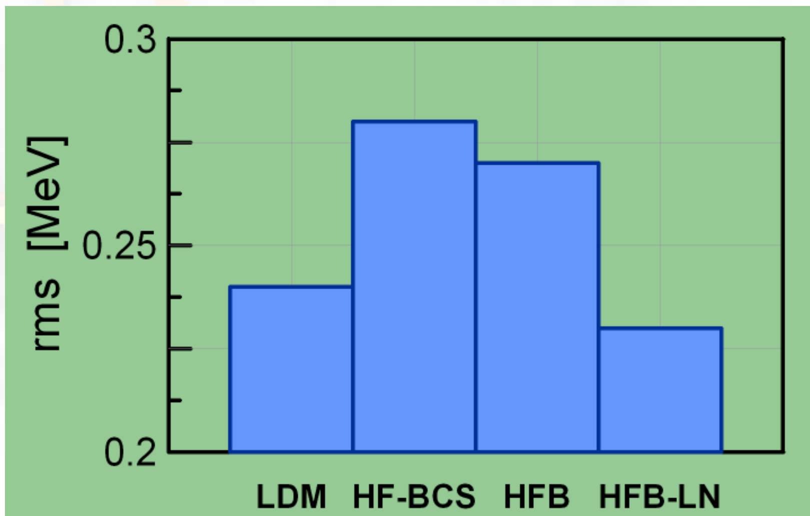
Satula, Dobaczewski, Nazarewicz, PRL 81, 3599 (1998).

The uNclear Nuclear Pairing – UNEDF Collaboration



- Mass tables for 2,400 nuclei have been analyzed using different forms and methodologies for the pairing mechanism

- New functional forms for the pairing interaction have been proposed



$$\Delta^{(3)} = \frac{1}{2}(-1)^N\left[B(N-1)+B(N+1)-2B(N)\right]$$

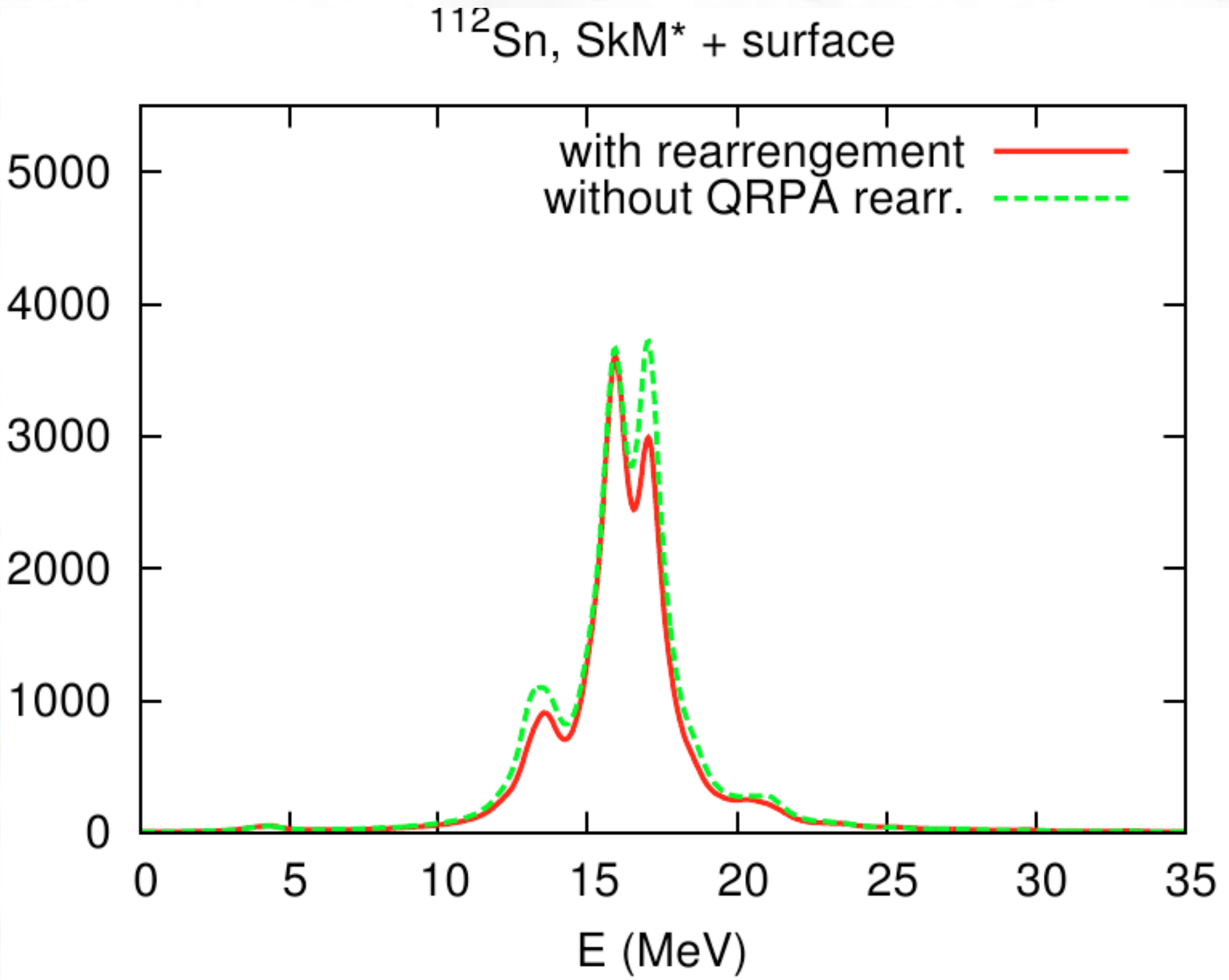
Bertsch, Bertulani, Nazarewicz, Schunck, Stoitsov, PRC 79, 0343306 (2009)

QRPA: The Role of the Rearrangement Term

Avogadro, Bertulani, PRC 88, 044319 (2013)

$$h = \frac{\delta E_{kin}}{\delta\rho} + \frac{\delta E_{skyrme}}{\delta\rho} + \frac{\delta E_{pair}}{\delta\rho} + \frac{\delta E_{Coul}}{\delta\rho}$$

- **Fully self consistent EWSR = 99.2%**
- **Without Rearrangement in EWSR =116%**

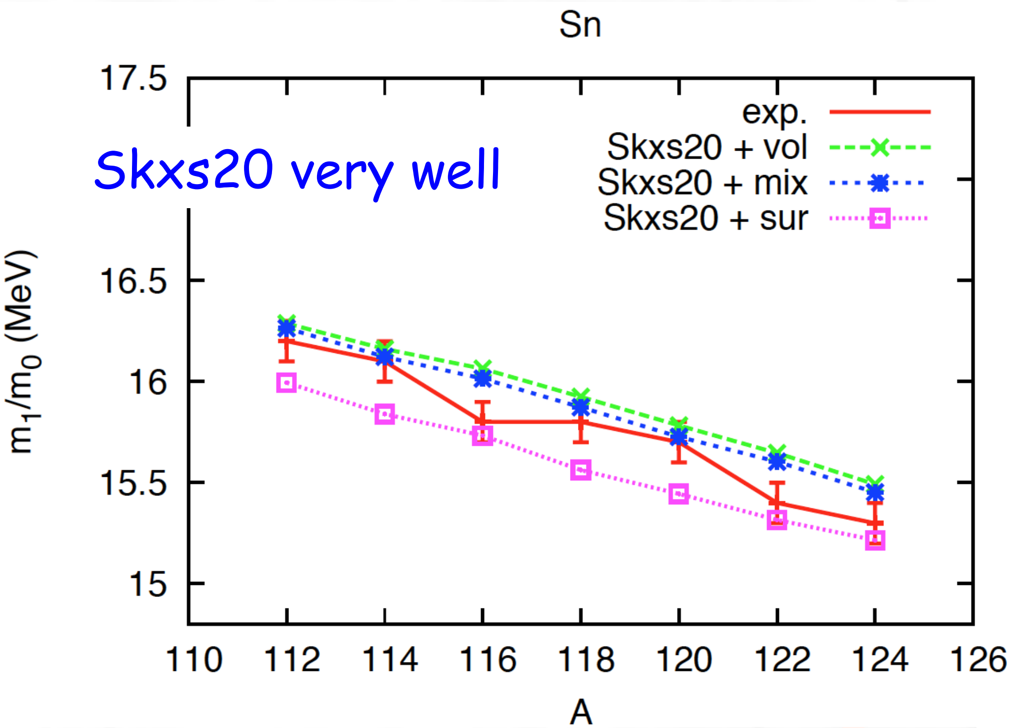


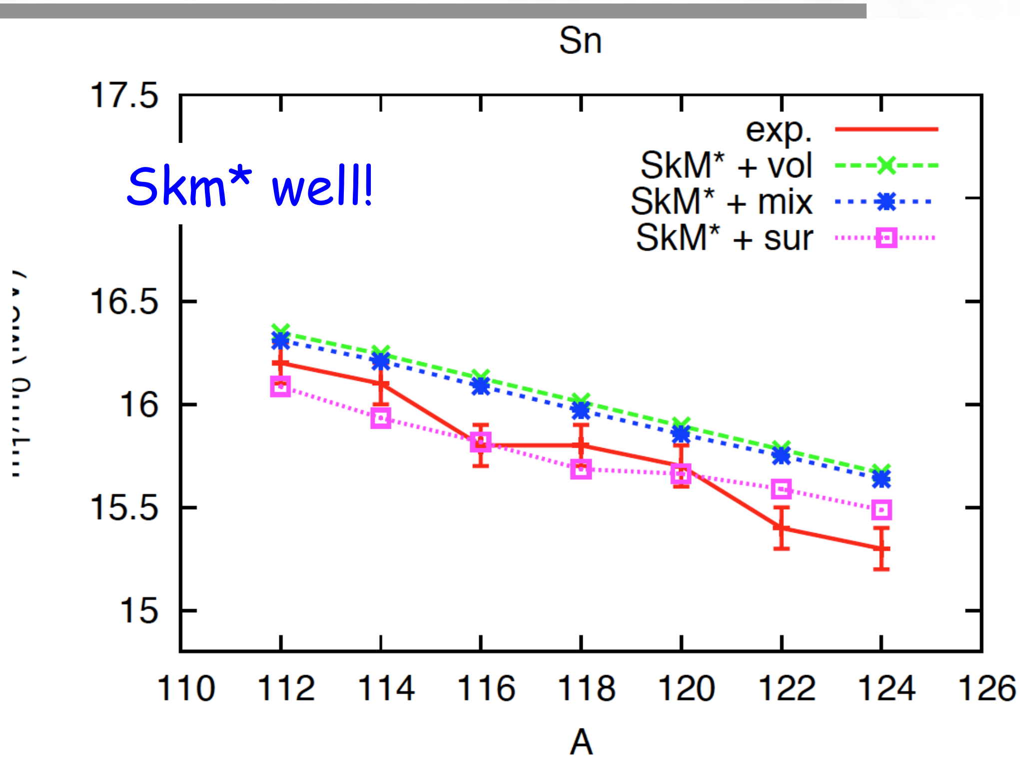
$$\frac{\delta h_{rearr}}{\delta\rho} = \frac{\delta}{\delta\rho}\left(\frac{\delta E_{pair}}{\delta\rho}\right)$$

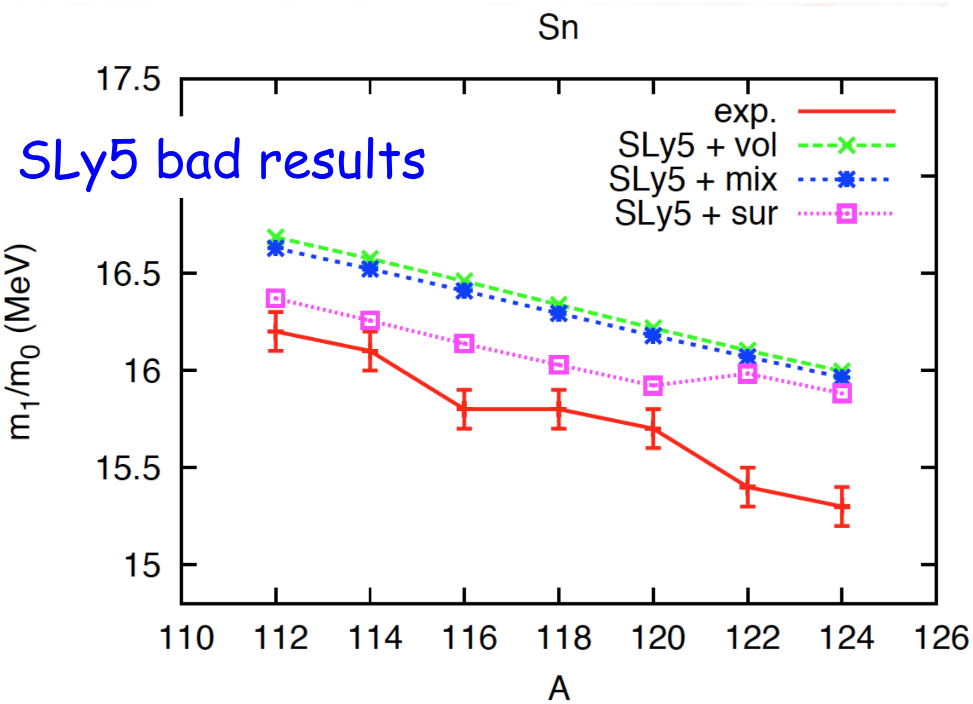
≠ 0 if E_{pair} depends on density

Calculations without rearrangements tend to return higher centroids respect to the fully self-consistent case.

Dependence on Functional







$$S(E) = \sum_j \left| \langle 0 | F_0 | j \rangle \right|^2 \delta\left(E - E_0\right)$$

$$F_0 = \sum_{i=1}^{A} r_i^2$$

$$m_k = \int_0^{\infty} E^k S(E)\, dE$$

Isovector pairing

$$v(\mathbf{r},\mathbf{r}') = v_0\left[1-\eta\left(\frac{\rho}{\rho_0}\right)^{\gamma}\right]\delta\left(\mathbf{r}-\mathbf{r}'\right)$$

$$v_{\text{pair}}^{\text{MSH}}(\mathbf{r},\mathbf{r}') = v_0\left[1-\left(1-\delta\right)\eta_s\left(\frac{\rho}{\rho_0}\right)^{\alpha_s}-\delta\eta_n\left(\frac{\rho}{\rho_0}\right)^{\alpha_n}\right]\delta(\mathbf{r},\mathbf{r}')$$

Margueron, Sagawa, Hagino, PRC 76, 064316 (2007)

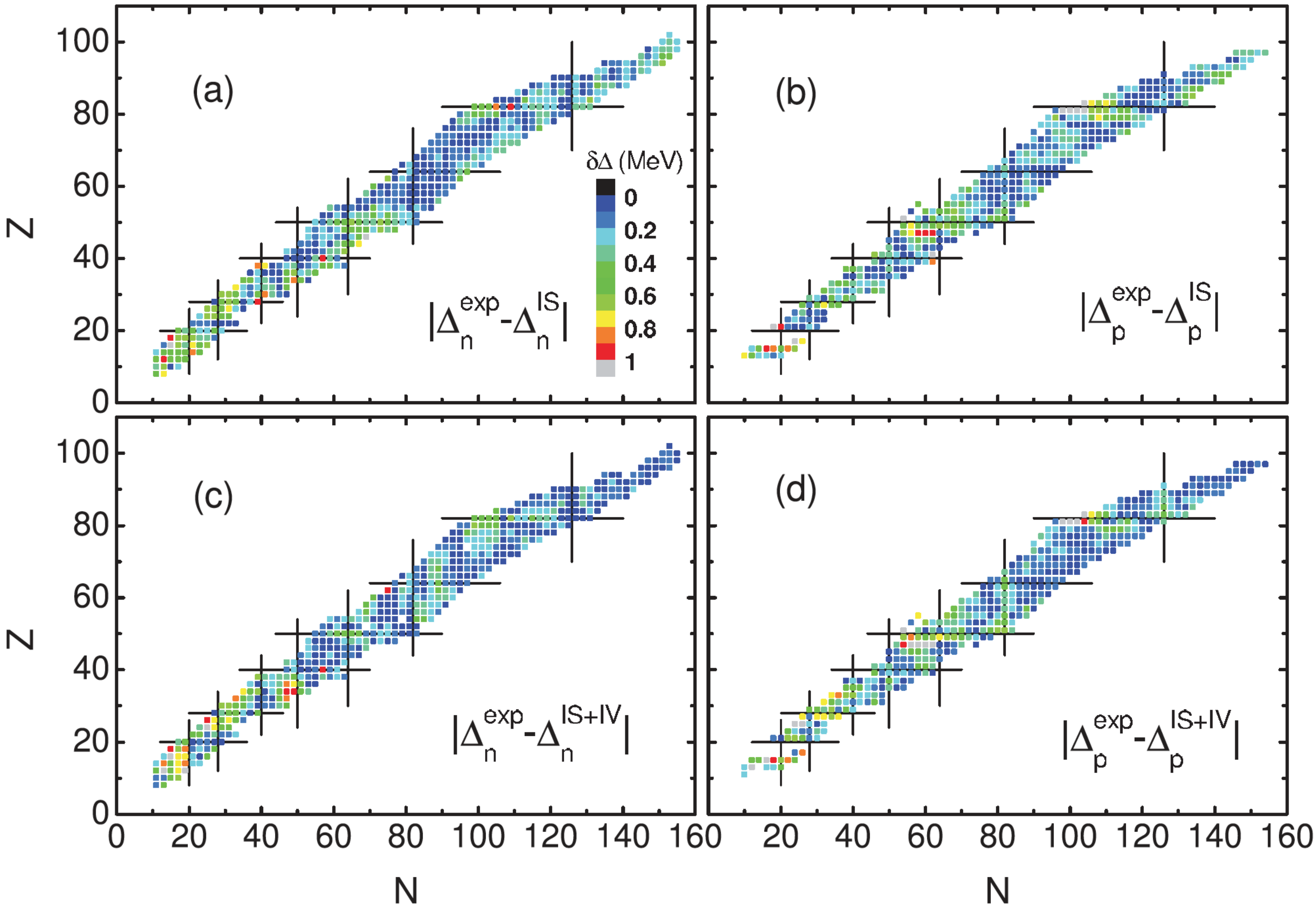
$$\rho = \rho_n + \rho_p$$

$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

$$v_{\text{pair}}^{\text{MSH}}(\mathbf{r},\mathbf{r}') = v_0\left[1-\left(\eta+\eta_1\tau_3\delta\right)\frac{\rho}{\rho_0}-\eta_2\left(\delta\frac{\rho}{\rho_0}\right)^{2}\right]\delta(\mathbf{r},\mathbf{r}')$$

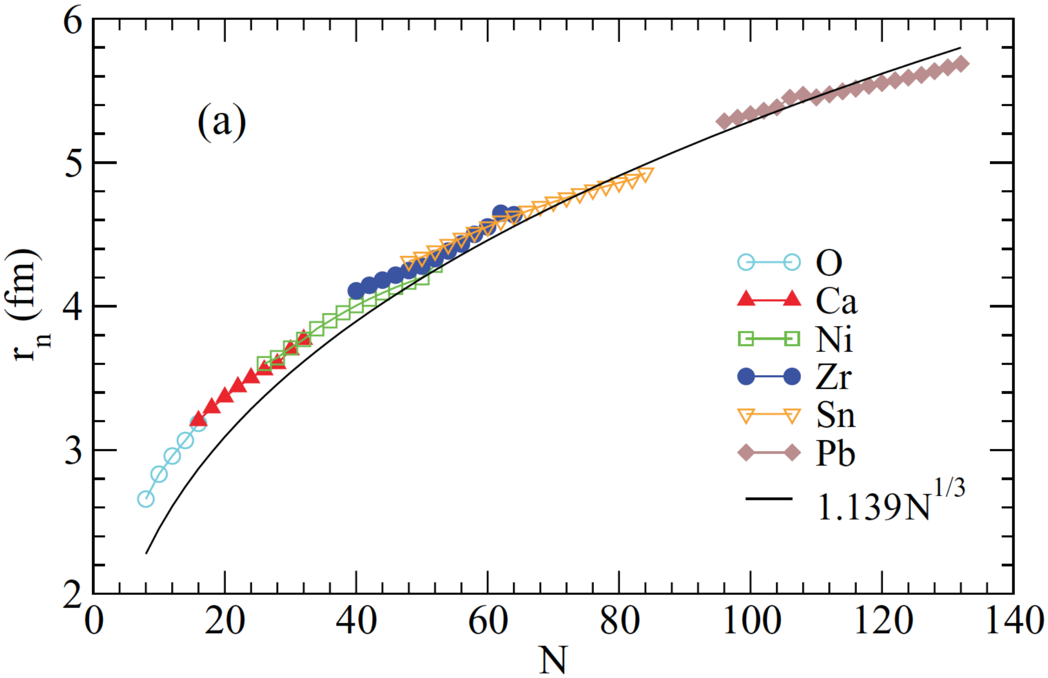
Yamagami, Shimizu, Nakatsukasa, PRC 80, 064301 (2009)

Isovector pairing – Good globabl fits to pairing gaps

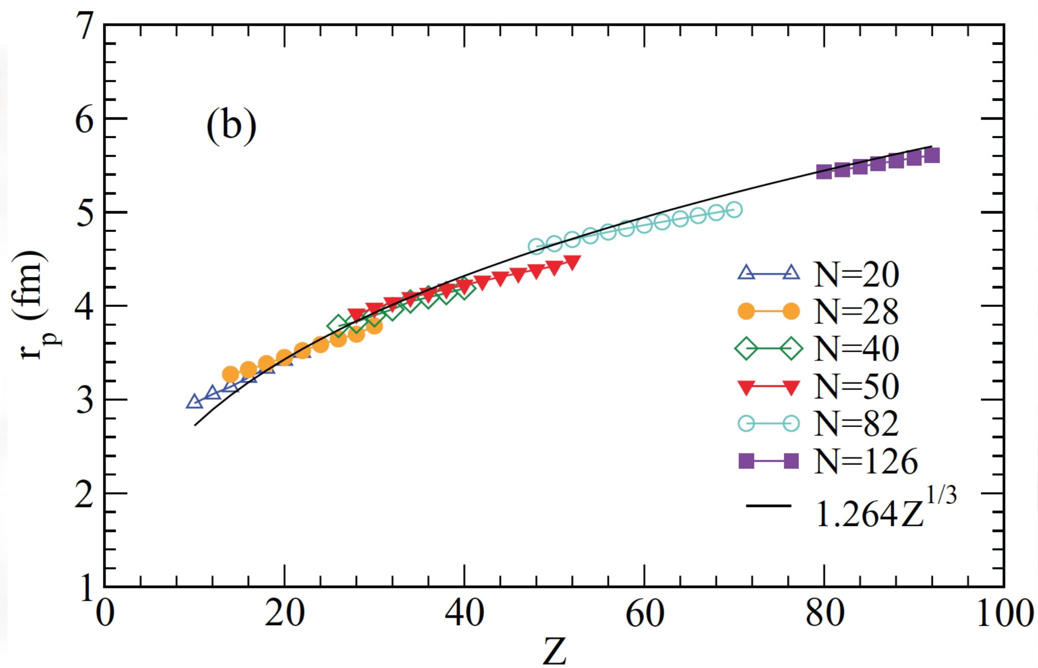


Bertulani, Liu,, Sagawa, PRC 85, 014321 (2012)

Isovector pairing – Reasonable Nuclear Radii

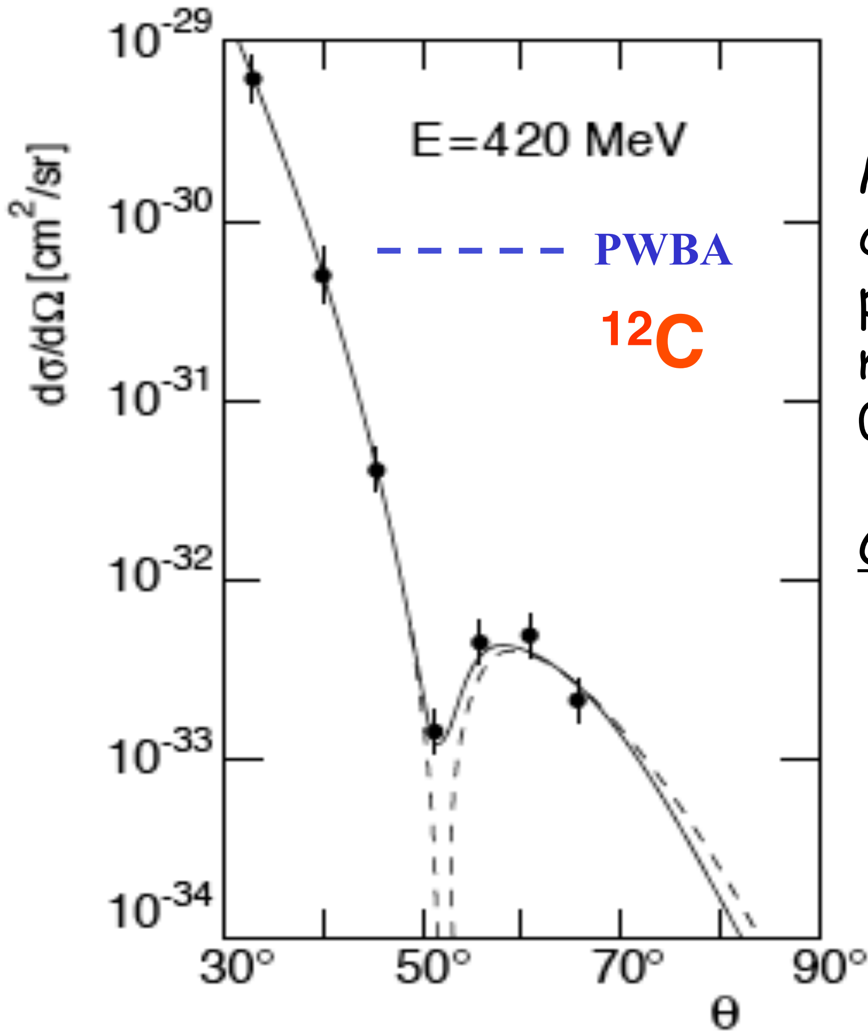


MSH IS+IV pairing



Bertulani, Liu,, Sagawa, PRC 85, 014321 (2012)

Radii and Skins

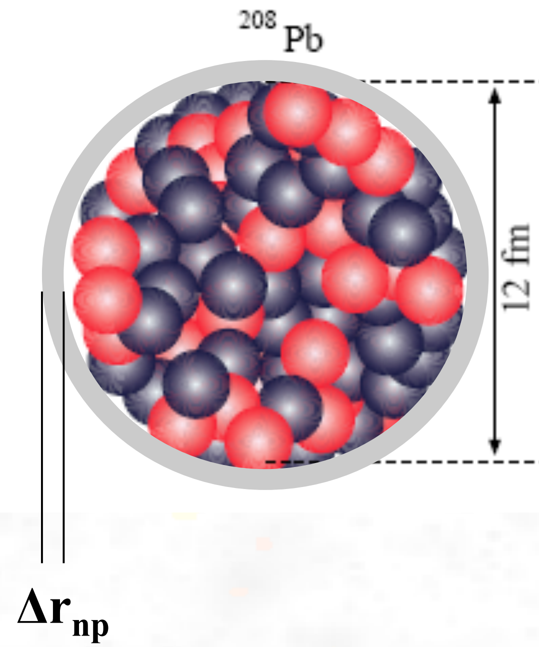


$$\frac{d\sigma}{d\Omega} = \frac{\sigma_M(\theta)}{Z^2}\left|F_{charge}(q)\right|^2$$

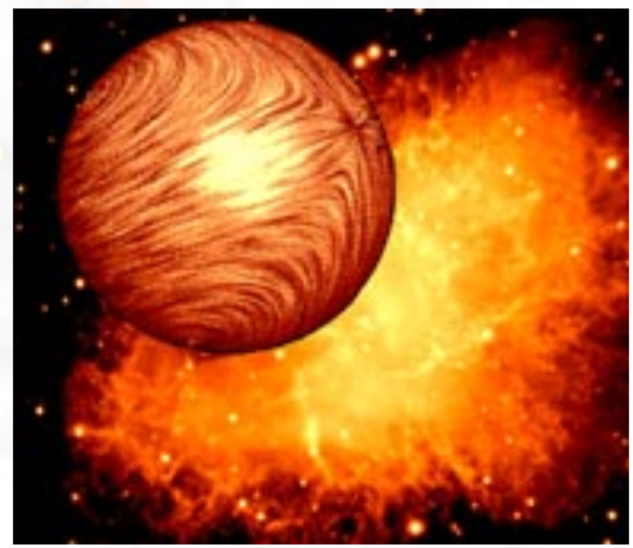
Hofstadter, 1953

Measuring R_n in ^{208}Pb constrains the pressure of neutron matter at ~ $2/3\rho_0$ = 0.1 fm^{-3}.

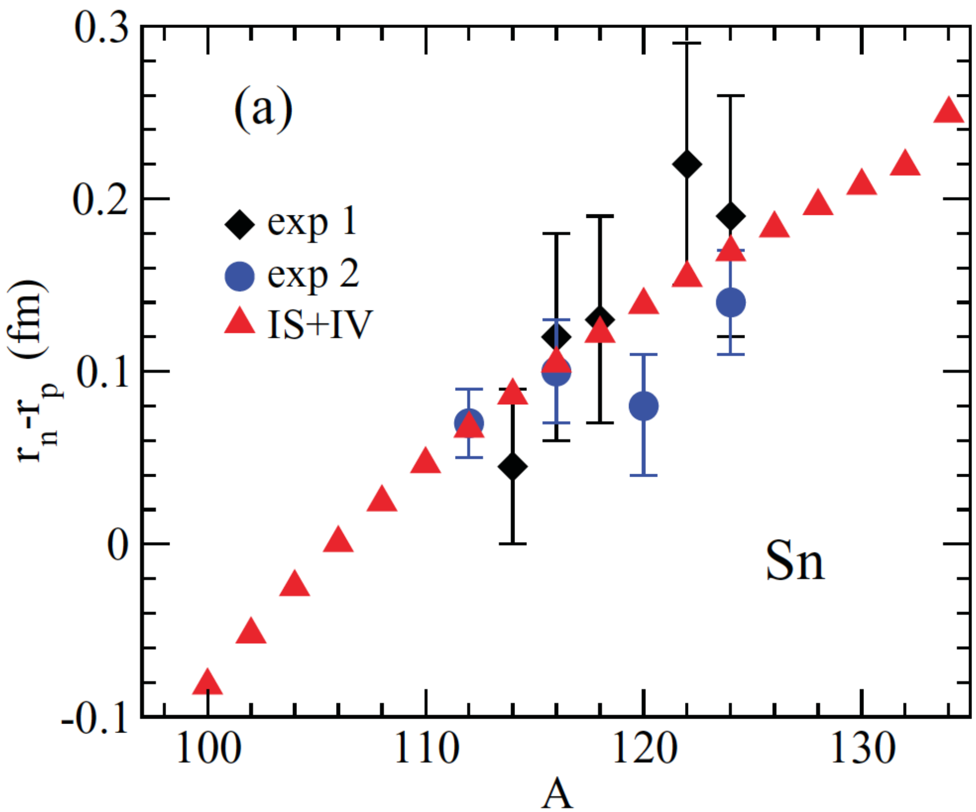
C. Horowitz



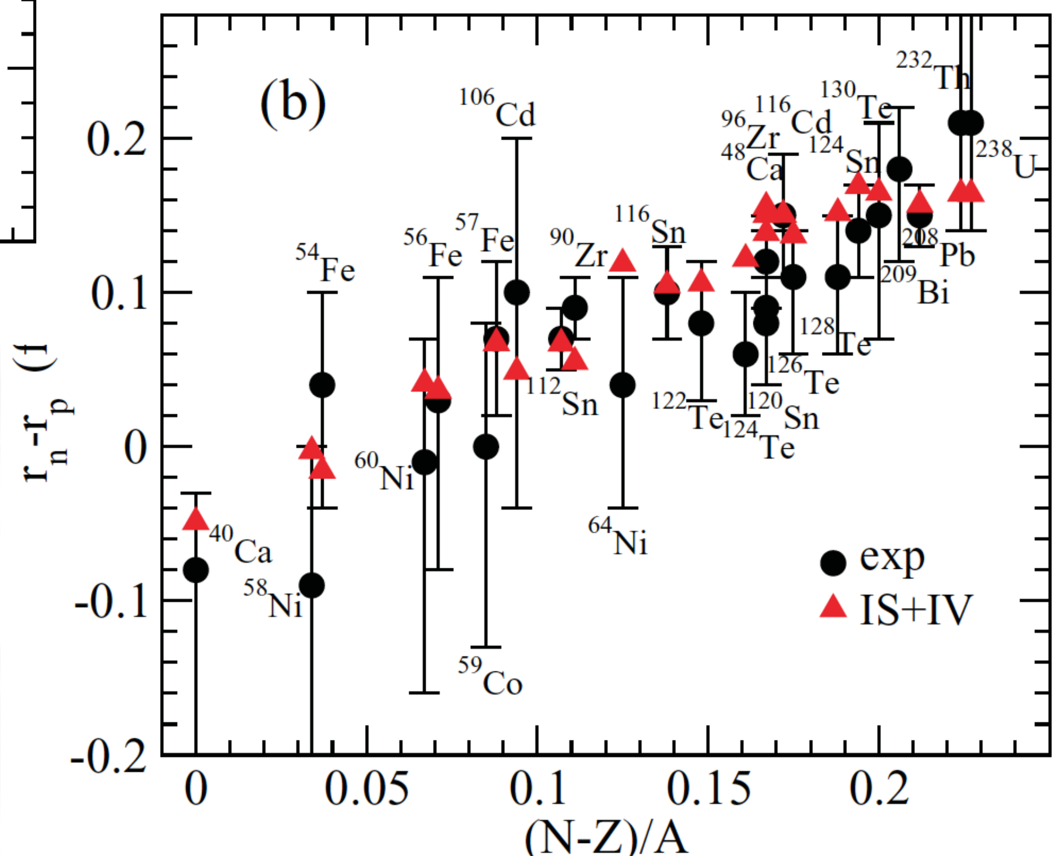
Neutron stars



Isovector pairing – Reasonable Nuclear Skins



MSH IS+IV pairing



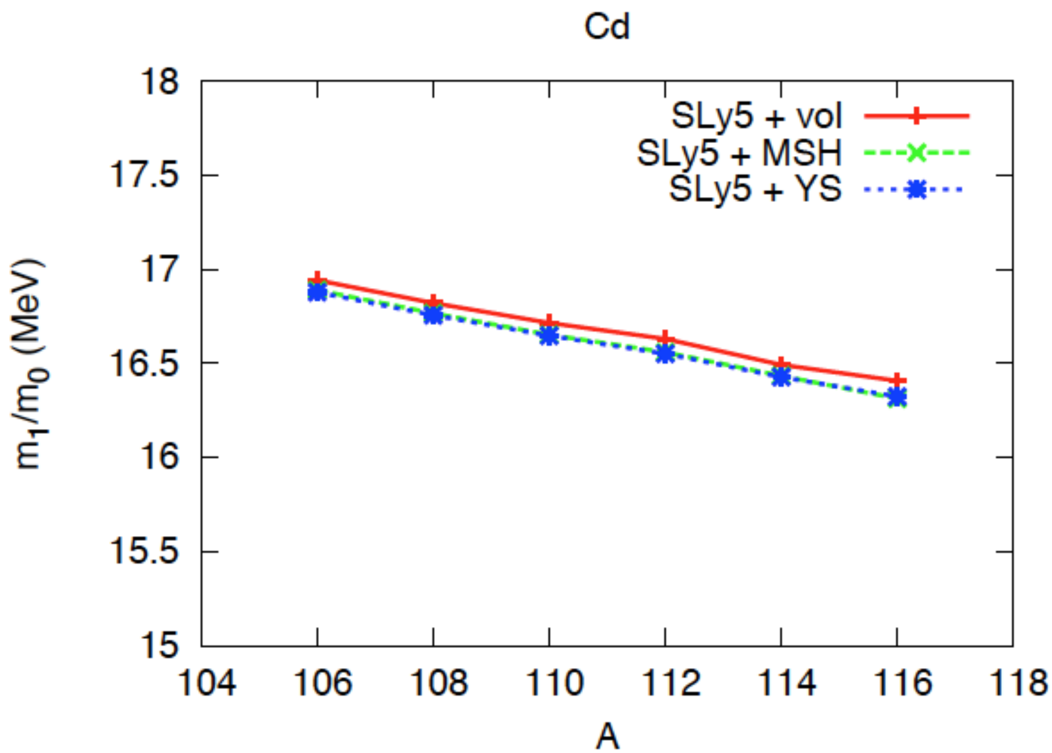
Radii from spin-dipole resonances
Krasznahorkay et al., PRL 82, 3216 (1999)

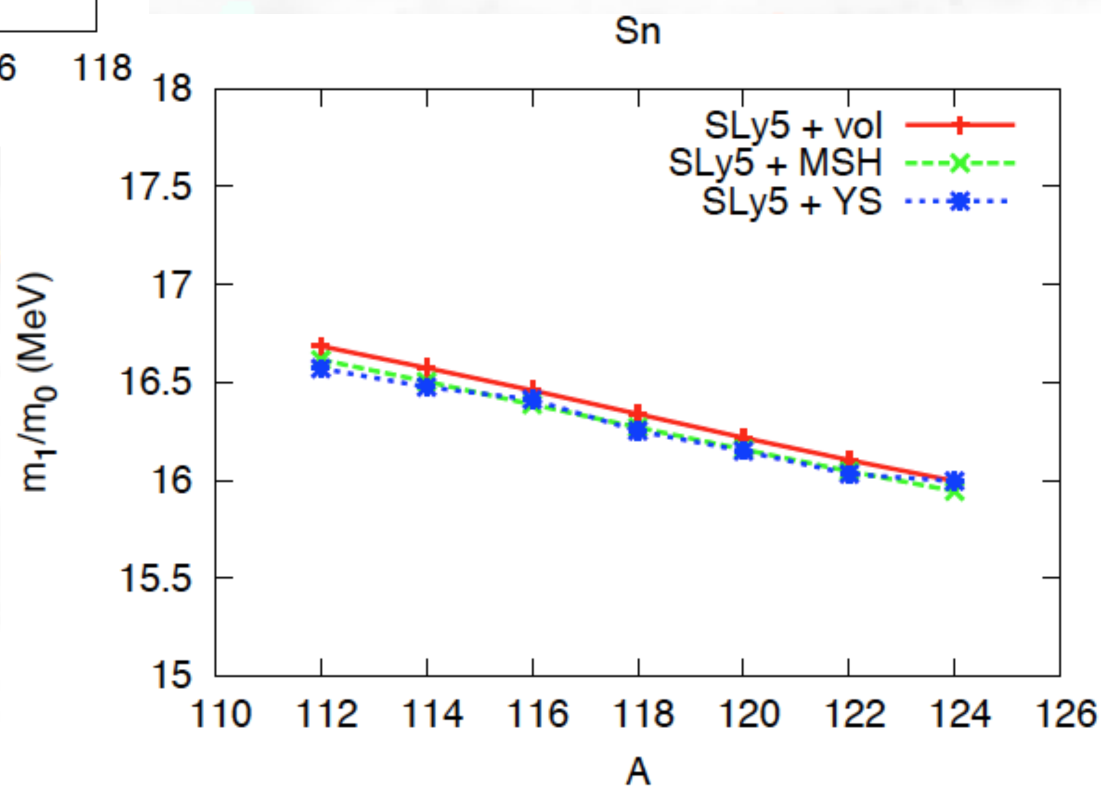
&

Antiprotonic atoms
Trzcinskaet al., PRL 87, 082501 (2001)

Bertulani, Liu,, Sagawa, PRC 85, 014321 (2012)

Isovector pairing – Improves Centroids of ISGMR





Isovector pairing – ISGMR – Comparison to Recent Data

	nucleus	ph	pp	diff.
TAMU/ RCNP	$^{204-206-208}$Pb	SLy5	all	< 0.1
TAMU/ RCNP	^{144}Sm	SkM*	*volume*	- 0.1
TAMU/ RCNP	^{90}Zr	SLy5	all	+ 0.2
TAMU	^{92}Zr	SLy5	*volume*	- 0.4
	^{94}Zr	Skxs20	*surface*	+ 0.8
TAMU	^{92}Mo	SLy5	*volume*	- 1.6
	^{94}Mo	Skxs20	*surface*	+ 0.0
RCNP	$^{112-114-118-120}$Sn [4]	Skxs20	*mixed*	< 0.1
	$^{122-124}$Sn [4]	Skxs20	*surface*	< 0.1
	^{116}Sn [4]	SkM*	*surface*	< 0.1
TAMU	$^{112-124}$Sn [35]	Skxs20	*surface*	≈ 0.8
	^{116}Sn [35]	Skxs20	*surface*	+ 0.2
RCNP	$^{106-110-112-114-116}$Cd [6]	Skxs20	*surface*	< 0.1
TAMU	$^{110-116}$Cd [46]	Skxs20	*surface*	≈ 0.9

A=92 very stiff!

Avogadro, Bertulani, PRC 88, 044319 (2013)

Conclusions

EOS & Pairing Conundrum

- Nuclear pairing evidently improves masses, separation energies and staggering effects in microscopic approaches: HF + BCS or HFB.

- Inclusion of pairing complicates determination of best Skyrme models. E.g., 20% of the nuclei well explained with the SLy5 interaction, and 10% with the SkM* interaction.

- Isovector pairing improves nuclear properties ← at the cost of additional parameters

- Detailed studies using HFB + QRPA + isovector pairing with comparison with newest data on ISGMR (RCNP and TAMU)
  →
  ISGMR is better reproduced with the soft interaction Skxs20 ($K_\infty \approx 202$ MeV), in contrast with the generally accepted value for $K_\infty \approx 230$ MeV.