Embedded Lensing Time Delays, Fermat Potential, and the Integrated Sachs-Wolfe Effect

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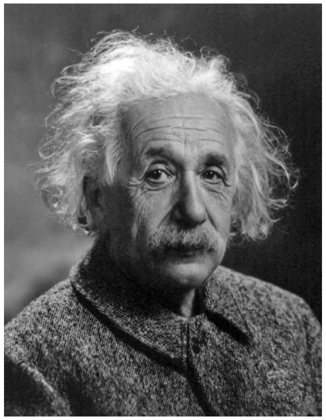
Outline

1. Swiss Cheese model and Embedded Lens Theory
2. Fermat's Least Time Principle and Fermat Potential
3. A Simple Formula Computing Integrated Sachs-Wolfe Effect using Fermat Potential
4. Study Anti-Lensing and CMB lensing by Cosmic Voids using Embedded Lens Theory

Swiss-Cheese Models



(Einstein in 1947)

- **Swiss Cheese Model** (Einstein & Strauss 1945 Rev. Mod. Phys.) was originally invented to embed *static* gravitational field of a spherical mass condensation (a star, or a galaxy) into an expanding homogeneous FLRW universe. It is an exact solution of Einstein's field equations.
- How to build a simple Swiss cheese model?

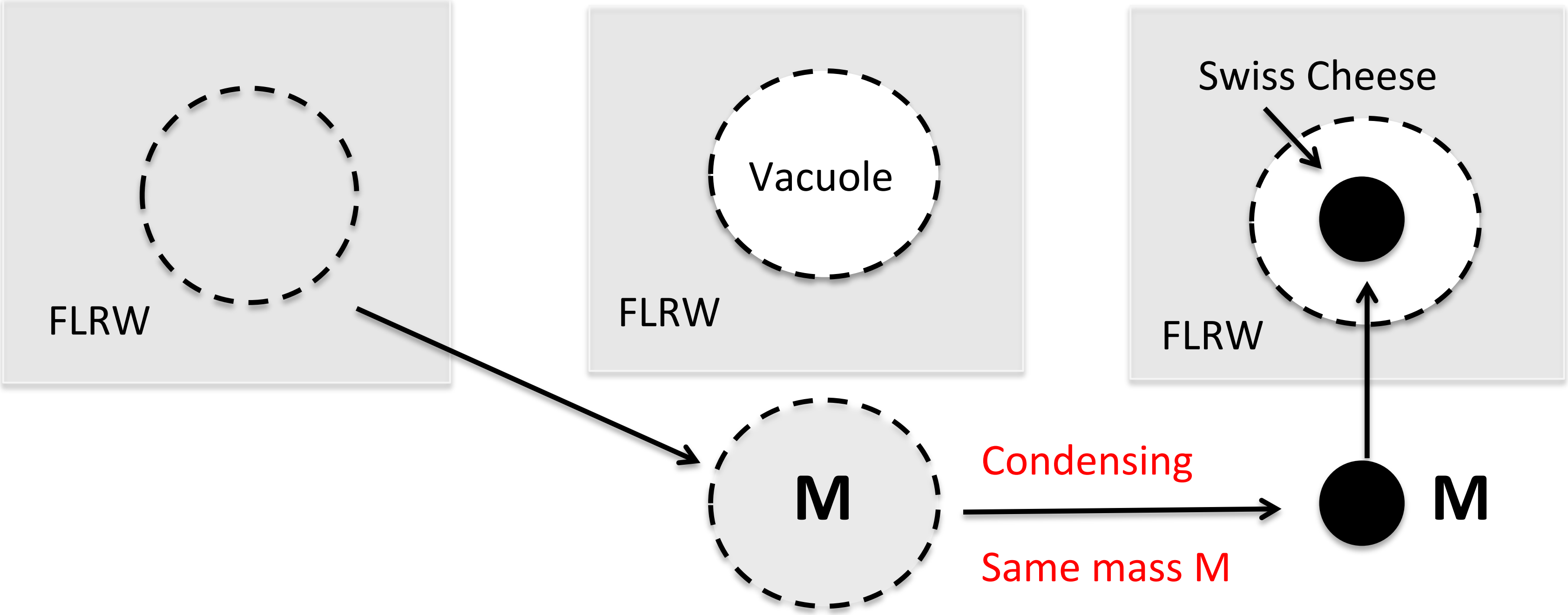


Fig. 1 Build a simple Swiss cheese model

Embedded Lens Theory

- **Embedded Lens Theory** treats a Swiss cheese condensation as a gravitational lens and studies its geometrical optics (Swiss cheese optics).
- **Our goal** is to develop a lens theory which (1) respects **Einstein**'s general relativity; (2) is **accurate** enough to evaluate the effect of Λ on lensing light bending (e.g., Rindler & Ishak, Phys. Rev. D 2007); and (3) **simple** enough to allow practical use.

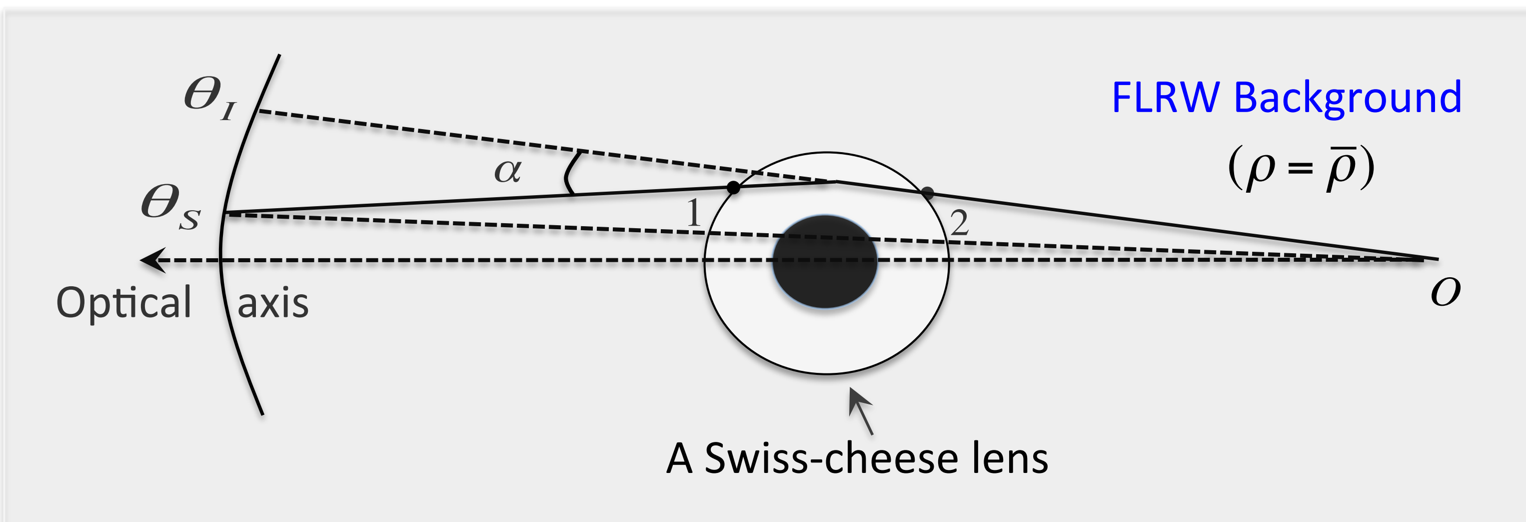


Fig. 2. Embedded Lens Geometry

Our Recent Papers (2010-2013)

1. Kantowski, Chen, & Dai, 2010, ApJ, 718, 913
2. Chen, Kantowski, & Dai, 2010, PRD, 82, 043005
3. Chen, Kantowski, & Dai, 2011, PRD, 84, 083004
4. Kantowski, Chen, & Dai, 2012, PRD, 86, 043009
5. Kantowski, Chen, & Dai, 2013, PRD, 88, 083001
6. Chen, Kantowski, & Dai, 2013a, ApJ, submitted, arXiv1310.6351
7. Chen, Kantowski, & Dai, 2013b, ApJ, submitted, arXiv1310.7574

- By 2013, the previous mentioned research goals have largely been realized.
- Dr. Kantowski will give a review of the embedded lens theory tomorrow.
- I concentrate on our most recent results (side products): investigating the ISW effect, CMB and Void lensing using the embedded lens theory.

Integrated Sachs-Wolfe (ISW) Effect

- **ISW effect** (Sachs & Wolfe 1967 ApJ): the *secondary* anisotropy of the CMB caused by *time-evolving gravitational potential*:

$$\Delta T(\hat{n}) = \frac{2}{c^2}\bar{T}\int_{t_L}^{t_0}\dot{\Phi}(t,r(t),\hat{n})\,dt \qquad (\dot{\Phi}\text{ the rate of the potential})$$

- **Important** because it probes the dynamics of the dark energy, in particular, its negative pressure nature, compensating geometrical probes such as SNe Ia and BAO.
- **Difficult** theoretically because there is not simple way to evaluate the potential, and observationally complicated by primordial anisotropies (larger!) and cosmic variance on large scales.
- Swiss cheese model was unique in studying the late-time ISW effect caused by individual non-linear structures (Rees & Sciama 1968, Nature).

Detect ISW Effect via CMB-LSS Correlation

Author	CMB	LSS Tracer	Wavelength	Method	Claimed detection
Boughn & Crittenden (2002)	COBE	XRB	X-ray	D2	No
Giannantonio et al. (2008)	W3			D2	2.7σ
Boughn & Crittenden (2004, 2005)	W1	XRB/NVSS	X-ray/radio	D2	"tentative" (2–3σ)
Fosalba et al. (2003)	W1	SDSS DR1		D2	2σ (low z) 3.6σ (high z)
Cabré et al. (2006)	W3	SDSS DR4	Optical	D2	$>2\sigma$
Giannantonio et al. (2008)	W3	SDSS DR6		D2	2.2σ
Sawangwit et al. (2010)	W5	SDSS DR5		D2	"marginal"
López-Corredoira et al. (2010)	W5	SDSS DR7		D2	"No detection"
Giannantonio et al. (2006)	W3	SDSS Quasars	Optical	D2	2σ
Giannantonio et al. (2008)	W3	SDSS Quasars		D2	2.5σ
Xia et al. (2009)	W5	SDSS Quasars		D2	2.7σ
Scranton et al. (2003)	W1			D2	$>2\sigma$
Padmanabhan et al. (2005)	W1			D1	2.5σ
Granett et al. (2009)	W3	SDSS LRG	Optical	D1	2σ
Giannantonio et al. (2008)	W3			D2	2.2σ
Sawangwit et al. (2010)	W5	SDSS LRG, 2SLAQ		D2	"marginal"
Sawangwit et al. (2010)	W5	AAOmega LRG		D2	Null
Fosalba & Gaztañaga (2004)	W1	APM	Optical	D2	2.5σ
Afshordi et al. (2004)	W1			D1	2.5σ
Rassat et al. (2007)	W3	2MASS	NIR	D1	2σ
Giannantonio et al. (2008)	W3			D2	0.5σ
Francis & Peacock (2010b)	W3			D1	"weak"
Boughn & Crittenden (2002)	COBE			D2	No
Nolta et al. (2004)	W1			D2	2.2σ
Pietrobon et al. (2006)	W3	NVSS	Radio	D3	$>4\sigma$
Vielva et al. (2006)	W3			D3	3.3σ
McEwen et al. (2007)	W3			D3	$>2.5\sigma$
Raccanelli et al. (2008)	W3			D2	2.7σ
McEwen et al. (2008)	W3			D3	$\sim 4\sigma$
Giannantonio et al. (2008)	W3			D2	3.3σ
Hernández-Monteagudo (2010)	W3			D1	$<2\sigma$
Sawangwit et al. (2010)	W5			D2	"marginal" ($\sim 2\sigma$)
Corasaniti et al. (2005)	W1			D2	$>2\sigma$
Gaztañaga et al. (2006)	W1			D2	2σ
Ho et al. (2008)	W3	Combination	Combination	D1	3.7σ
Giannantonio et al. (2008)	W3			D2	4.5σ

Table 1. ISW detection through correlation of CMB with local tracer of mass in different wavelengths: X-ray, Optical, NIR, and radio. Refer to Dupe et al. 2011 A&A 534, 51.

ISW Detection by Void-Stacking Method

- Cosmic voids are under-dense regions in the Universe (refer to the talks by Paul and Alice yesterday)
- Temperature of a CMB photon will be reduced after traversing a void (i.e., void yields cold spot).
- ISW effect can also be detected by stacking a large number of voids.
- Granett et al. 2008 ApJL claimed a detection at 4σ by stacking 50 largest voids at redshift ~0.5.

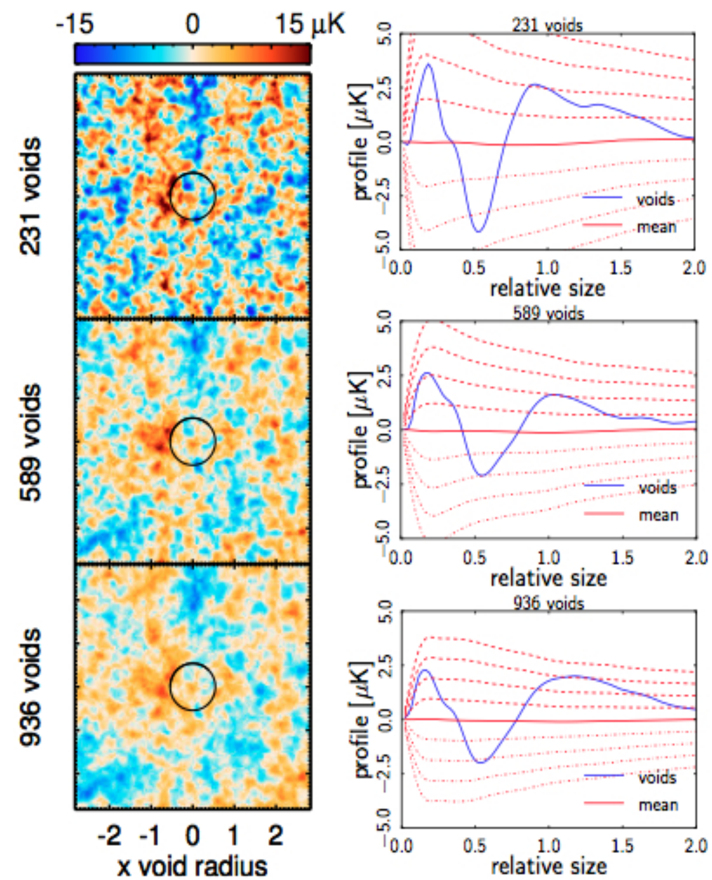
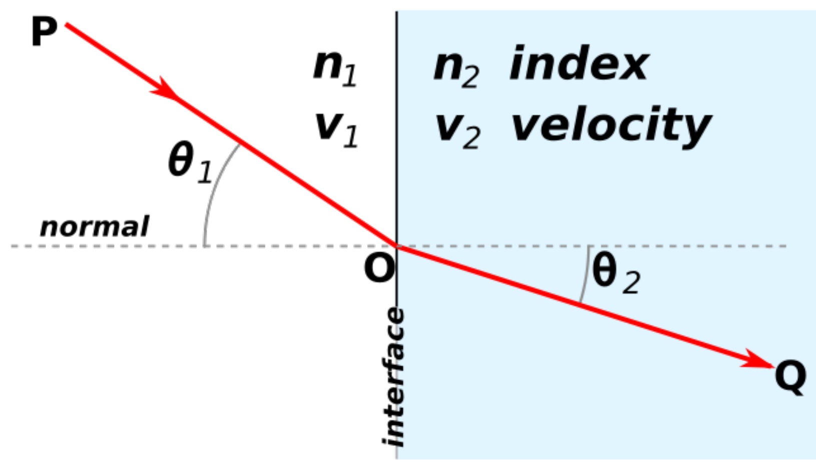


Fig.3 Planck Detection of ISW by Void-Stacking

Fermat's Least Time Principle

- **Fermat's principle**: the light path between two points is the one that takes the *shortest* time.
- For example: Snell's law can be proved using Fermat's principle



Pierre de Fermat
(1601-1665)

Fig. 4. Snell's Law: $\frac{Sin\ \theta_1}{Sin\ \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$

Fermat's Principle In Gravitational Lens Theory

- **Lensing Time Delay**: a lensed photon arrives at us at a delayed time. The total time delay contains a geometrical and a potential part (Cooke & Kantowski 1975, ApJL), $\Delta T = \Delta T_{geo} + \Delta T_{pot}$
- **Fermat potential:** the total time delay is also called the Fermat potential because minimizing it gives the gravitational lens equation (Schneider 1985, A&A; Blandford & Narayan 1986, ApJ).
- Kantowski et al. (2013) derived the Fermat potential for *embedded* spherical lenses and proved the Fermat principle for the embedded lens theory:

$$\phi(\theta_S,\theta_I,z_d) = (1+z_d)\frac{D_d D_s}{D_{ds}}\left[\frac{(\theta_S-\theta_I)^2}{2} - \theta_E^2\int_x^1 \frac{f(x',z_d)-f_{\rm RW}(x')}{x'}dx'\right] \quad (1)$$

$$\frac{\delta\phi}{\delta\theta_I} = 0 \quad \Rightarrow \quad \text{Embedded lens equation} \quad y = x - \left(\frac{\theta_E}{\theta_M}\right)^2 \frac{f(x)-f_{\rm RW}(x)}{x} \quad (2)$$

Note : y, x are normalized source and image angle, and

$$f(x) = M(<\theta_I)/M; \quad f_{RW} = 1-(1-x^2)^{3/2}$$

ISW Effect via Fermat Potential

- For an *individual* embedded spherical lens, the ISW effect can be expressed as (Chen, Kantowski, & Dai 2013a)

$$\frac{\Delta T}{\bar{T}}(\theta_I) = H(z_d)\frac{\partial \phi}{\partial z_d}, \tag{3}$$

$$\phi(\theta_I, z_d) \text{ is the Fermat potential, } z_d \sim t_d \sim R(t_d)$$

- Equivalently, in terms of the density contrast δ of the Swiss cheese lens,

$$\frac{\Delta T}{T} = \frac{6r_s}{c/H_d}\int_x^1 \frac{dx'}{x'}\int_0^{x'} dy\, y \int_0^{\sqrt{1-y^2}} d\zeta\, \delta(\sqrt{y^2+\zeta^2}, R_d)\left[1 - \frac{R_d}{\delta}\frac{\partial \delta}{\partial R_d}\right] \tag{4}$$

$$\delta = (\rho - \bar{\rho})/\bar{\rho}$$

Remark 1. The above expression is by far the simplest formula for computing the ISW effect caused by a single spherical inhomogeneity (Dyer, 1976 ApJ; Martinez-Gonzalez et al., 1900 ApJL; Inoue & Silk 2006 ApJ).
Remark 2. Need input astrophysical evolution (collapsing/exploding) theories (e.g.,Bertschinger 1985a, 1985b, ApJS), but no Fourier transform.

Example: ISW Effect Caused by Compensated Voids (Chen, Kantowski, Dai 2013a, arXiv1310.6351)

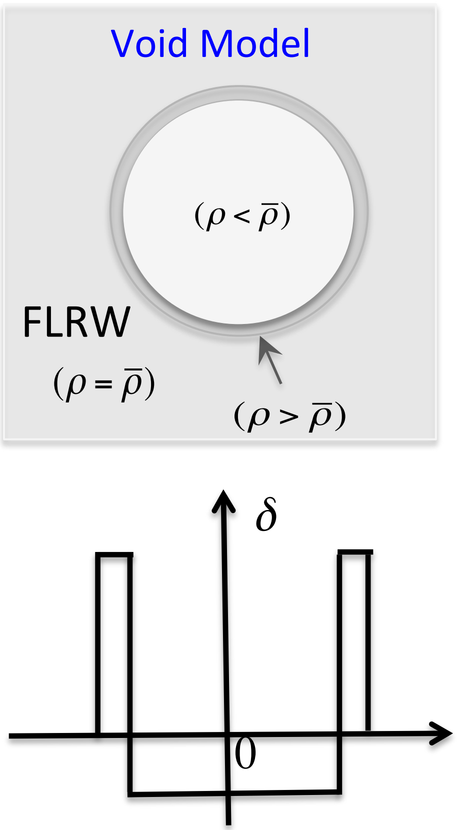


Fig. 5. A simple void model: Homogenous under-dense interior compensated by an over-dense thin shell.

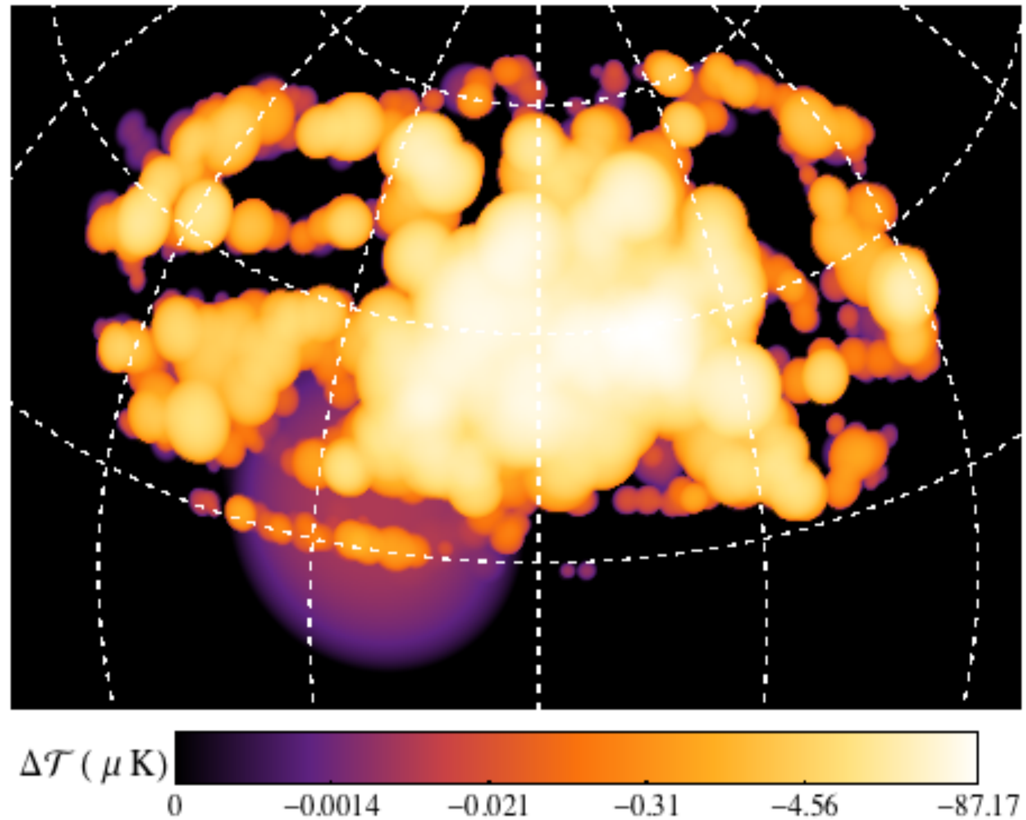


Fig. 6. ISW cold spots using void catalog of Sutter et al. 2012, ApJ assuming the simple void model.

$$\frac{\Delta T}{T} = \delta \frac{2r_s}{3c/H(z_d)}(1-x^2)^{3/2}$$

Anti-Lensing by Cosmic Void

- Anti-lensing: light is repelled (instead of attracted) when traversing an under-dense region (a void)

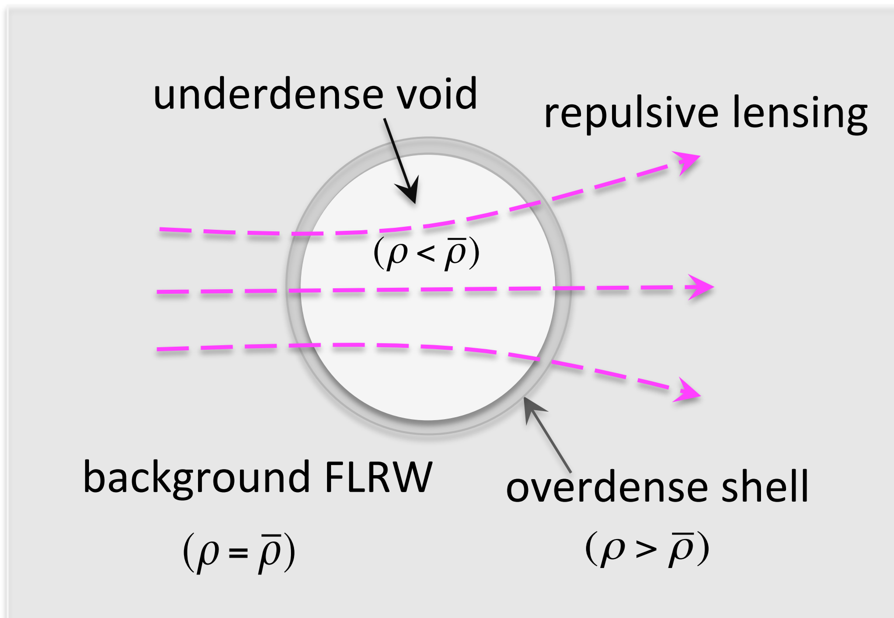


Fig. 7 Anti-Lensing by A Void

Embedded Anti-Lensing by Cosmic Voids
(Chen, Kantowski & Dai 2013b, arXiv1310.7574)

- Embedded theory can be used to study anti-lensing by cosmic voids.
- Embedded void lens equation for the simple compensated void model:

$$\boldsymbol{y} = \boldsymbol{x}\,[1+\zeta(1-\boldsymbol{x}^2)^{1/2}], \qquad (5)$$

$$\zeta = \xi(1+z_d)^2\Omega_{\rm m}^{2/3} r_s^{1/3}\frac{2D_{ds}D_d}{(c/H_0)^{4/3}D_s}$$

Example. For a large void of radius 4 degree at redshift 0.5, with density contrast δ = -0.9 (very deep), the lens strength is about 0.01, the bending angle is at sub-arcminute level

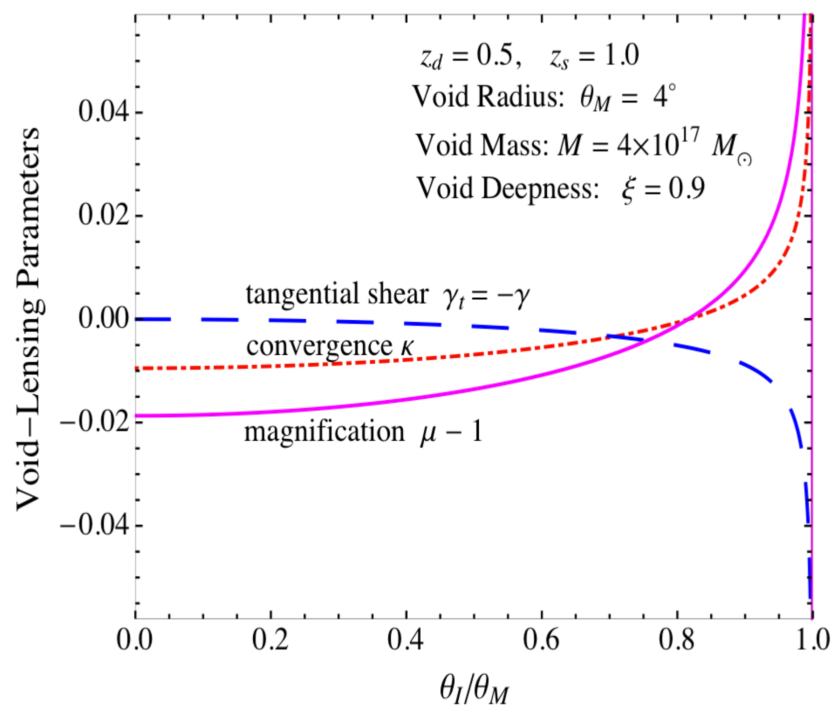


Fig. 8 Weak Void-Lensing Parameters

Simulated Void-Lensed CMB Map

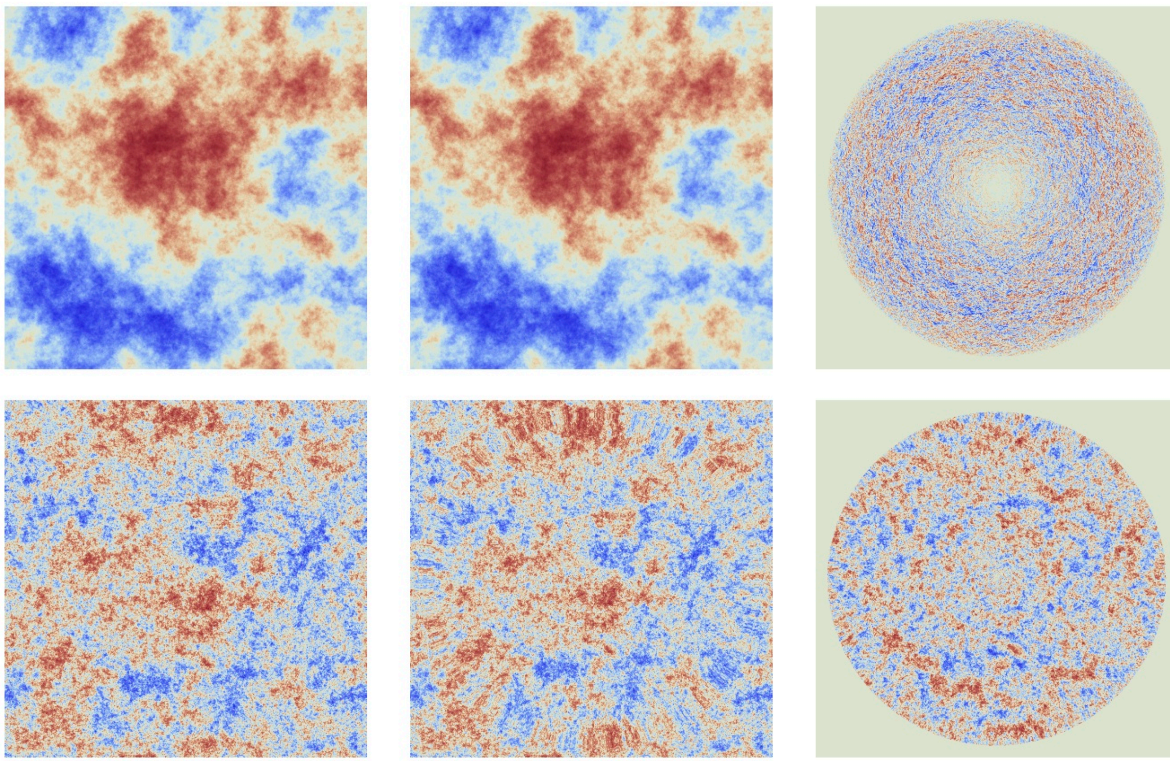


Fig. 9. Simulation of Void lensed CMB. The lens strength is amplified by a factor of 20 for the 2nd row.

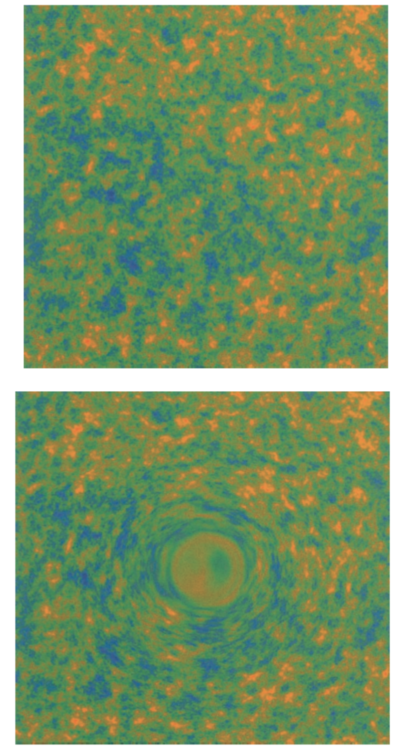


Fig 10. CMB lensing by clusters (W. Hu 2001, ApJL)

Conclusions

1. We developed "embedded lens theory", it is simple enough to allow practical use.
2. We presented a simple formulae for the ISW effect, and modeled ISW effect caused by cosmic voids.
3. We applied embedded lens theory to study weak and CMB lensing by cosmic voids.
4. We invite you to further test and use our theory, and we expect many useful applications in future.