Cosmology in One Dimension: Correlation, Power Spectra and Void Geometry

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Model Assumptions

- Starting from matter fluctuations, the structure of the universe presents now a hierarchical structure
- We will use a toy model to follow its evolution after recombination and later, so Newtonian dynamics applies
- We restrict to 1D gravitating system
- And use an N-body description

System description 1/2

- Equation of motion

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = E(x,t)$$

- Introducing Scaled Space and Time

$$x = C(t)x' \quad \mathrm{d}t = A^2(t)\mathrm{d}t'$$

with

$$C(t) = (\alpha\omega_{J0}t)^{2/3} \text{ and } A^2(t) = (\alpha\omega_{J0}t)$$

- Transform the equation of motion

$$\frac{\mathrm{d}^2 x'}{\mathrm{d}t'^2} + \frac{1}{3}\alpha\omega_{J0}\frac{\mathrm{d}x'}{\mathrm{d}t'} - \frac{2}{9}(\alpha\omega_{J0})^2 x' = E'(x',t')$$

- The force proportional to x' will be taken as given by a neutralizing background

System description 2/2

- Here, we consider a planar perturbation. So $i = 1, N$ infinite plane sheets.
- For an initially 1-D planar problem we have

$$\frac{\mathrm{d}^2 x_i''}{\mathrm{d}t'^2} + \frac{1}{\sqrt{2}}\omega_{J0}\frac{\mathrm{d}x_i''}{\mathrm{d}t'} - \omega_{J0}^2 x_i'' = E_i''$$

- For an initially 1-D cylindrical problem we have

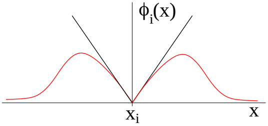
$$\frac{\mathrm{d}^2 x_i''}{\mathrm{d}t'^2} + \frac{1}{2}\omega_{J0}\frac{\mathrm{d}x_i''}{\mathrm{d}t'} - \omega_{J0}^2 x_i'' = E_i''$$

- For an initially 1-D spherical problem we have

$$\frac{\mathrm{d}^2 x_i''}{\mathrm{d}t'^2} + \frac{1}{6}\omega_{J0}\frac{\mathrm{d}x_i''}{\mathrm{d}t'} - \omega_{J0}^2 x_i'' = E_i''$$

- The friction coefficient α depends on the initial system geometry
- System with periodic boundary conditions
  $\rightarrow$ periodic potential exactly solved

Periodic system



- For a single particle i, the potential $\phi_i(x)$ reads :

$$\begin{aligned}\phi_i(x) &= \frac{\sigma}{2}\,|x-x_i|\,\exp(-\kappa|x-x_i|)\\ &= -\frac{\sigma}{2}\,\frac{\mathrm{d}}{\mathrm{d}\kappa}\exp(-\kappa|x-x_i|)\end{aligned}$$

M. Kiessling : 2003, Adv. Appl. Math. **31**

- For a periodic system of length $2L$, taking into account all replica :

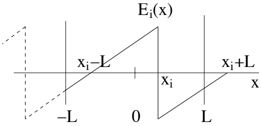
$$\begin{aligned}\phi_i(x) &= -\frac{\sigma}{2}\sum_n \frac{\mathrm{d}}{\mathrm{d}\kappa}\exp(-\kappa|x-(x_i-2nL)|)\\ &\sim \frac{\sigma}{2}\left[|x-x_i|\exp(-\kappa|x-(x_i-2nL)|)+\frac{1}{\kappa^2 L}-\frac{1}{2}(x-x_i)^2\right]\end{aligned}$$

- Substracting the neutralizing background contribution is required to obtain a convergent potential

$$\phi_{BG}(x) = -\int_{-\infty}^{\infty}\rho_{BG}(x')\,|x-x'|\exp(-\kappa|x-x'|)\,\mathrm{d}x' = -\frac{\sigma}{2\kappa^2 L}$$

with $\rho_{BG} = -\frac{\sigma}{2L}$.

Periodic system



- The total field for a single particle reads, taking $\kappa \to 0$

$$E_i(x) = -\frac{\sigma}{2}\mathrm{sign}(x - x_i) + \frac{\sigma}{2L}(x - x_i)$$

- Summing over all particles, the field for x in the primitive cell reads

$$E(x) = \sum_i E_i(x) = \frac{\sigma}{2}\left[N_{right}(x) - N_{left}(x) + \frac{N}{L}(x - x_c)\right]$$

$x_c = \frac{1}{N}\sum_{i=1}^{N} x_i$: center of mass, keeps the field constant when a particle leaves the system while another enters form the other side.

B.N. Miller and J.L. Rouet : 2010, PRE **82**, 6

Periodic system : symmetry based derivation

- Poisson's equation for a single particle, background included :

$$\frac{\mathrm{d}E_i}{\mathrm{d}x} = -\sigma\,\delta(x - x_i) + \frac{\sigma}{2L}$$

- The general solution reads

$$E_i(x) = -\sigma\,\Gamma(x - x_i) + \frac{\sigma}{2L}\,x + C$$

- Global neutrality on $\pm L$ gives $C = \frac{\sigma}{2} - \frac{\sigma}{2L}\,x_i$

$\Rightarrow$ So

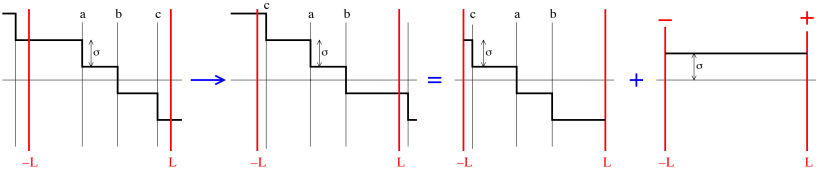
$$E_i(x) = -\frac{\sigma}{2}\mathrm{sign}(x - x_i) + \frac{\sigma}{2L}\,(x - x_i)$$

Periodic system : Polarized boundaries

- Another point of view is to write

$$E_i = \frac{\sigma}{2}\,[N_{right}(x) - N_{left}(x)] + E_B$$

where E_B is the boundary polarization field.



Particle c goes out from right and enters left

$\frac{\sigma}{2}\,[N_{right}(x) - N_{left}(x)]$

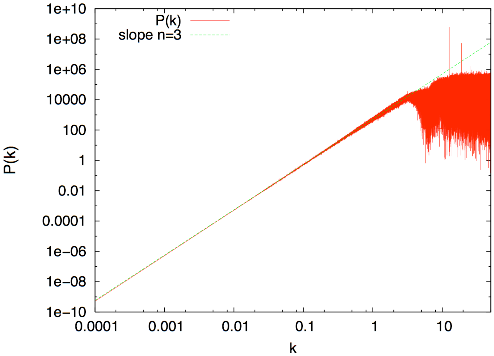
E_B

- Plasma case identical

Numerical Simulation : Initial Condition

- Importance of P(k) to characterize fluctuations
- Power law provides scale-free behavior of primordial Gaussian density fluctuations
- Particles are shifted from their equilibrium position in order to have $P(k) \sim k^n$
- Velocities are connected to the displacement according to the growing mode of the system (trajectories in phase space)
- Figures given for the RF-model and $n = 3$
  - The RF-model is a mathematicaly consistent 1D model
  - $n = 3$ is the most chaotic choice, corresponds to $n = 1$ in 3D (J.A. Peacock, Cosmological Physics)

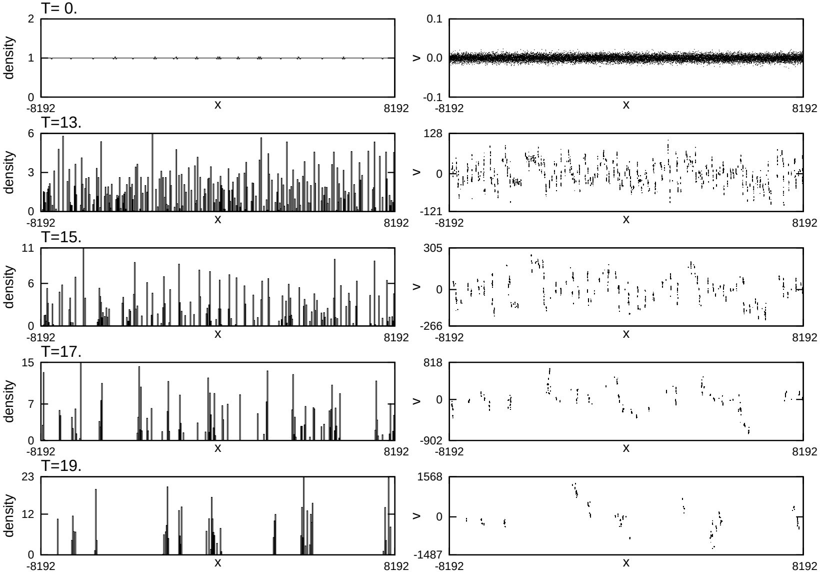
$P(k)$, $n = 3$ at $T = 0$



$\rightarrow$ $P(k) \sim k^n$ with $n = 3$

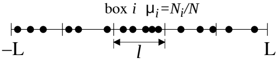
- Require $n > 0$ to obtain a hierarchical structure
- Simulation with $N = 65535$ particles

A Simulation Result



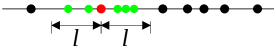
Multifractal Analysis

- Box Counting Method



$$B(l,q) = \frac{1}{N_{box}(l)} \sum_{i=1}^{N_{box}} \mu_i(l)^q \sim l^{\tau_q}$$

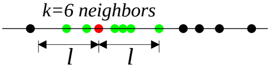
- Correlation-Integral (or Point-Wise Dimension method)



$$Z(l,q) = \frac{1}{N} \sum C_i(l)^{q-1} \sim l^{\tau_q}$$

$NC_i(l)$ number of particles at a distance l

- Density-reconstruction (or k-neighbor method)

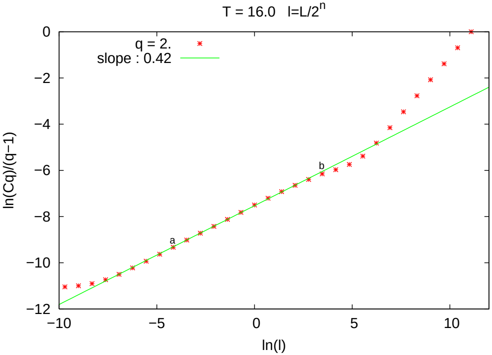


$$W(p,\tau_q) = \frac{1}{N} \sum R_i(p)^{-\tau_q} \sim p^{q-1}$$

R_i contains $k = pN$ points

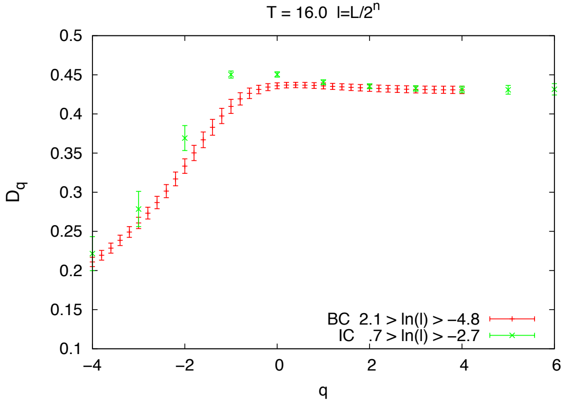
- $\tau_q = (q-1)\, D_q$ where D_q is the generalized dimension of order q.
- $q > 0$ emphasizes high density region, $q < 0$ emphasizes low density region
- D_2 is the correlation dimension

Simulation results : Box counting



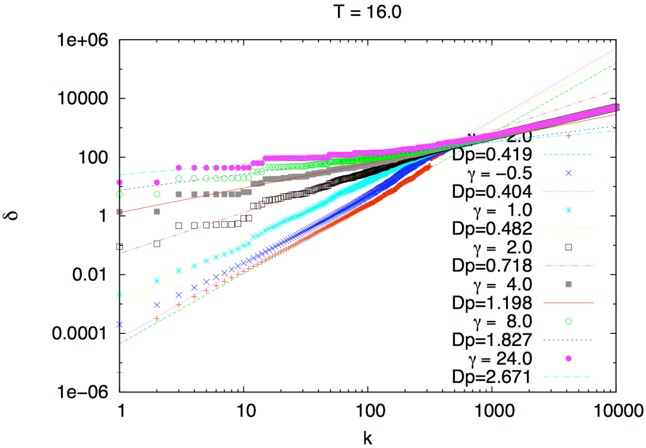
- A scaling range around $l = 1$ thanks to the friction
- Two trivial scaling ranges
  - for large l : the slope is 1 due to homogeneity
  - for small l : the slope is 0 due to the discretzation (finite number of particles)

Simulation results : Box counting and Correlation-Integral



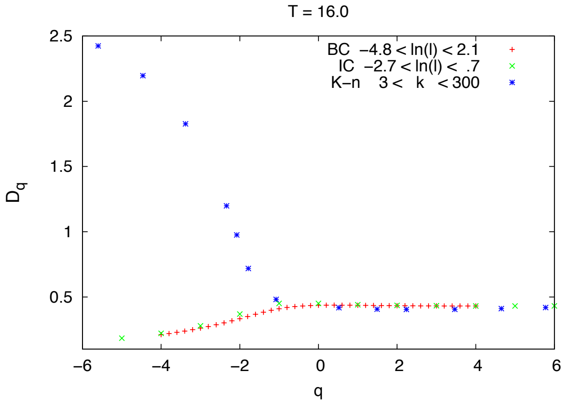
- Increasing curve !
- For $q \geq 0$ the curve decreases very slightly
- Similar results with Box counting and Correlation-Integral methods

Simulation results : Density-Reconstruction



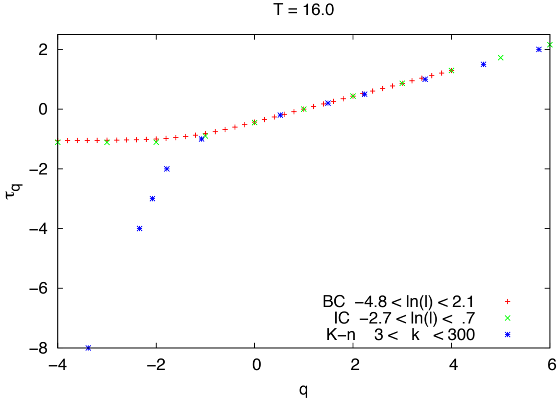
- Two scaling ranges
- The cutoff is increasing with time
- All curves gather for large k which correspond to homogeneity

D_q : Comparison of the 3 methods



- D_q is Decreasing for the DR method
- Similar results with Box counting and Correlation-Integral method for $q > 0$

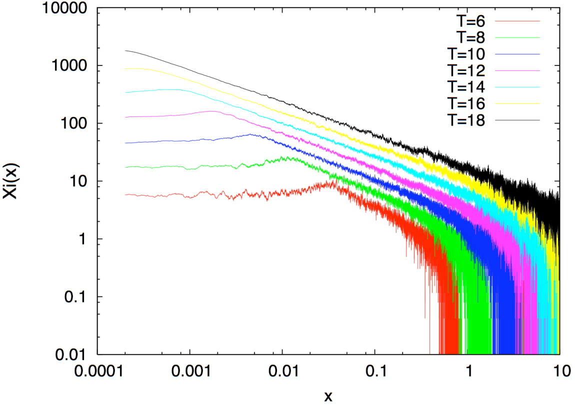
τ_q : Comparison of the 3 methods



- $\tau_q = (q - 1)\, D_q$
- The curve τ_q suggests a Bi-fractal

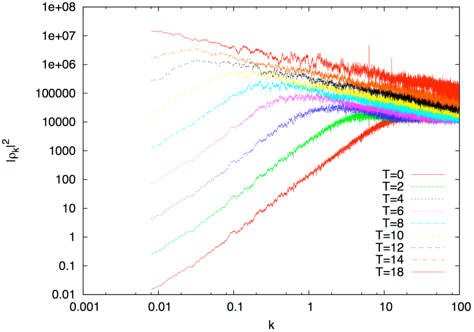
(R. Balian and R. Schaeffer : 1989, Astron. Astrophys. **226** 1)

Correlation : Time evolution



- The slope n gives $D_2 = 1 + n$
- For $T = 16$ $n = -.58$ for $D_2 = .40$

Power spectrum evolution



Transition between linear and non-linear regime :
$k_c(t) \sim \exp(-rt/(n_l - n))$

$$\text{with } r = \gamma(-1 + \sqrt{1 + 4/\gamma^2})$$

B.N. Miller, J.L. Rouet : 2010, Phys Rev E **82** 6, B.N. Miller, J.L. Rouet : 2010, JSTAT, P12028

Conclusion

- A 1D toy model including expansion and gravitation
- 1D models allow
  - to use the gravitational field without cutoff
  - to deal with a high number of particles (here 65535)
- Show a hierachical formation structure in μ-space
- The analysis is performed on the projection in configuration space
- Using large data sets, robust scaling regimes are observed for both low and high density region
- The apparent fractality that arises in observations is a projection from six dimensions
- Share similar fractal properties with observations and 3D simulations (apparent bifractal geometry)
- This remains true for other models (changing the friction coefficient) and other Initial Conditions
- The time evolution of the scalings (power spectrum, correlation) are consistant with universe observations