RADIO OBSERVATIONS OF THE TIDAL DISRUPTION SWIFT J1644+57

Video credit: NASA/Goddard Space Flight Center/CI Lab

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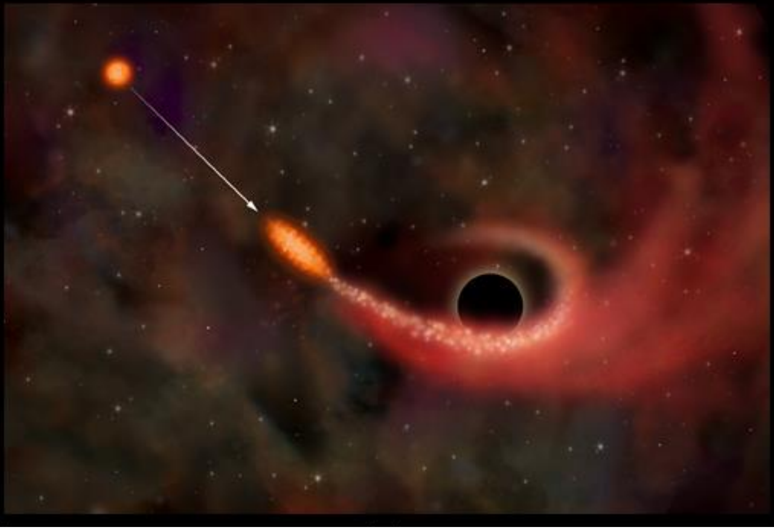
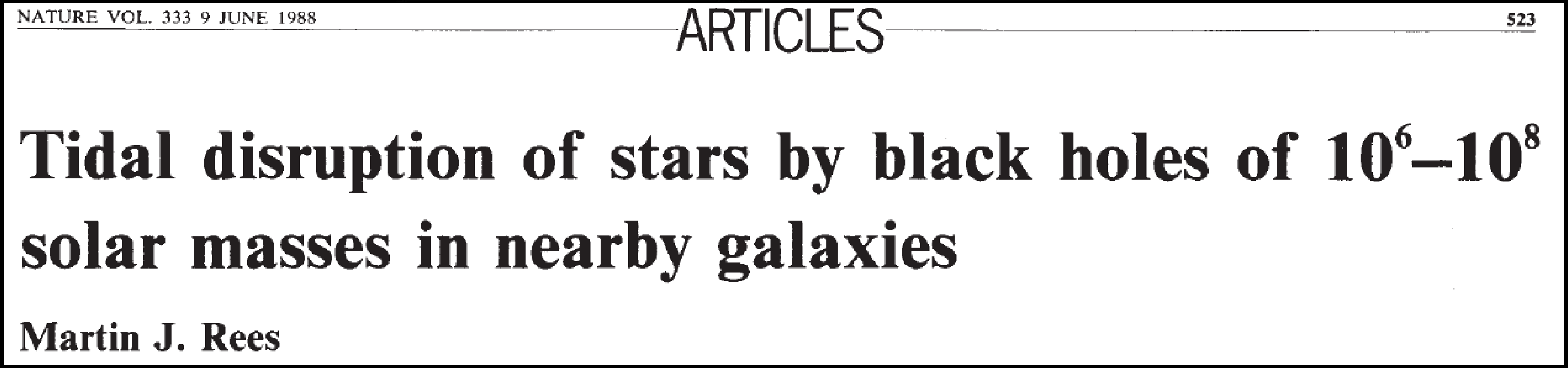
TX Symposium 2013

E. Berger, A. Soderberg, R. Sari, A. Loeb, R. Narayan, D. Frail, G. Petitpas, A. Brunthaler, R. Chornock, J. Carpenter, G. Pooley, K. Mooley, S. Kulkarni, R. Margutti, D. Fox, E. Nakar, N. Patel, M. Bietenholz, M. Rupen, R. Duran

Main Points

- Observations to-date of Sw1644+57 are consistent with a TDE interpretation
- Entire life cycle of a relativistic jet from birth to cessation observed (?)
- Radio observations contribute unique and complementary information (e.g. BH environment / density structure)
- Radio observations have implications for rates (e.g. off-axis events) of TDEs with relativistic jets

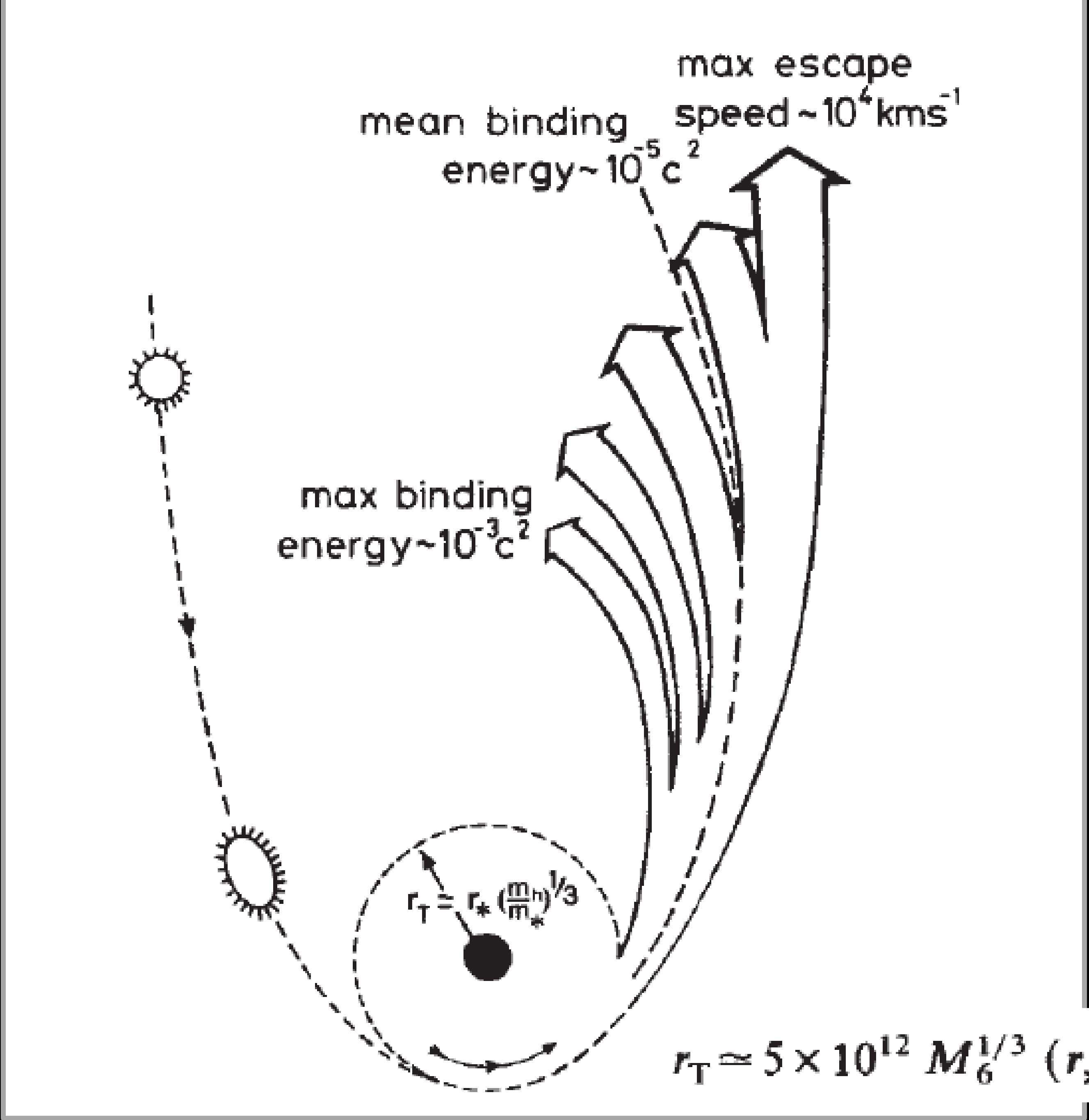




NATURE VOL. 333 9 JUNE 1988 ARTICLES 523

Tidal disruption of stars by black holes of 10^6–10^8 solar masses in nearby galaxies

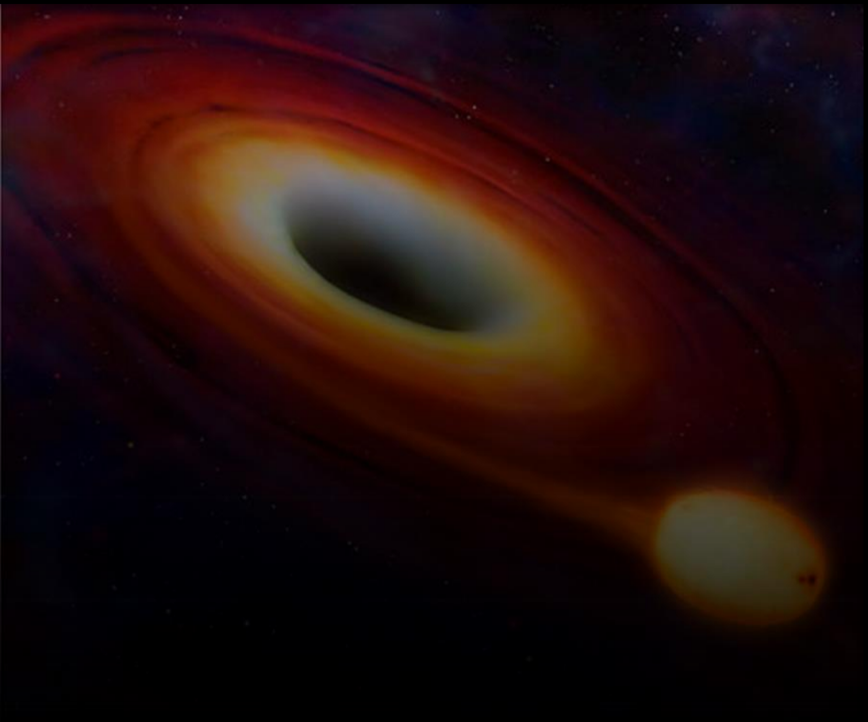
Martin J. Rees



$$r_\mathrm{T} \simeq 5 \times 10^{12}\, M_6^{1/3}\, (r_*/r_\odot)\, (m_*/m_\odot)^{-1/3}\ \mathrm{cm}$$

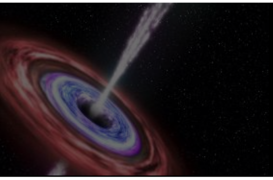
Rees 1988 (Nature)

Tidal Disruption:
Probing Previously Quiescent Supermassive Black Hole



Theoretical and observational:
Rees, Gezari, van Velzen, Komossa

Sw1644+57
J2058.4+0516 (Cenko et al. 2012)
X-ray selected (Bower et al. 2013)

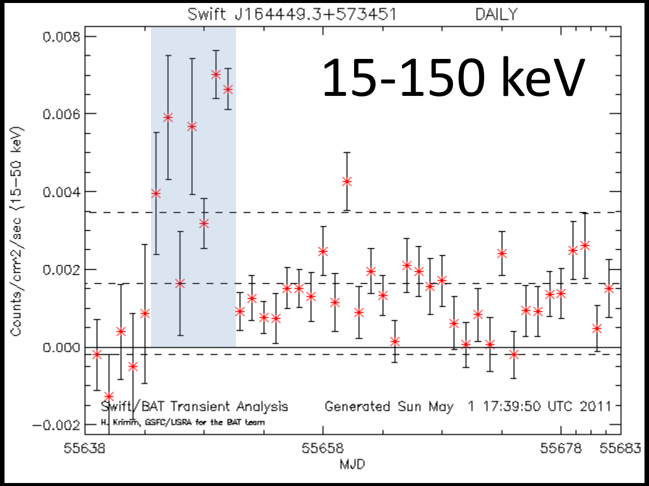


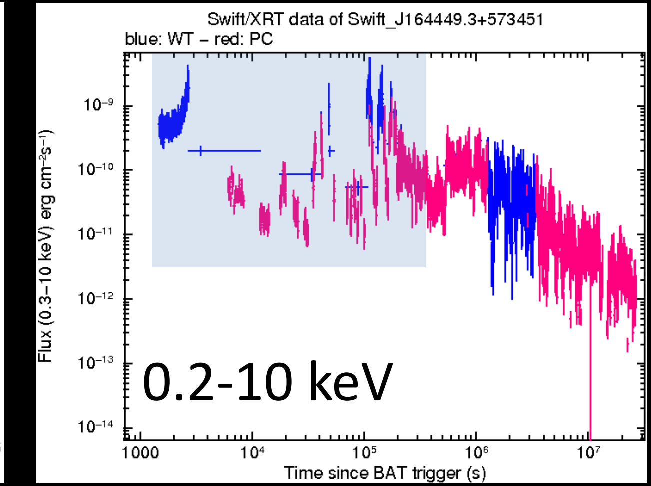
Sw1644+57: Overview

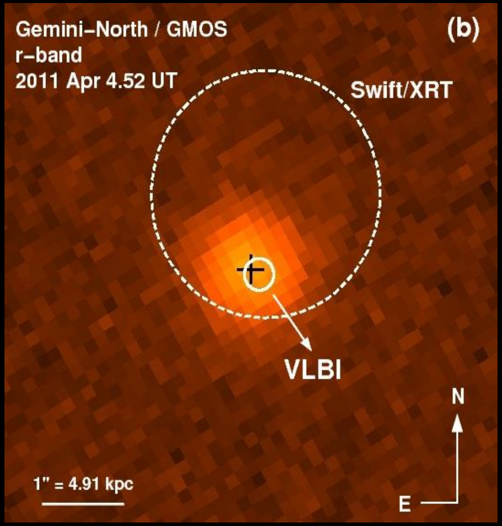
- *Swift* discovery on 2011 March 28.5 UT; multiple triggers; emission on 3/25

Burrows et al. Nature (2011), Levan et al. Science (2011)

- Long-lived X-ray source; $L \propto t^{-5/3}$ at >10 days; beaming of 0.1 rad; extinction







Early radio observations *(Zauderer et al. 2011, Nature, 476, 425)*:

- Linked the γ-ray/X-ray transient to the nucleus of a galaxy at $z = 0.354$
- Demonstrated a relativistic outflow (equipartition, interstellar scintillation)
- Tied the outflow formation time to the onset of γ-ray emission

Interpretation: tidal disruption of a star by a ~ few × 10^6 $M_\odot$ SMBH

see also Bloom et al. Science (2011), Castro-Tirado et al. 2012

Fallback to the Black Hole: Simple picture

- Energy distribution + Kepler's Laws give mass fallback rate

$$\dot{M}_{\text{fallback}} = \frac{dm}{dt} \sim \boxed{\frac{dm}{d\epsilon}}\,\frac{d\epsilon}{dt} \propto \frac{d\epsilon}{dt}$$

assume const
(square const-density star)

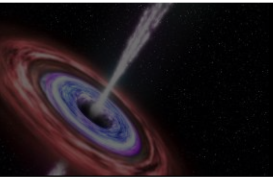
Keplerian potential: $\epsilon \sim \frac{GM_{\rm BH}}{a}$ $\quad P \propto a^{3/2}$ $\quad a \propto P^{2/3}$

$$\epsilon \propto P^{-2/3}$$

$$\frac{d\epsilon}{dt} \propto P^{-5/3}$$

$$\Rightarrow \dot{M}_{\text{fallback}} \propto t^{-5/3}$$

Slide Courtesy Linda Strubbe



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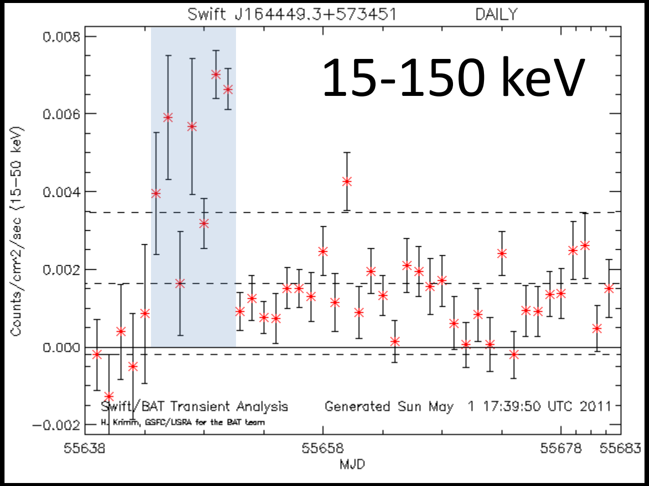

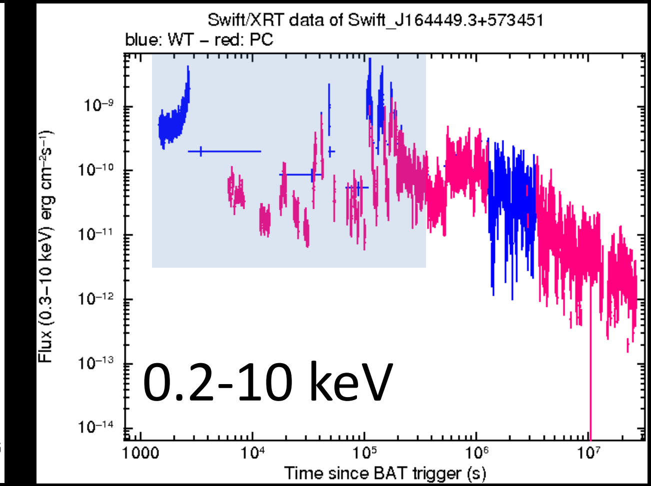

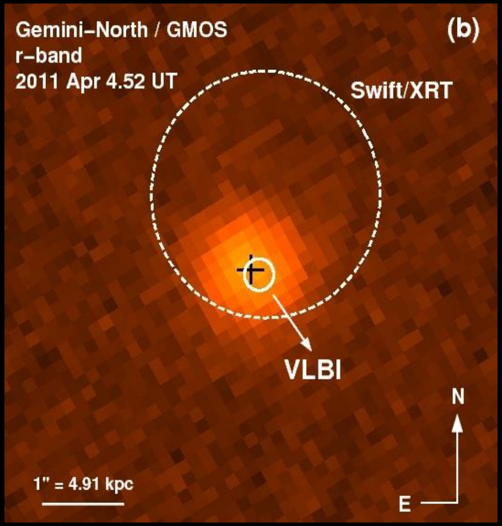

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Summary of Radio Monitoring Program

- Frequencies from < 1 GHz – 345 GHz
  - GMRT (< 1 GHz)
  - EVLA (L, C, X, K, Ka and Q bands)
  - VLBI (K band)
  - AMI-LA (15 GHz)
  - CARMA (95 GHz)
  - SMA (230, 345 GHz)

- Light curve sampled from 0.28 ~ 1000 days since Swift trigger / Fluxes published

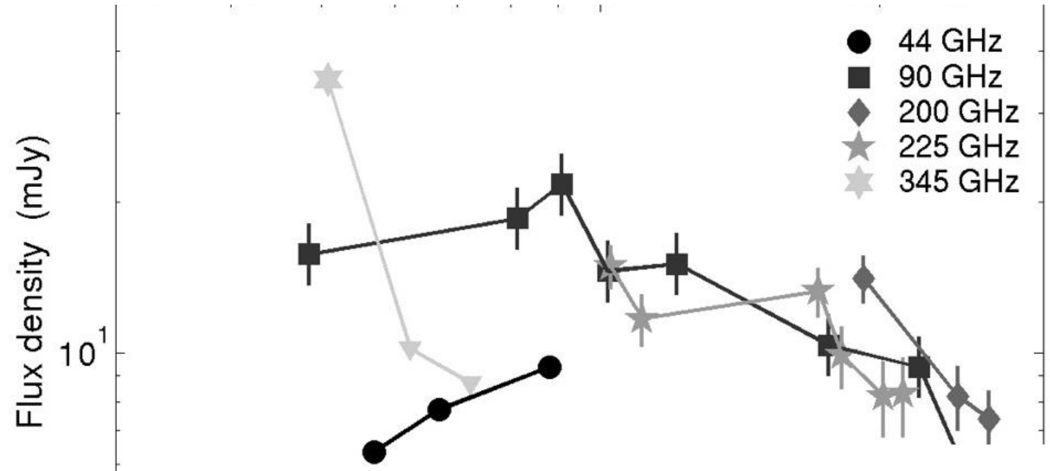
  Zauderer et al. 2011

  Berger et al. 2012, ApJ, Paper I

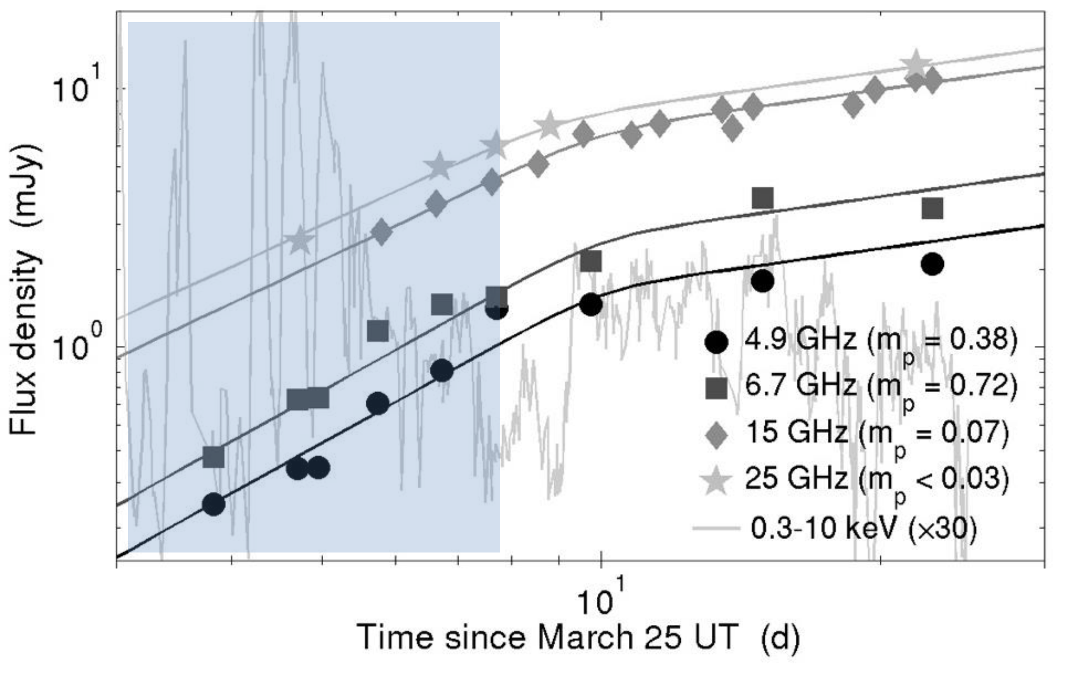
  Zauderer et al. 2013, ApJ, Paper II

- Full polarization calibration with EVLA (2 GHz bandwidth – R. Perley et al.)

Sw1644+57: Early Light Curve



Large variations in X-rays are not accompanied by variations in radio



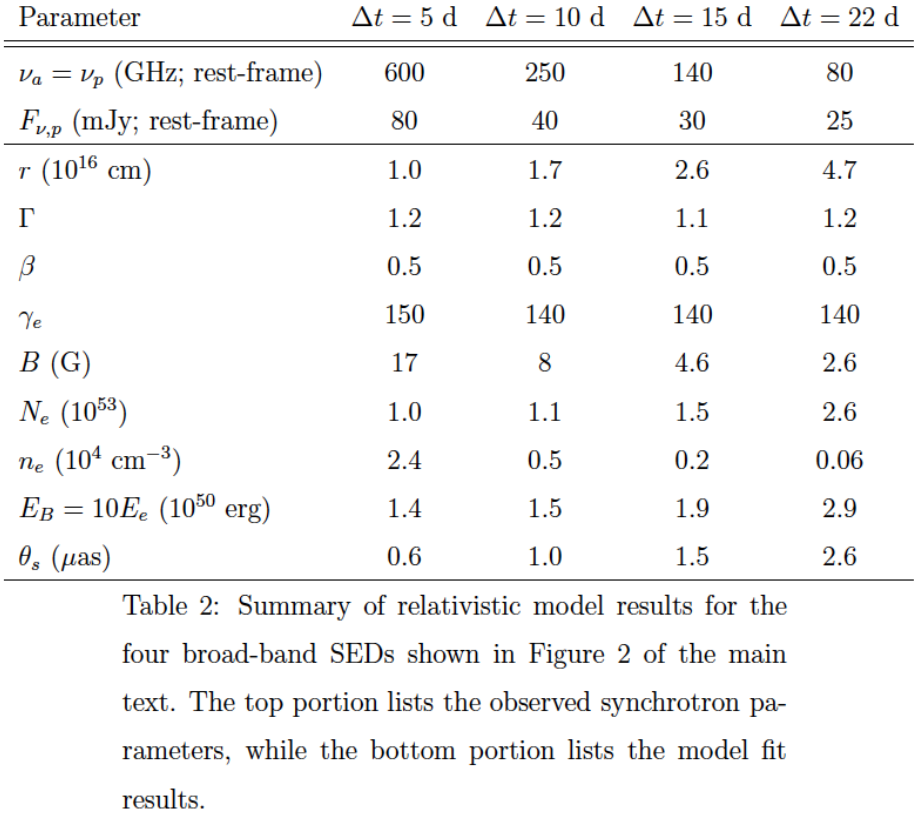
⇒ X-ray emission region distinct from radio emission region

Zauderer et al.

Sw1644+57: Synchrotron Modeling

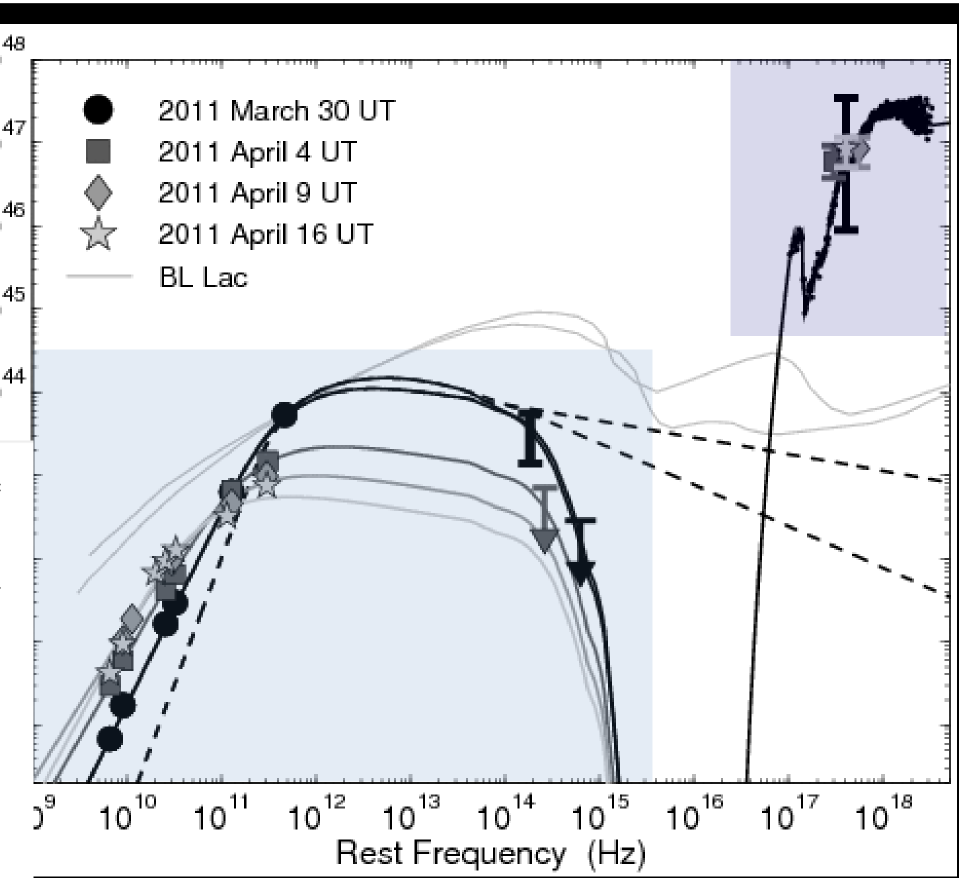
Assumptions: peak freq = abs. freq
equipartition, spherical

Calculate: radius, Lorentz factor, magnetic field strength, electron number density, angular size



Parameter	$\Delta t = 5$ d	$\Delta t = 10$ d	$\Delta t = 15$ d	$\Delta t = 22$ d
$\nu_a = \nu_p$ (GHz; rest-frame)	600	250	140	80
$F_{\nu,p}$ (mJy; rest-frame)	80	40	30	25
r (10^{16} cm)	1.0	1.7	2.6	4.7
Γ	1.2	1.2	1.1	1.2
β	0.5	0.5	0.5	0.5
γ_e	150	140	140	140
B (G)	17	8	4.6	2.6
N_e (10^{53})	1.0	1.1	1.5	2.6
n_e (10^4 cm^{-3})	2.4	0.5	0.2	0.06
$E_B = 10E_e$ (10^{50} erg)	1.4	1.5	1.9	2.9
θ_s (μas)	0.6	1.0	1.5	2.6

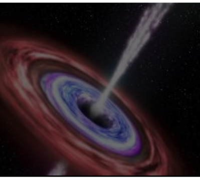
Table 2: Summary of relativistic model results for the four broad-band SEDs shown in Figure 2 of the main text. The top portion lists the observed synchrotron parameters, while the bottom portion lists the model fit results.



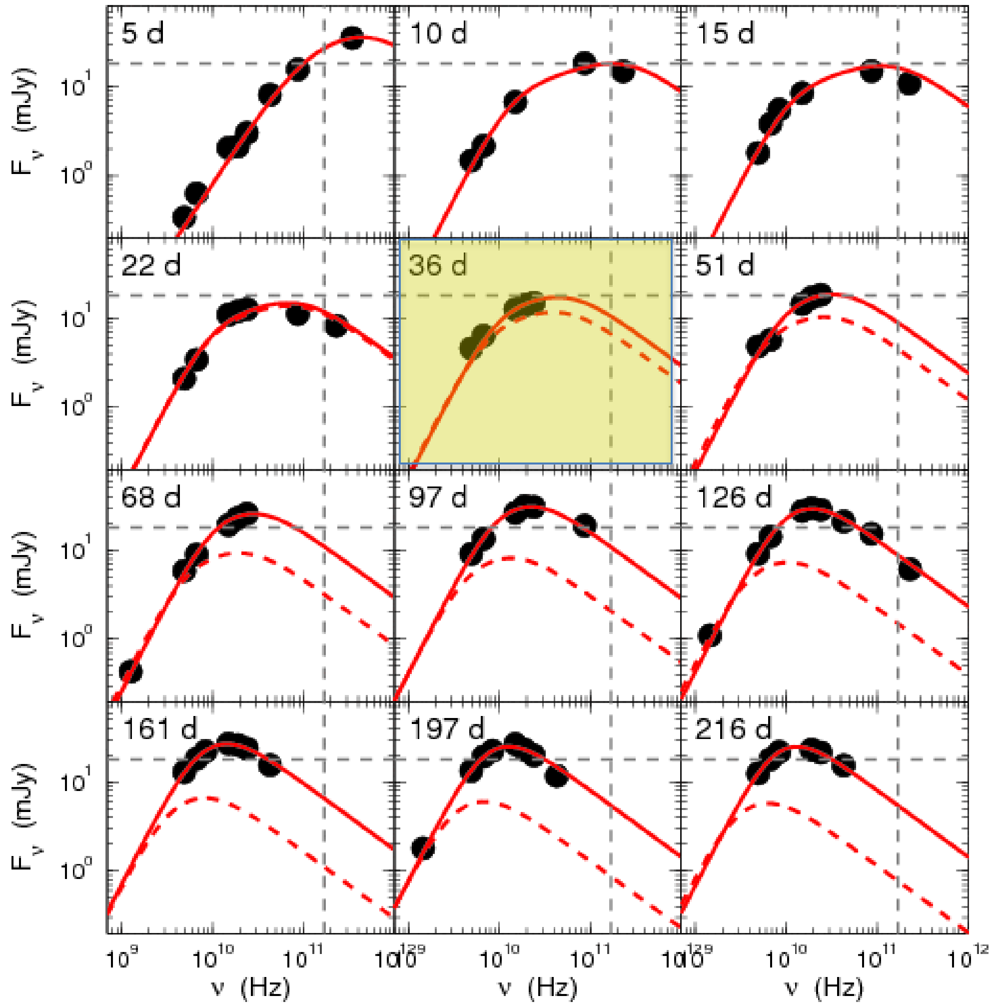
Zauderer et al.

- Radio spectrum ⇒ synchrotron
- optical limits ⇒ $A_V > 3$ mag
- L_X exceeds L_{syn} by ~10^3

See also Bloom et al. Science (2011), Castro-Tirado et al. 2012



Sw1644+57: Synchrotron Modeling



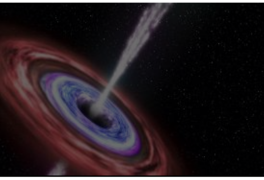
No assumptions about hydrodynamics

Model each “snapshot” spectral energy distribution with a synchrotron model (cf GRBs)

⇒ determine time evolution of *E*, *ρ*, *R*, *Γ*

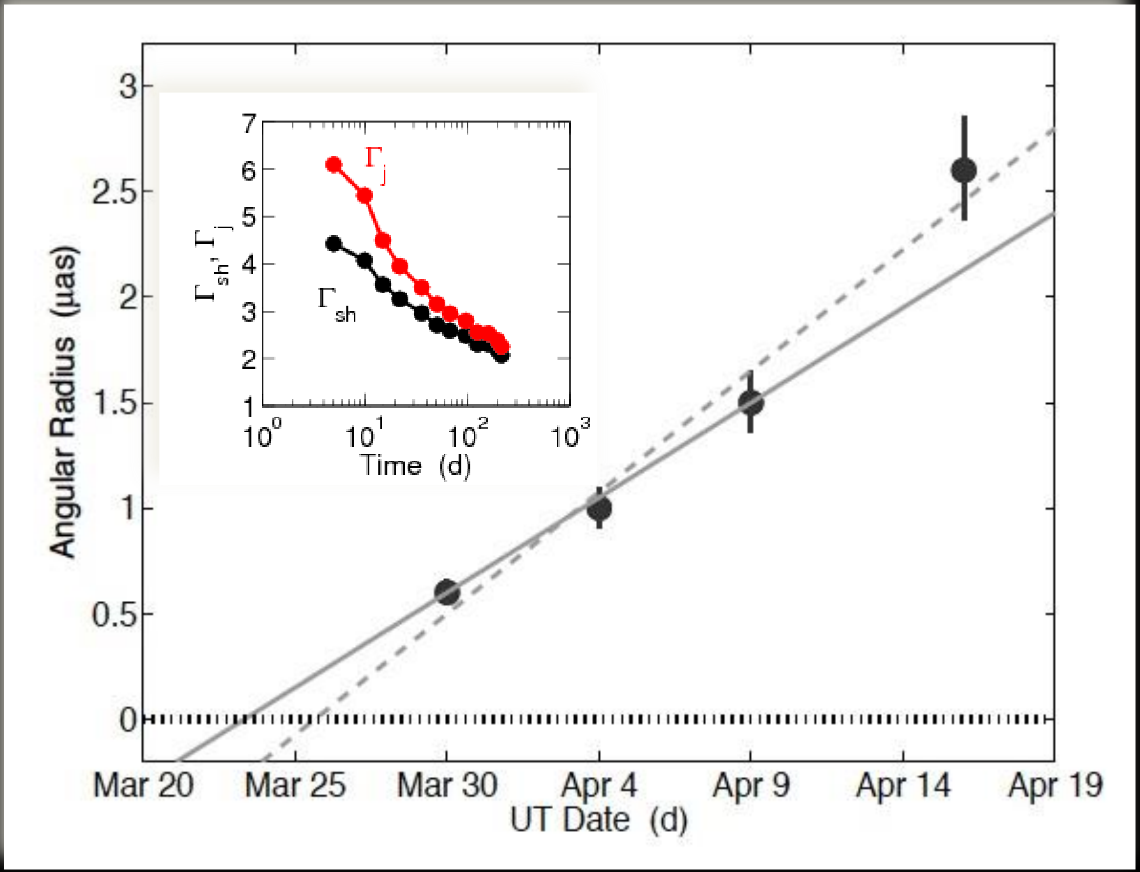
- - - - - - - - - - no change in *E*, *ρ*

Berger et al. ApJ (2012)



Relativistic Expansion?

1. Modeling



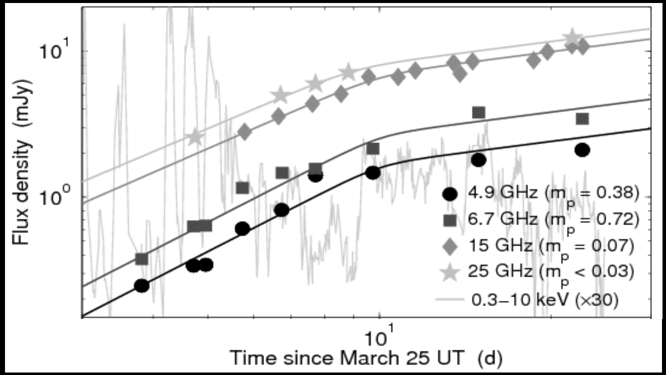
Zauderer et al.

Berger et al. ApJ (2012)

Formation epoch: March 23-26

$$\theta_{\rm eq} \approx 110\, d_{\rm L,Mpc}^{-1/19}\, F_{\nu,{\rm p,mJy}}^{9/19}\, \nu_{\rm p,GHz}^{-1}\ \ \mu as$$

2. Interstellar scintillation:



$$\nu_o \approx 10\ \mathrm{GHz}$$

$$\theta_{F,0} \approx 1\ \mu\mathrm{as}$$

for $\nu < \nu_0$

$$m_p \propto (\nu/\nu_0)^{17/30}\, (\theta_s/\theta_r)^{-7/6}$$

$$\theta_s \approx 5\ \mu\mathrm{as}$$

$$\Gamma \approx \mathrm{few}$$

Sw1644+57: Radio Monitoring

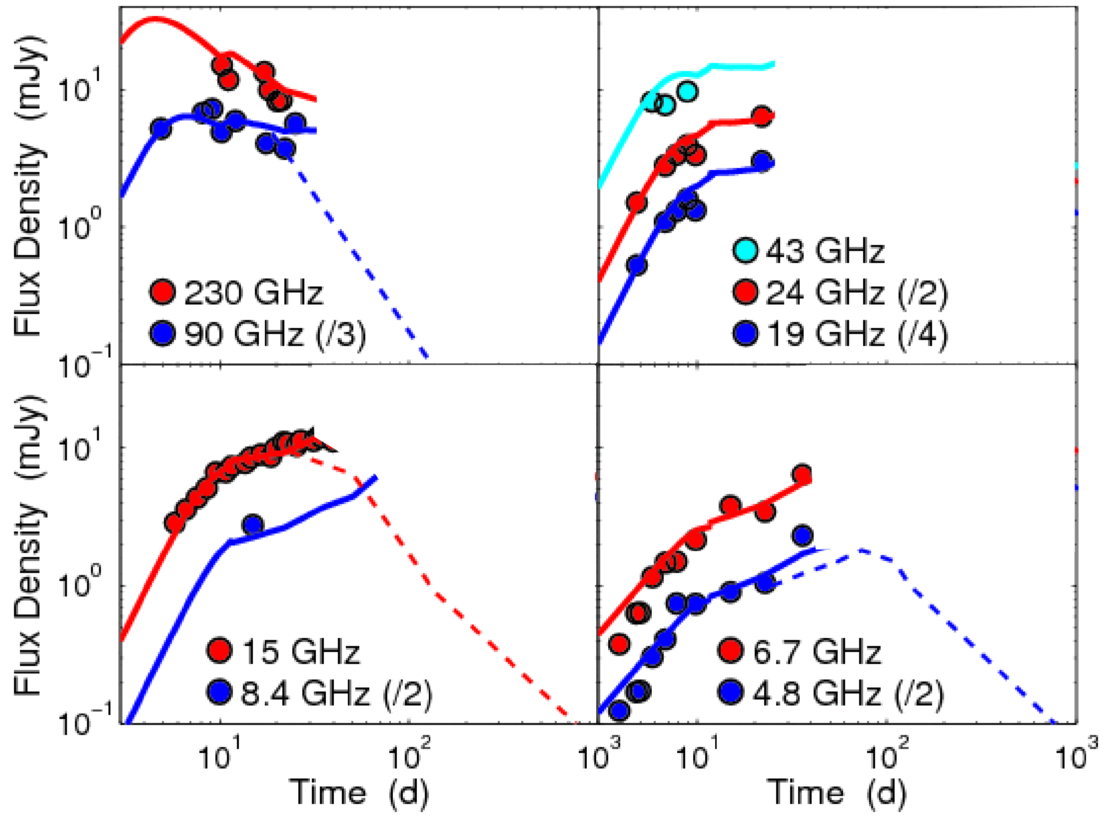
Long-term radio study provides several unique opportunities:

- Structure and evolution of a relativistic jet (flux, polarization, VLBI)
- Pristine environment of a (previously-dormant) SMBH

SMA
CARMA
EVLA
AMI-LA

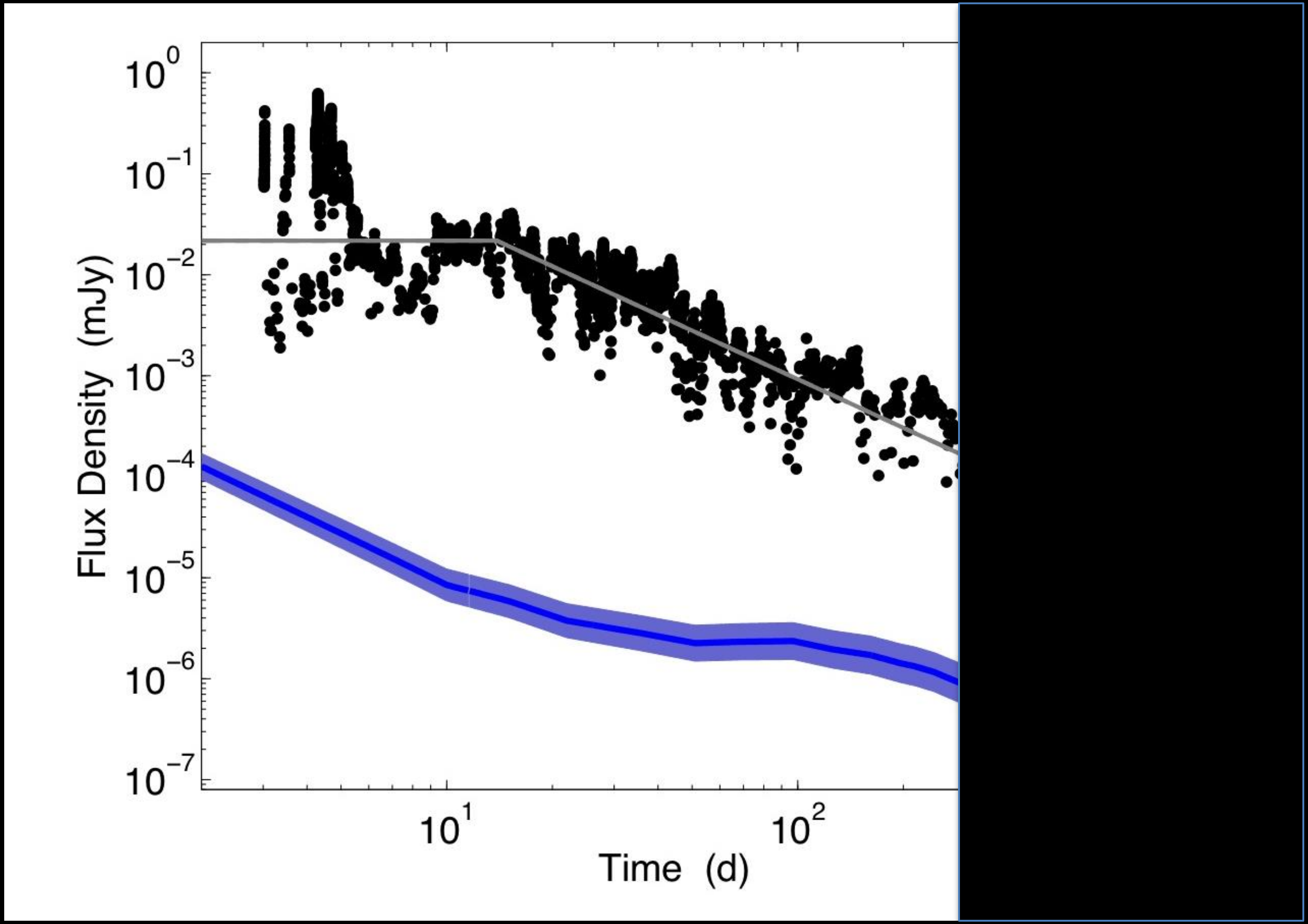
Observed radio evolution reveals much longer rise time and brighter emission than initially expected

⇒ increase in *E*, *ρ*, both?



Berger et al. ApJ (2012)

Jet Shuts off?



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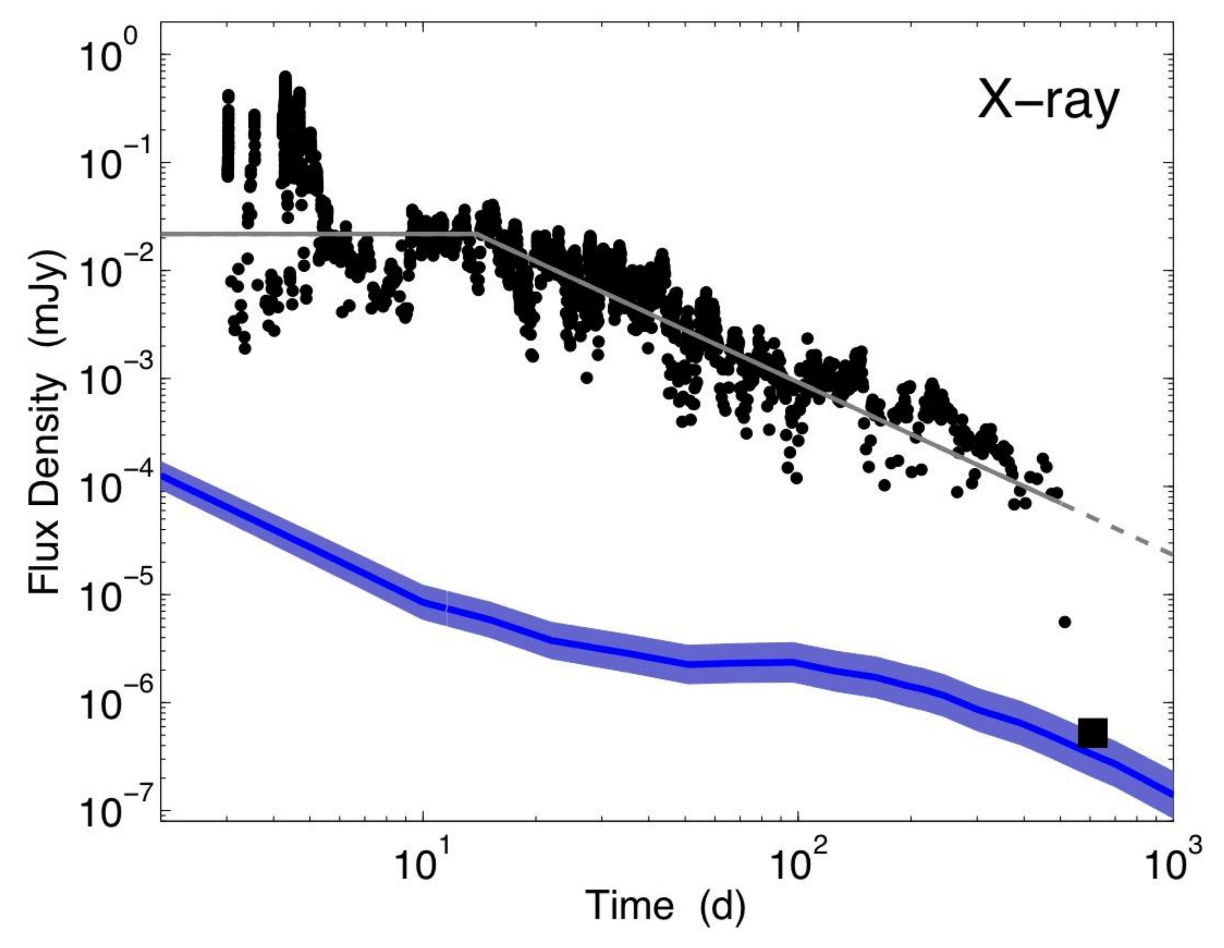
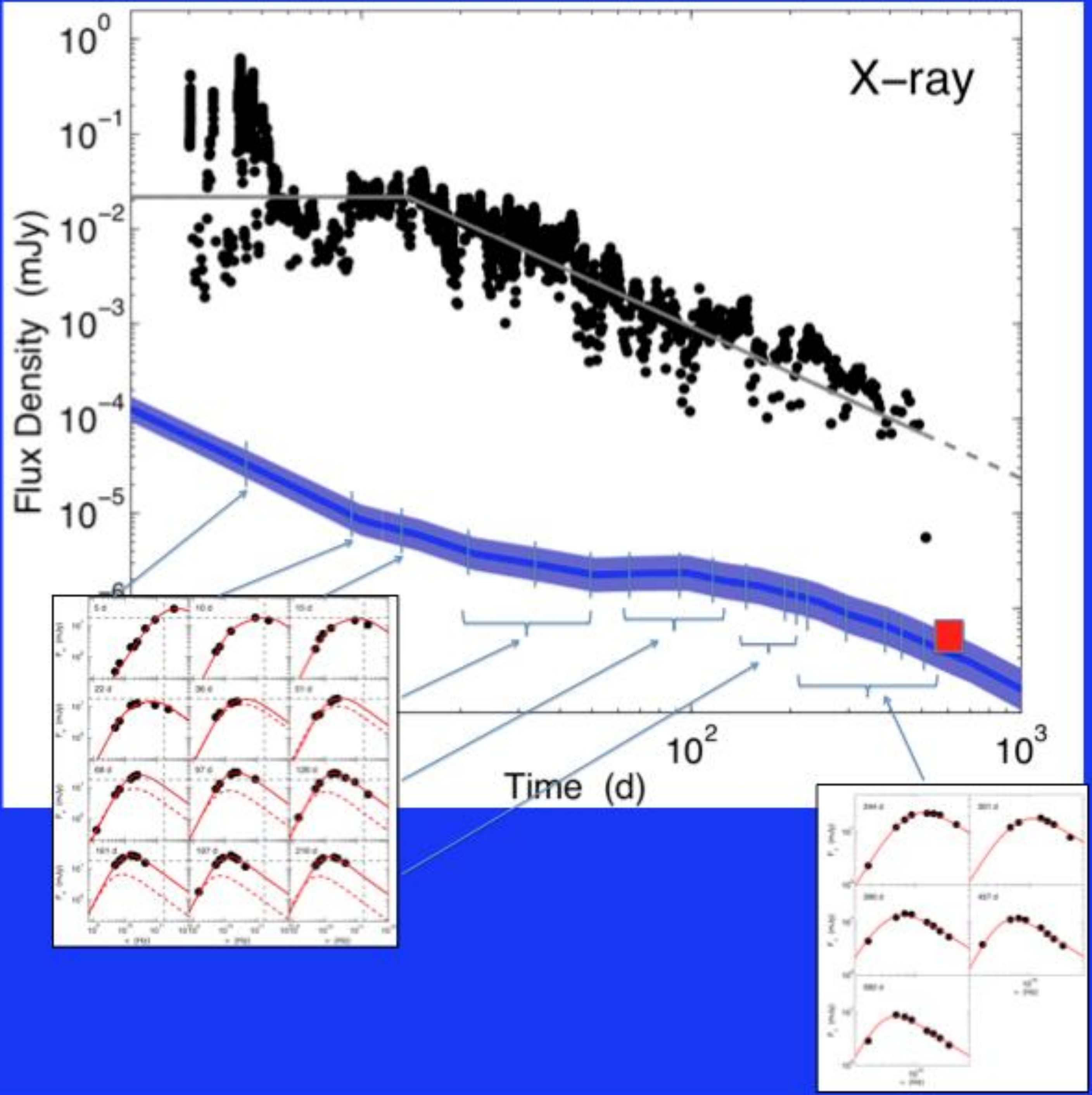


Zauderer et al. (2013)

Chandra X-ray detection
Levan and Tanvir
(November 2012)

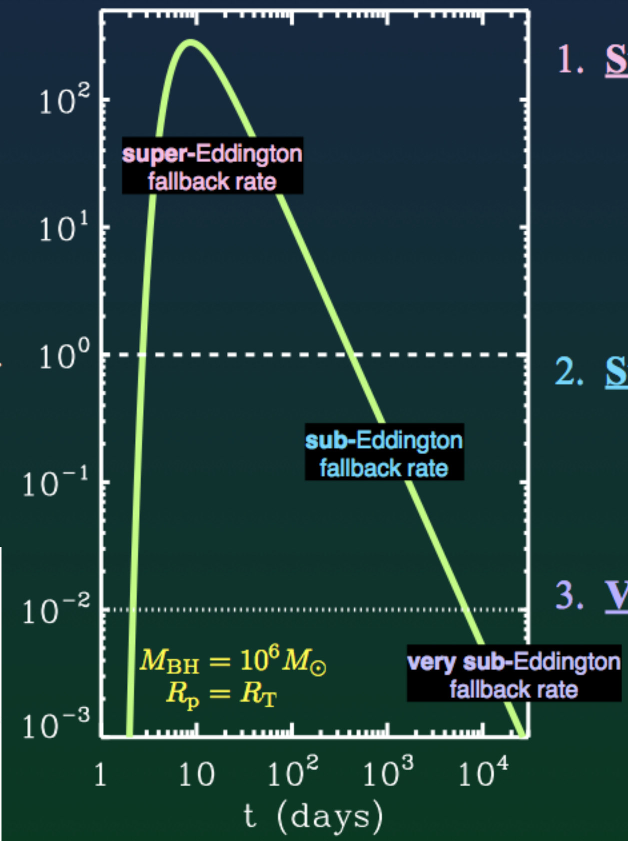


Chandra X-ray detection at dt ~610 days by A. Levan and N. Tanvir (November 2012) Atel 4610

Fallback & Observable Signatures

$$\dot{M}_{\rm fallback} \sim \frac{M_*}{t_{\rm fallback}} \left(\frac{t}{t_{\rm fallback}} \right)^{-5/3}$$

As fallback rate declines with time, 3 Phases of Evolution:

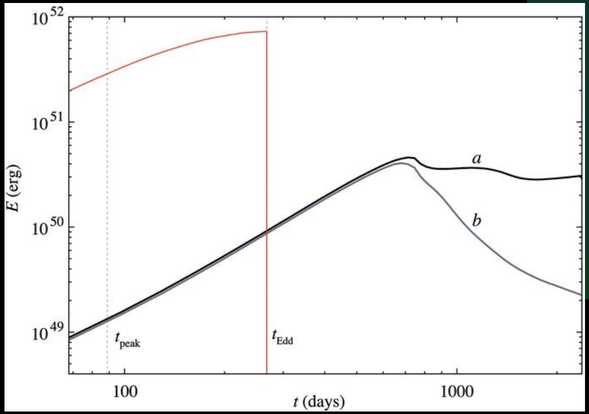


1. **Super-Eddington fallback**: ~weeks - months
   $\dot{M}_{\rm fallback} \gg \dot{M}_{\rm Edd}$
   Physics is uncertain, but likely advective disk + powerful outflows (+ jet?)

2. **Sub-Eddington fallback**: ~months - year
   $\dot{M}_{\rm fallback} \lesssim \dot{M}_{\rm Edd}$
   Thin accretion disk

3. **Very Sub-Eddington fallback**: ~several yr - decades
   $\dot{M}_{\rm fallback} \lesssim 10^{-2} \dot{M}_{\rm Edd}$
   Radiatively inefficient flow: hot disk + jet

The timescale for falling below Eddington is similar to predictions by de Colle et al. 2012 (below).



Slide courtesy Linda Strubbe (Toronto)

Information from Radio / Future Work

Localization

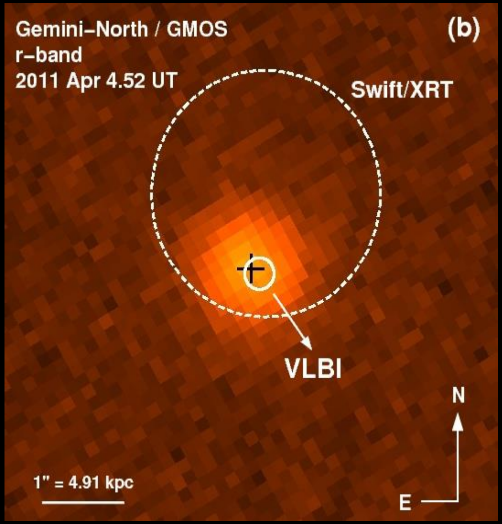
Circumburst density

Velocity / energy scale

-Beaming

-radius constraints and size evolution

Magnetic field strength/ outflow line-of-sight orientation (via polarization)



Summary

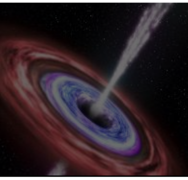
- Radio observations to-date of Sw1644+57 are consistent with a TDE interpretation

- We have potentially observed an entire life cycle of a relativistic jet from birth to cessation

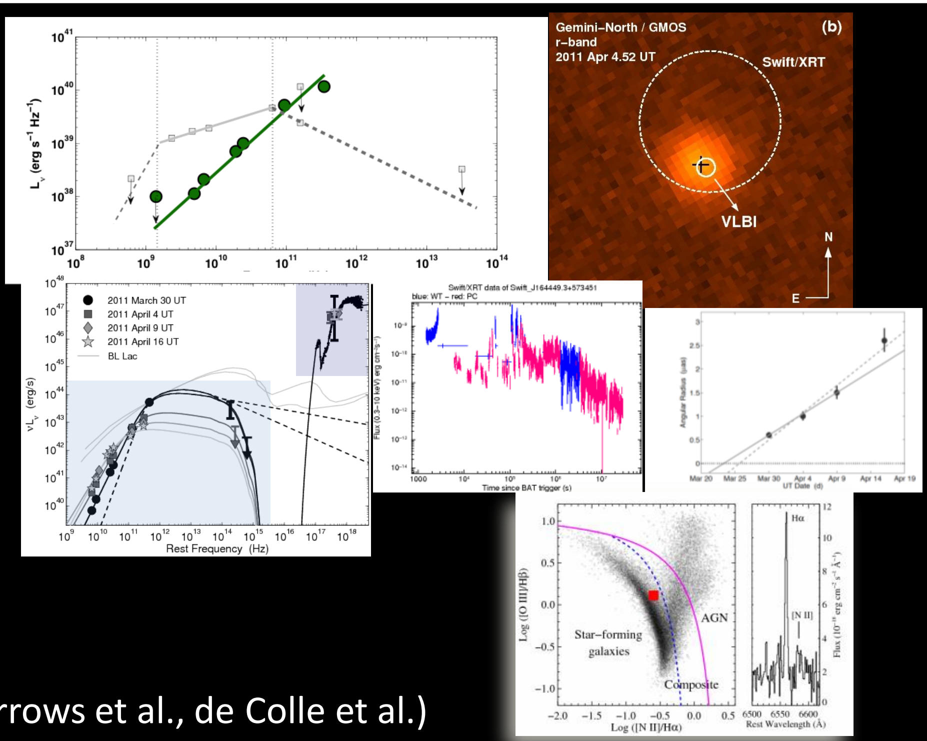
- Radio observations contribute unique and complementary information to higher energy observations

    - Continued radio observations are crucial (esp. if X-ray emission continues dropping) as the only way to continue monitoring the source

- Radio observations have implications for rates (e.g. off-axis events) of TDEs with relativistic jets



Sw1644+57: Progenitor Possibilities



Non-TDE:

(1) Long GRB with Nuclear Origin ?

- spectral information unusual
- coincidence with nucleus of inactive galaxy
- Early gamma-ray/X-ray flares and long-lived X-ray emission inconsistent
- only mildly relativistic

(2) AGN / Blazar

- typical lifetimes of ~million years
- spectral diagnostics
- no prior signatures in X-ray or radio

TDE variations:

- million solar mass black hole disrupting a star (e.g. Burrows et al., de Colle et al.)
- Less massive BH disrupting a white dwarf (e.g. Piran)
- Variations in BH spin or mass (e.g. Kesden, Gultekin)
- Precession of jet (e.g. N. Stone)
- Eccentricity of disrupted star (Hayasake)
- Deeply plunging (Cannizzo, Troja, Lodato)
- ???

Radio emission consistent with synchrotron emission, mildly relativistic, evolution inconsistent with single injection event, small measured polarization

Bower et al. X-ray selected TDEs

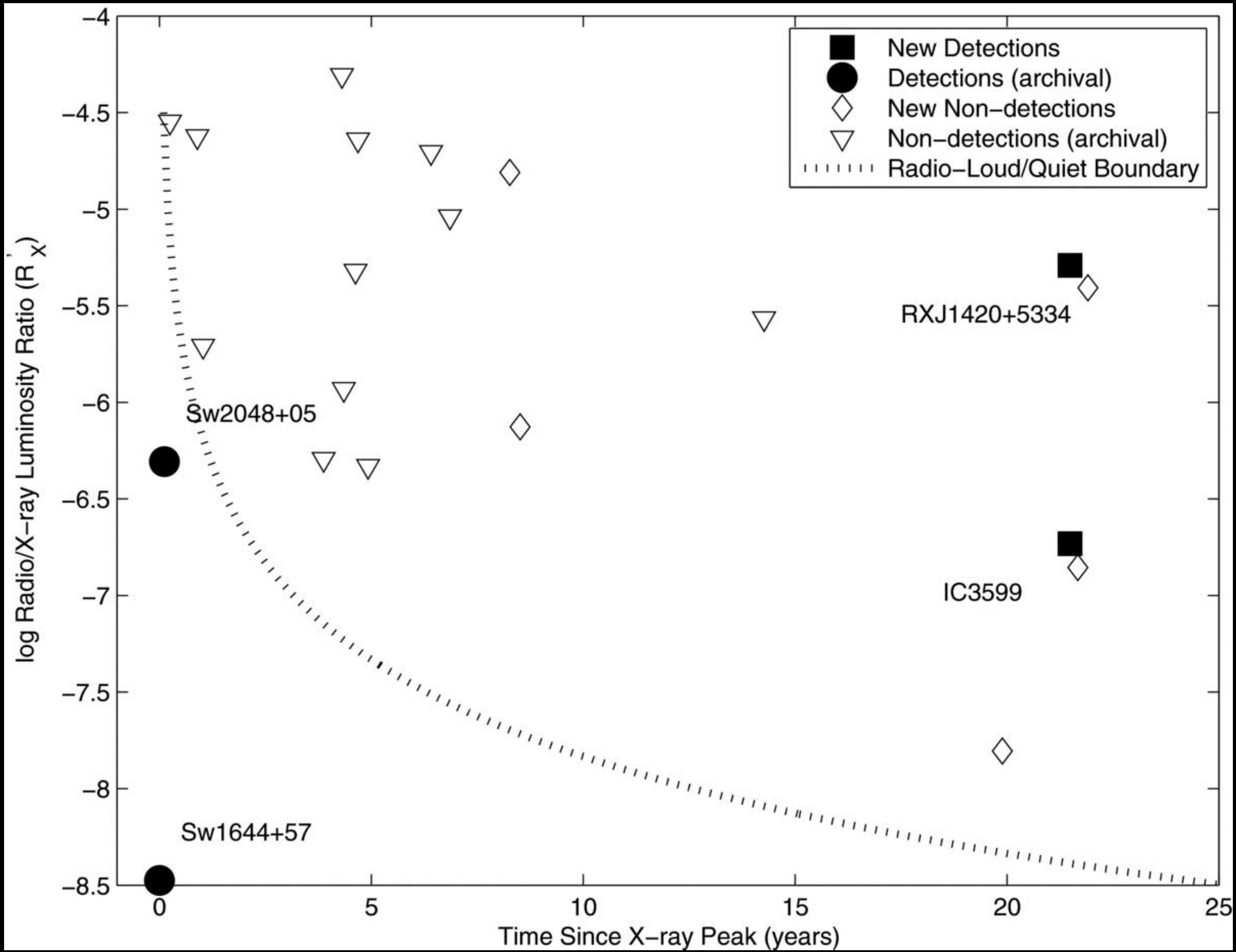
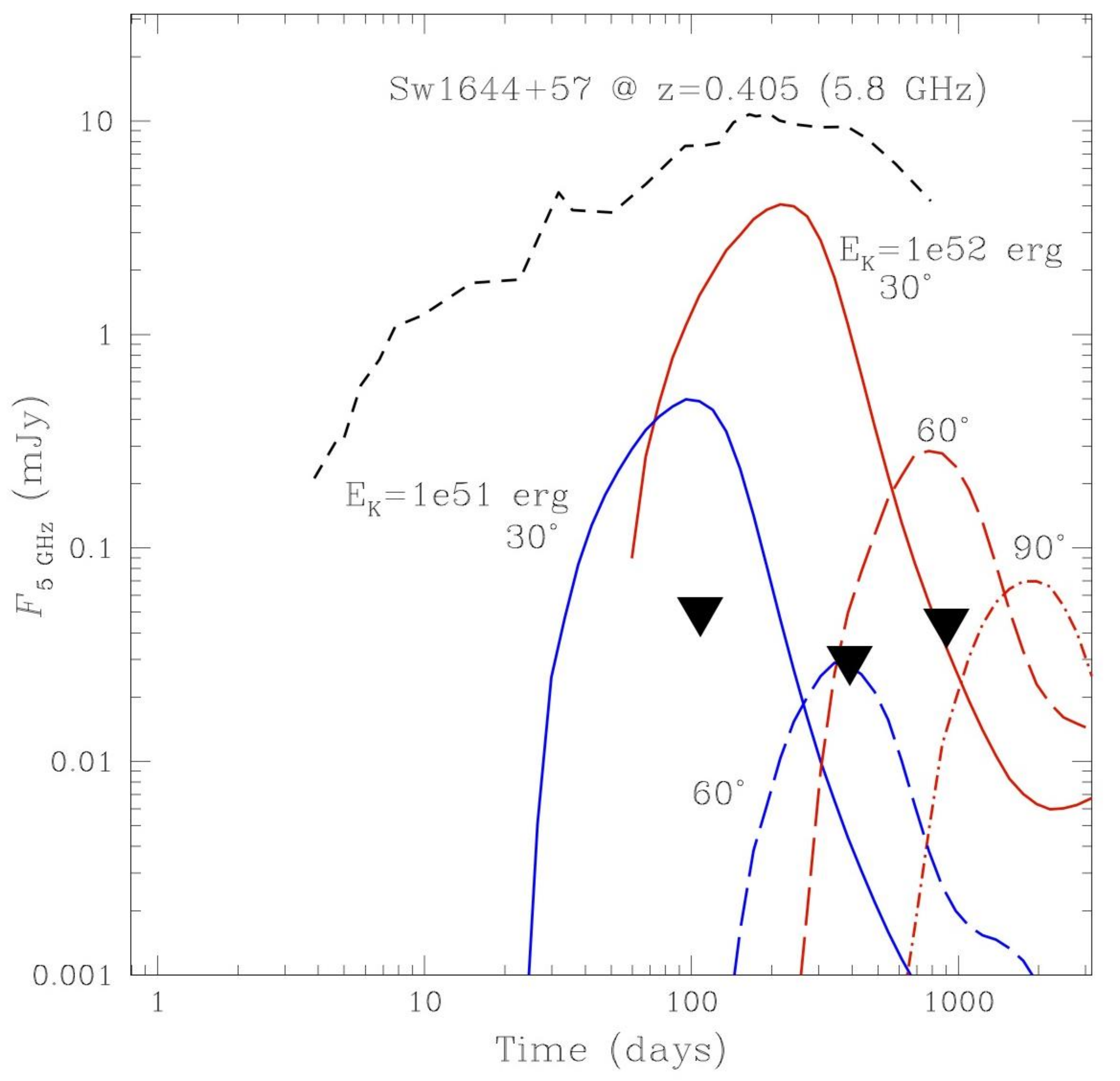


Figure 4 from Late-time Radio Emission from X-Ray-selected Tidal Disruption Events
Geoffrey C. Bower et al. 2013 ApJ 763 84 doi

Off-axis events?



Off-axis radio emission from hydrodynamic simulations via NYU Afterglow Library BOXFIT Code (van Eerten et al. 2012, ApJ 749, 44)

A TDE candidate from PanSTARRs Ryan Chornock et al. (in prep)

Van Velzen 2011
rate of TDEs from SDSS

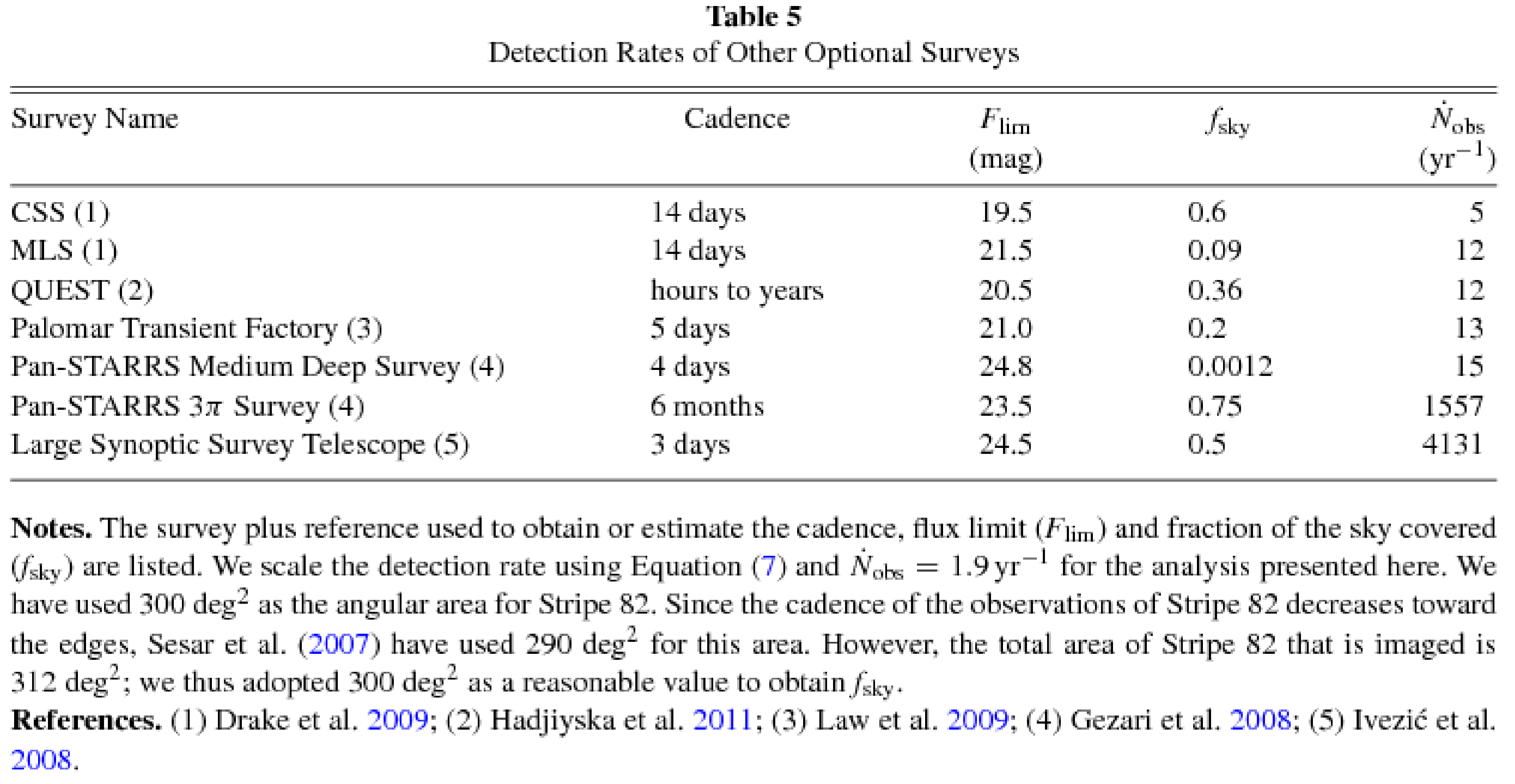


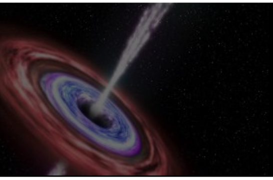
Table 5

Detection Rates of Other Optional Surveys

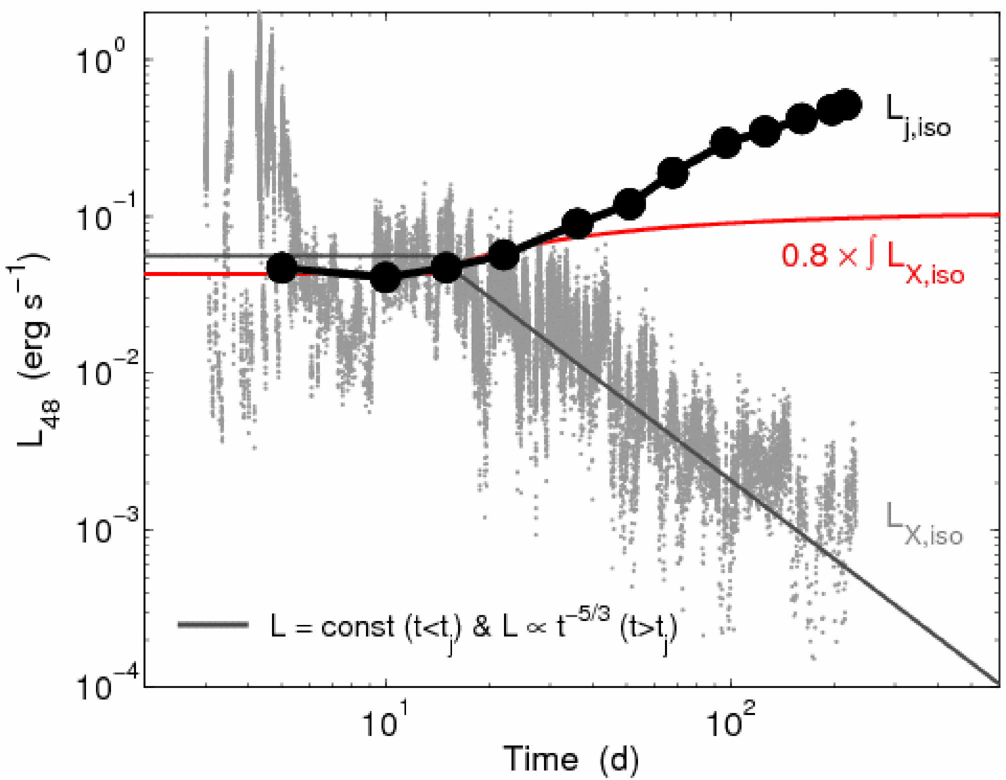
Survey Name	Cadence	$F_{\rm lim}$ (mag)	$f_{\rm sky}$	$\dot{N}_{\rm obs}$ ($\rm yr^{-1}$)
CSS (1)	14 days	19.5	0.6	5
MLS (1)	14 days	21.5	0.09	12
QUEST (2)	hours to years	20.5	0.36	12
Palomar Transient Factory (3)	5 days	21.0	0.2	13
Pan-STARRS Medium Deep Survey (4)	4 days	24.8	0.0012	15
Pan-STARRS 3π Survey (4)	6 months	23.5	0.75	1557
Large Synoptic Survey Telescope (5)	3 days	24.5	0.5	4131

Notes. The survey plus reference used to obtain or estimate the cadence, flux limit ($F_{\rm lim}$) and fraction of the sky covered ($f_{\rm sky}$) are listed. We scale the detection rate using Equation (7) and $\dot{N}_{\rm obs} = 1.9\,\rm yr^{-1}$ for the analysis presented here. We have used 300 deg^2 as the angular area for Stripe 82. Since the cadence of the observations of Stripe 82 decreases toward the edges, Sesar et al. (2007) have used 290 deg^2 for this area. However, the total area of Stripe 82 that is imaged is 312 deg^2; we thus adopted 300 deg^2 as a reasonable value to obtain $f_{\rm sky}$.

References. (1) Drake et al. 2009; (2) Hadjiyska et al. 2011; (3) Law et al. 2009; (4) Gezari et al. 2008; (5) Ivezić et al. 2008.



Jet Energetics



Berger et al. ApJ (2012)

Is this ubiquitous for relativistic jets, Blandford-Znajek mechanism, GRBs?

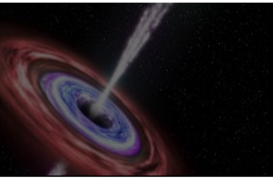
$$
\begin{aligned}
E_{\rm j,iso} &= \Delta t_j \times L_{\rm j,iso} \\
&\approx 10^6\,{\rm s} \times L_{\rm j,iso} \\
&\approx 5 \times 10^{53}\,{\rm erg\ at\ 200\,d}
\end{aligned}
$$

The increase in jet energy cannot be explained by on-going injection from the accreting SMBH

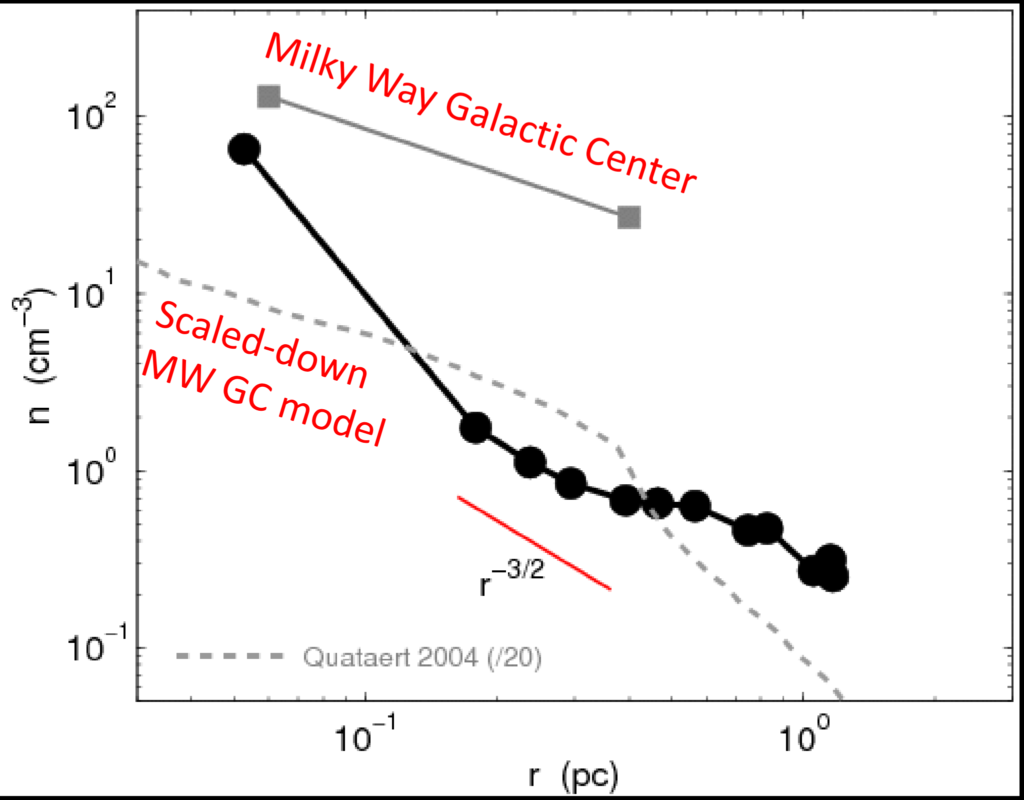
(as seen in X-rays w/ $L \propto t^{-5/3}$).

Instead, relativistic jet launched w/ a distribution of Lorentz factors explains the increase in E and evolution of R

$$E_{\rm j}(>\Gamma_{\rm j}) \propto \Gamma_{\rm j}^{-2.5}$$



Parsec-Scale Environment



Berger et al. ApJ (2012)

Radial profile is roughly $\rho \propto R^{-3/2}$

Lower density relative to Galactic Center indicative of lower SFR?

The density profile around a dormant SMBH at z = 0.354 measured with better spatial resolution than for the Milky Way Galactic Center

TDE Background

Year	Work	Summary	Prediction
1976 1988	Frank & Rees Rees	Tidal disruption of stars by black holes of 10^6–10^8 solar masses in nearby galaxies	-stars in galactic nuclei can be captured or tidally disrupted -some debris ejected at high speeds, some captured -UV or soft X-ray radiation from the innermost region of the accretion disk
2000	Ayal, Livio and Piran	Tidal disruption of a main sequence star by a SMBH	2 consecutive Sne-like events
2004	Bogdanovic et al.	-Simulations of 2 TDEs using relativistic smoothed particle hydrodynamic code -calculated the physical conditions and radiative processes in the debris	Time variable H-alpha emission -don’t see double-peak expected from rotating disk as debris doe not settle into a stable disk at early times -line profiles depend sensitively on the orientation of the tail relative to the line of sight, vary on short time scales
2009	Strubbe & Quataert		A radiatively driven wind, existing as a consequence of the super-Eddington fallback, may dominate emission of TDE at early time
2010	Giannios & Metzger	Radio emission *posited*; peak at ~1 year	Radio emission peaks at t~1 year

TDE Observations prior to 2011

Year	Work	Summary	Observations
2006, 2008, 2009	Gezari et al.	**UV/Optical Detections of Candidate Tidal Disruption Events by *GALEX and CFHTLS***	2 luminous UV/optical flares from nuclei of apparently inactive early-type galaxies at z=.37 and 0.33 that have radiative properties of a flare from the tidal disruption of a star
1996 1999, 2003	Komossa & Bade Komossa et al.	Chandra and XMM Newton observations of X-ray flares	
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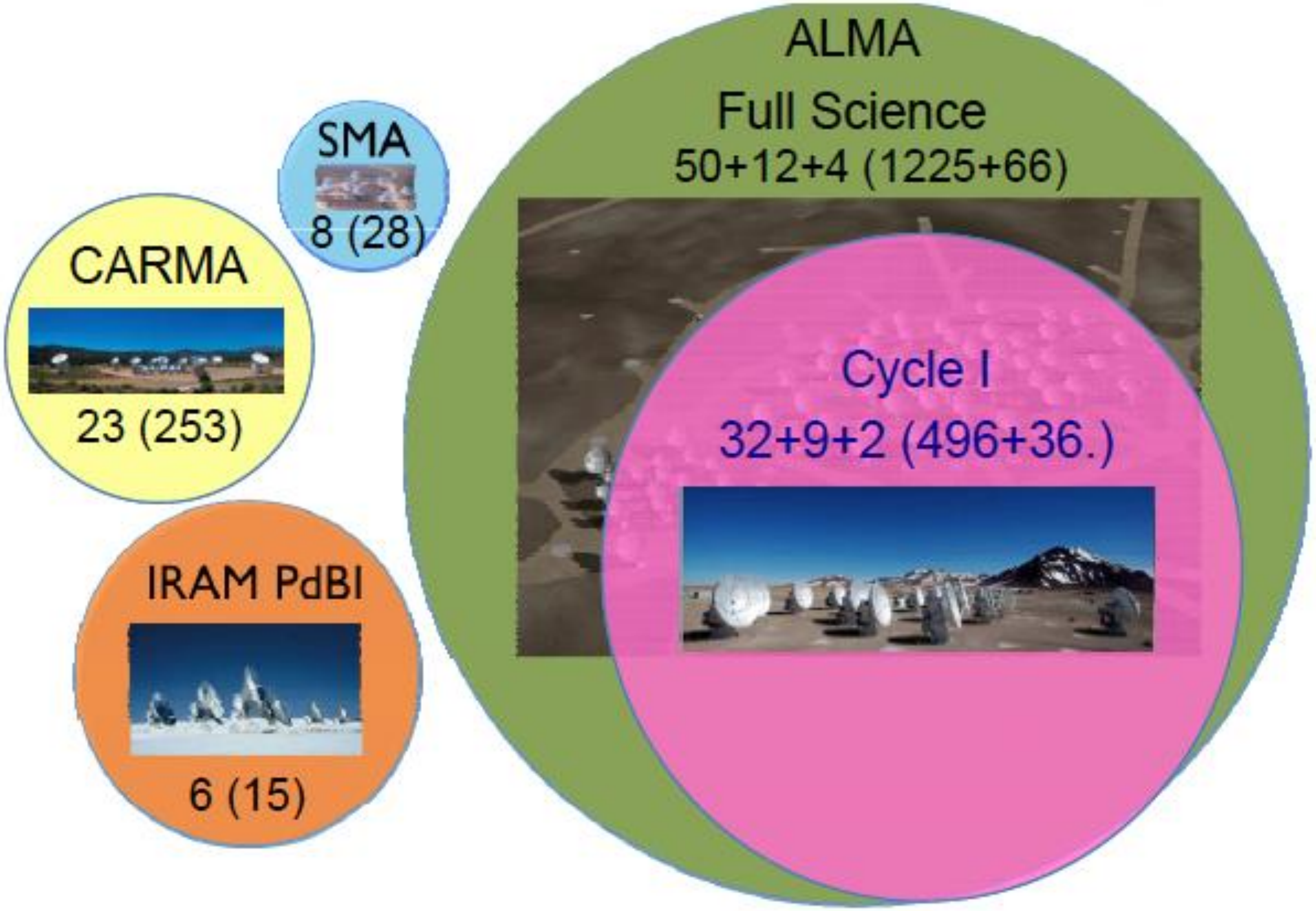
Thank you

Collecting Area & Baselines



SMA

8 (28)

CARMA

23 (253)

IRAM PdBI

6 (15)

ALMA

Full Science

50+12+4 (1225+66)

Cycle I

32+9+2 (496+36.)

Circles Show Collecting Area (sensitivity)

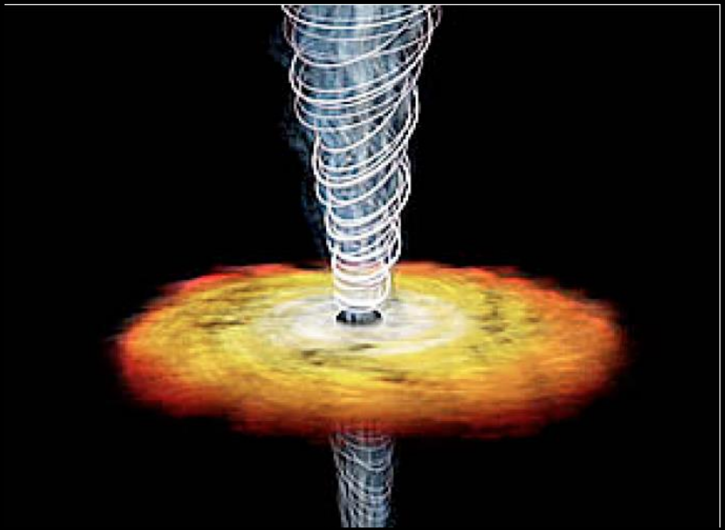
Captions give # of antennas and # of baselines (fidelity)

Sources of radio emission:

Synchrotron

Non-thermal coherent

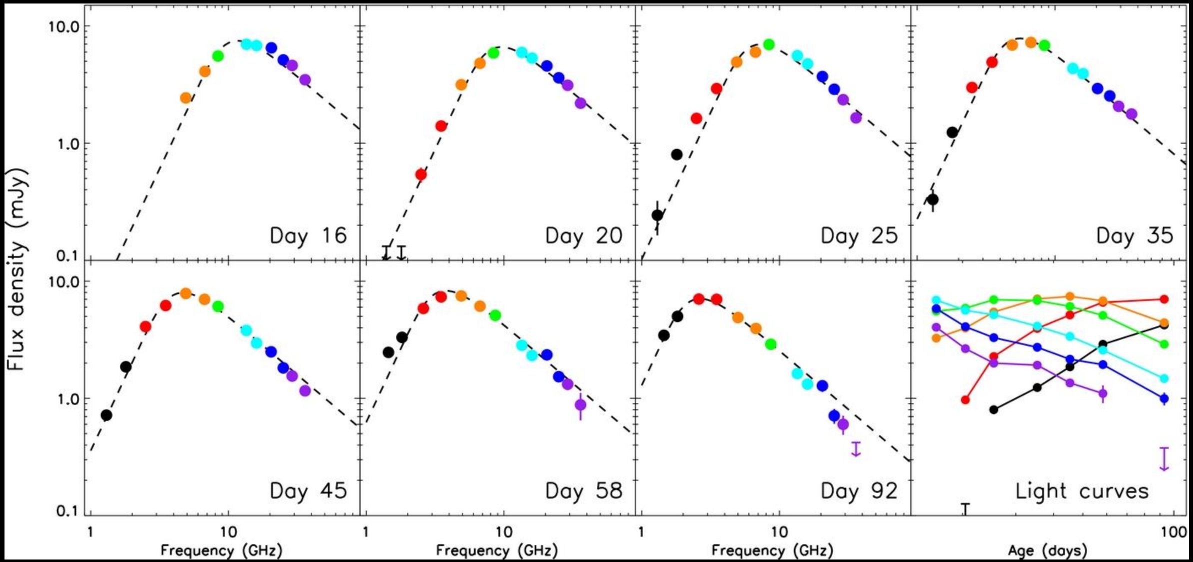
Thermal



- Gamma-ray bursts
- Supernovae
- X-ray binaries
- (Jetted) Tidal disruption events

Slide courtesy
Laura Chomiuk, MSU

Synchrotron Emission



SN 2011dh – Type IIb

Krauss et al. ApJL, 750, 250 (2012)

See also Horesh et al. (2012)

Soderberg et al. ApJ, 752, 78 (2012)

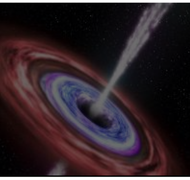
Timing of observations important

Chevalier, ApJ, 499, 810 (1998) “Synchrotron Self-Absorption in Radio Supernovae"

Granot & Sari, ApJ, 568, 820 (2002) “The Shape of Spectral Breaks in Gamma-Ray Burst Afterglows”

Duran, Nakar & Piran, arXiv:1301:6759 (2013) “Radius constraints and minimal equipartition energy of relativistically moving synchrotron sources”

See also NYU Afterglow library &
Hydrodynamic simulations (UCSC)



Radio Monitoring of TDE Sw1644+57

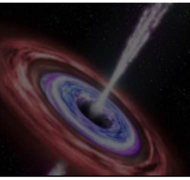
- Summary of radio emission mechanism

- Brief overview of discovery of GRB110328A / Sw 1644+57 and 2.5 year radio monitoring campaign
  - Track evolution of environment and total energy
  - Suppression of optical emission due to extinction
  - Relativistic: modeling / scintillation

- X-rays radio coming from distinct emission regions
  - Early X-ray emission cannot be from forward shock or re-processing, time scale of quick cutoff also inconsistent

- In 3 years, full cycle – birth/cessation of relativistic jet (X-rays/radio)

- Sw 1644+57 as prototype for TDE with on-axis relativistic jet, future search for off-axis and determination of rate of jet production



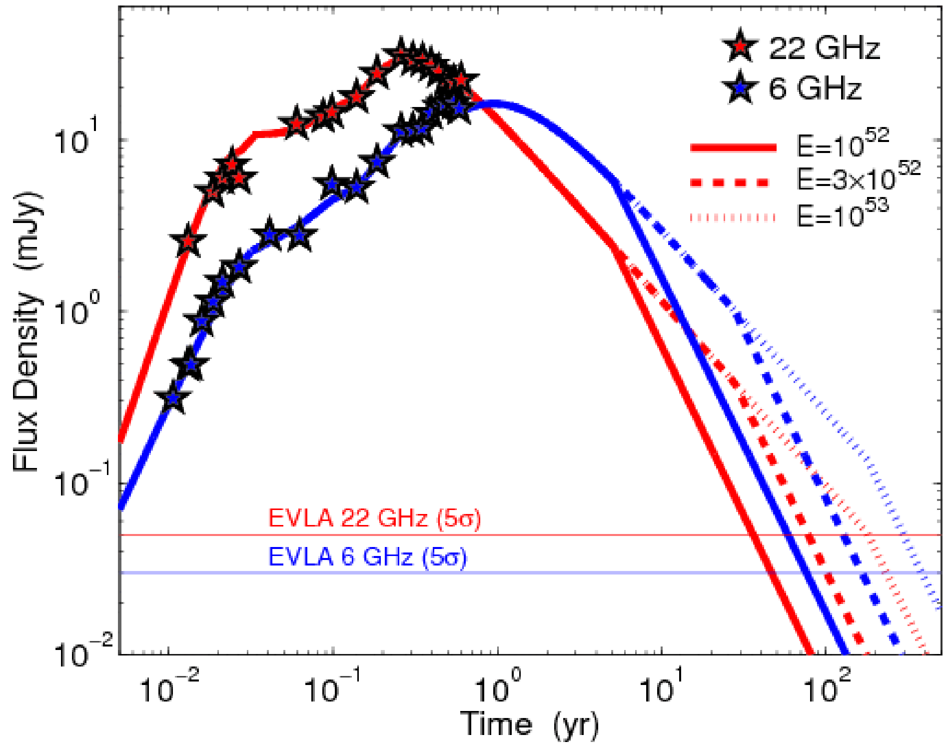
A Radio View of the Birth of a Relativistic Jet

Radio observations…

- Provide support for TDE scenario
    - Positional coincidence with nucleus of an inactive galaxy
    - Unlike typical long GRBs
    - Energetics agree with million solar mass BH disrupting solar-type star
- Establish presence of a relativistic outflow, tying onset to gamma-ray emission
- Probe environment around previously quiescent SMBH
- Demand an explanation for a seeming “energy injection” at late times
- Place constraints on jet opening angle
- Arise from different mechanism than X-ray emission until late times?

- May be a place to find more TDEs in the future?

VLBI

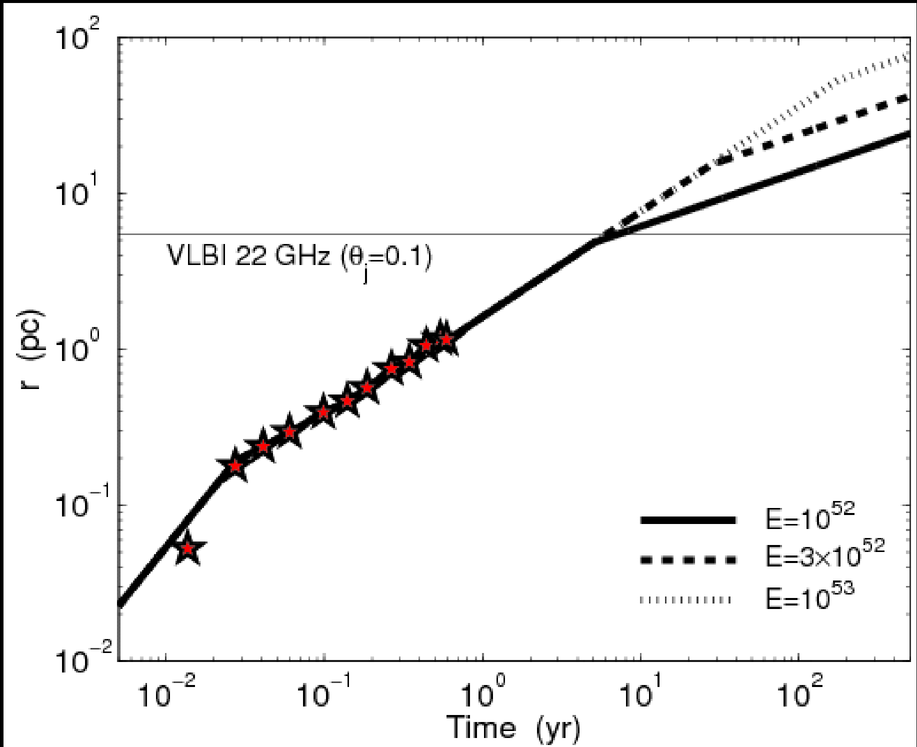


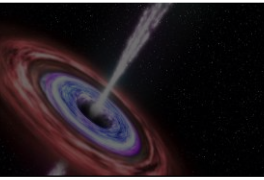
Berger et al. ApJ (2012)

Depending on total E_j, the radio emission will remain detectable with EVLA for ~100-500 years

Radio source will be resolvable with VLBI in ~3-5 years if stays collimated; ~1-2 years if it spreads

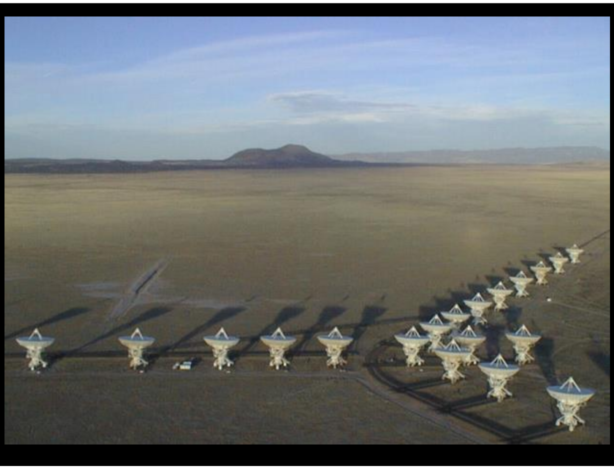
Berger et al. ApJ (2012)

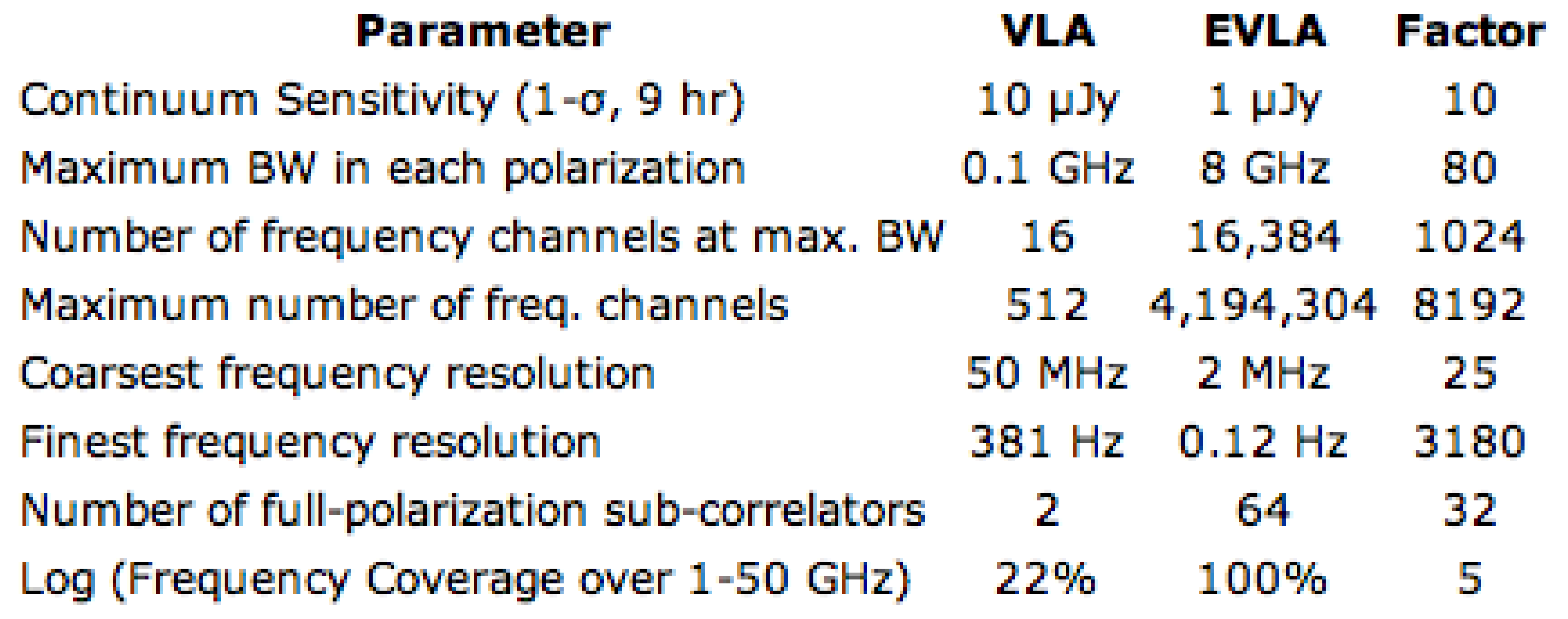




Karl G. Jansky VLA upgrade

$$\Delta I_m = \frac{SEFD}{\eta_c \sqrt{n_{\rm pol} N(N-1) t_{\rm int} \Delta\nu}}$$





Parameter	VLA	EVLA	Factor
Continuum Sensitivity (1-σ, 9 hr)	10 μJy	1 μJy	10
Maximum BW in each polarization	0.1 GHz	8 GHz	80
Number of frequency channels at max. BW	16	16,384	1024
Maximum number of freq. channels	512	4,194,304	8192
Coarsest frequency resolution	50 MHz	2 MHz	25
Finest frequency resolution	381 Hz	0.12 Hz	3180
Number of full-polarization sub-correlators	2	64	32
Log (Frequency Coverage over 1-50 GHz)	22%	100%	5

Physics of Tidal Disruption

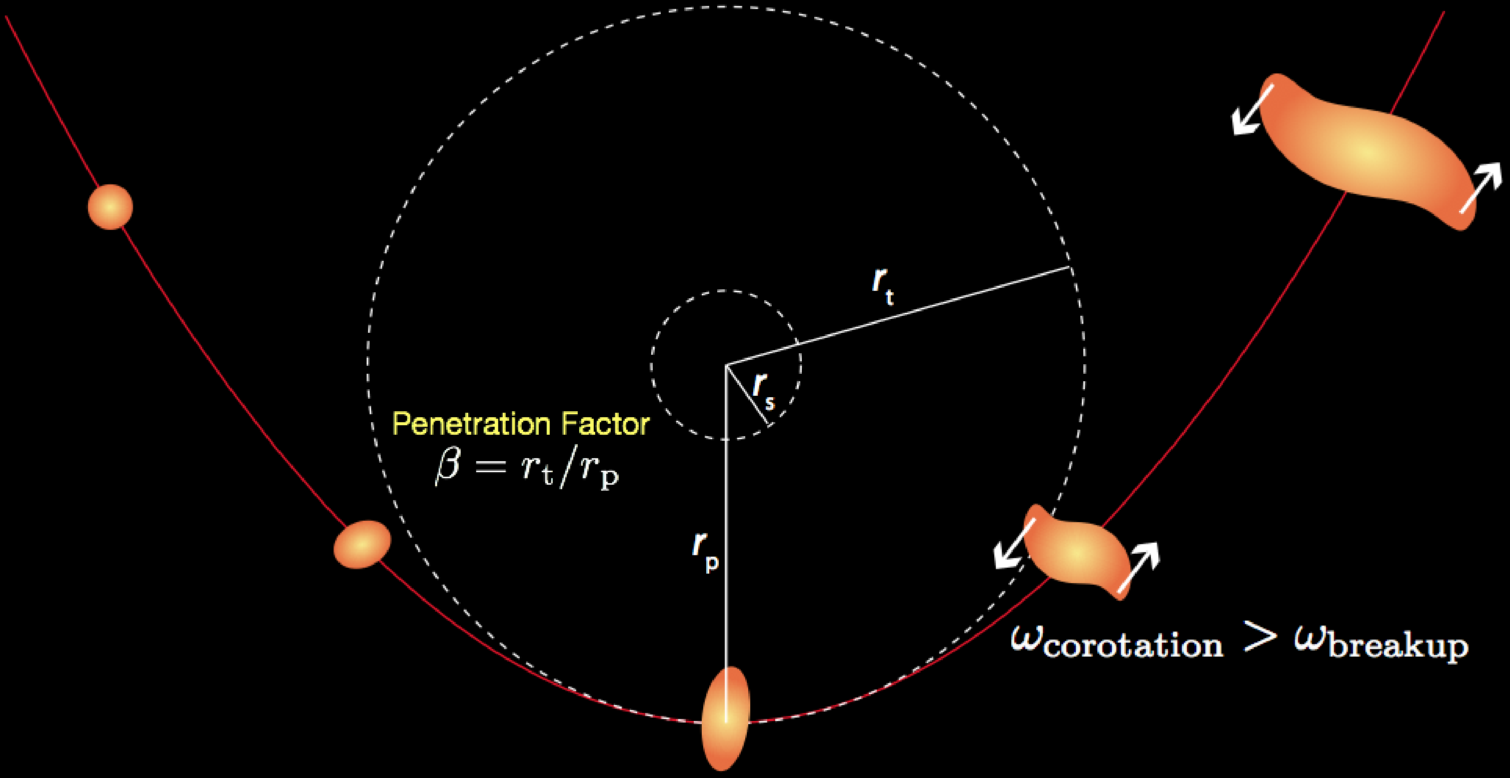
Schwarzschild Radius

$$r_s = 3 \times 10^{11} M_6 \text{ cm}$$

Tidal Radius

$$r_t = 7 \times 10^{12} \left(\frac{r_*}{r_\odot}\right) \left(\frac{M_*}{M_\odot}\right)^{-1/3} M_6^{1/3} \text{ cm}$$

Example: *Grazing encounter* (β ~ 1)



Penetration Factor

$$\beta = r_t / r_p$$

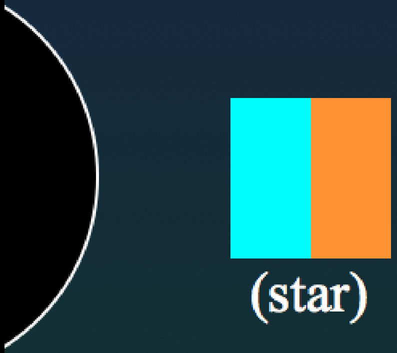
$$\omega_{\text{corotation}} > \omega_{\text{breakup}}$$

Fallback to the Black Hole: Simple picture

- Energy distribution + Kepler's Laws give mass fallback rate

$$\dot{M}_{\rm fallback} = \frac{dm}{dt} \sim \boxed{\frac{dm}{d\epsilon}}\,\frac{d\epsilon}{dt} \propto \frac{d\epsilon}{dt}$$

assume const
(square const-density star)

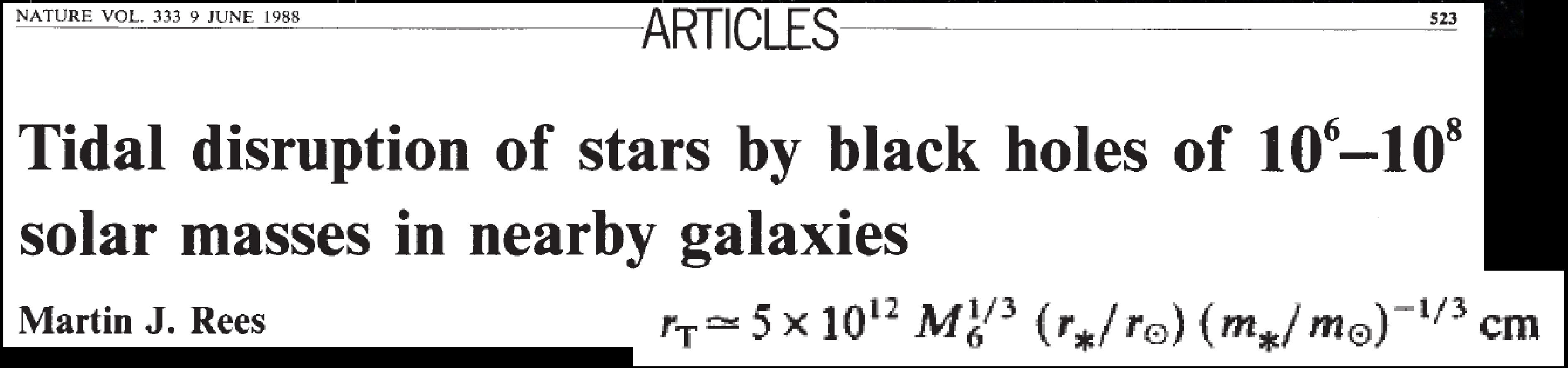

(star)

Keplerian potential: $\epsilon \sim \frac{GM_{\rm BH}}{a}$ $\quad P \propto a^{3/2}$ $\quad a \propto P^{2/3}$

$$\epsilon \propto P^{-2/3}$$

$$\frac{d\epsilon}{dt} \propto P^{-5/3}$$

$$\Rightarrow \dot{M}_{\rm fallback} \propto t^{-5/3}$$



NATURE VOL. 333 9 JUNE 1988

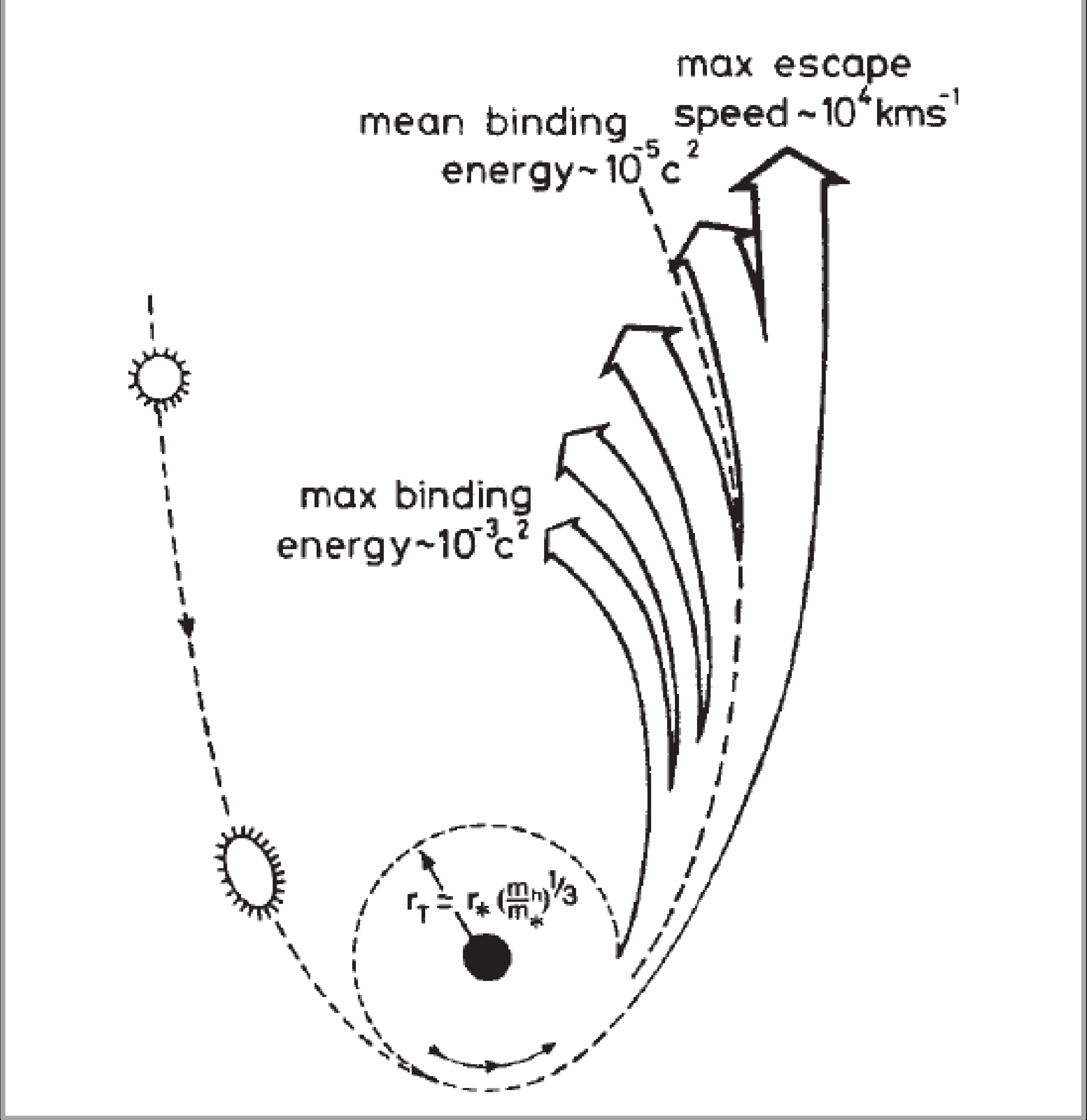
ARTICLES

523

Tidal disruption of stars by black holes of 10^6–10^8 solar masses in nearby galaxies

Martin J. Rees

$$r_T \simeq 5\times 10^{12}\, M_6^{1/3}\, (r_*/r_\odot)\,(m_*/m_\odot)^{-1/3}\ \mathrm{cm}$$



Rees 1988 (Nature)

TDE Background

Year	Work	Summary	Prediction
1976 1988	Frank & Rees Rees	Tidal disruption of stars by black holes of 10^6–10^8 solar masses in nearby galaxies	-stars in galactic nuclei can be captured or tidally disrupted -some debris ejected at high speeds, some captured -UV or soft X-ray radiation from the innermost region of the accretion disk
2000	Ayal, Livio and Piran	Tidal disruption of a main sequence star by a SMBH	2 consecutive Sne-like events
2004	Bogdanovic et al.	-Simulations of 2 TDEs using relativistic smoothed particle hydrodynamic code -calculated the physical conditions and radiative processes in the debris	Time variable H-alpha emission -don't see double-peak expected from rotating disk as debris doe not settle into a stable disk at early times -line profiles depend sensitively on the orientation of the tail relative to the line of sight, vary on short time scales
2009	Strubbe & Quataert		A radiatively driven wind, existing as a consequence of the super-Eddington fallback, may dominate emission of TDE at early time
2010	Giannios & Metzger	Radio emission *posited*; peak at ~1 year	Radio emission peaks at t~1 year

Year	Work	Summary	Observations
2006, 2008, 2009	Gezari et al.	**UV/Optical Detections of Candidate Tidal Disruption Events by *GALEX and CFHTLS***	2 luminous UV/optical flares from nuclei of apparently inactive early-type galaxies at z=.37 and 0.33 that have radiative properties of a flare from the tidal disruption of a star
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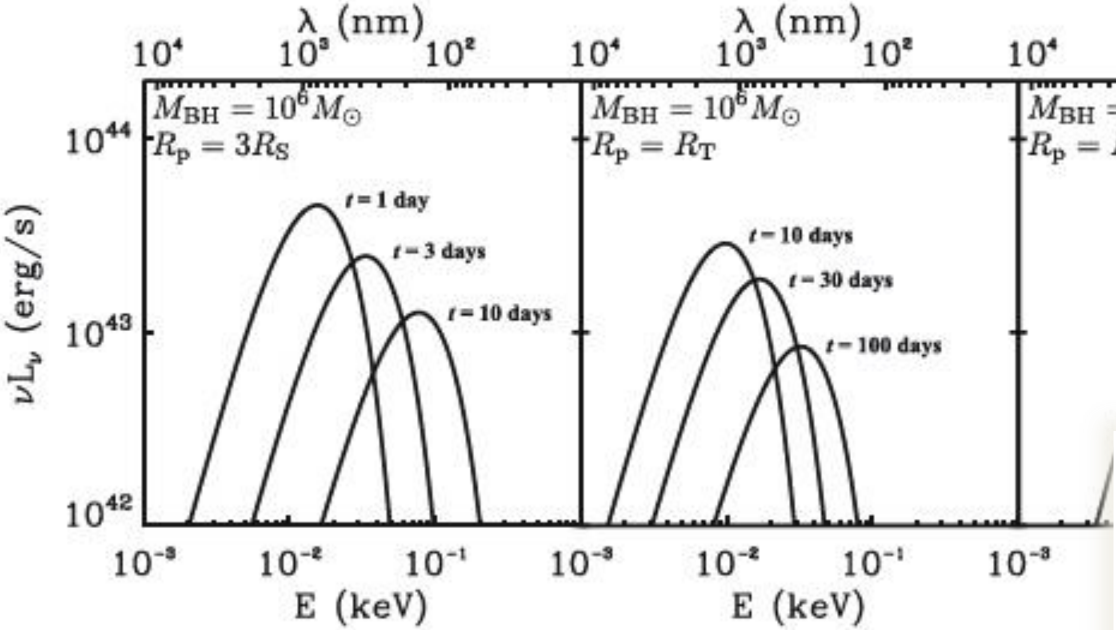
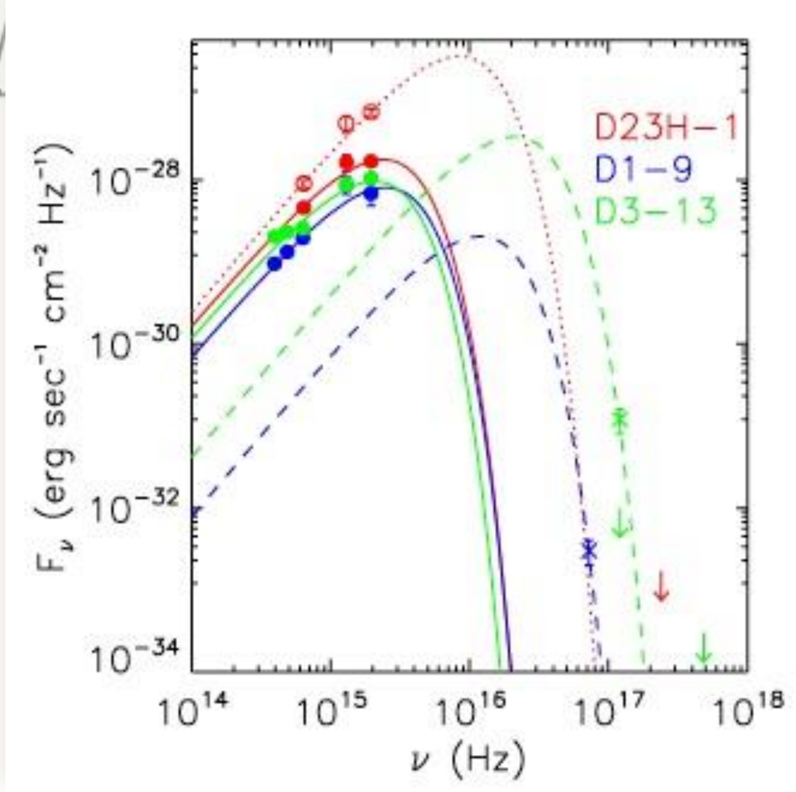
Theoretical Prediction



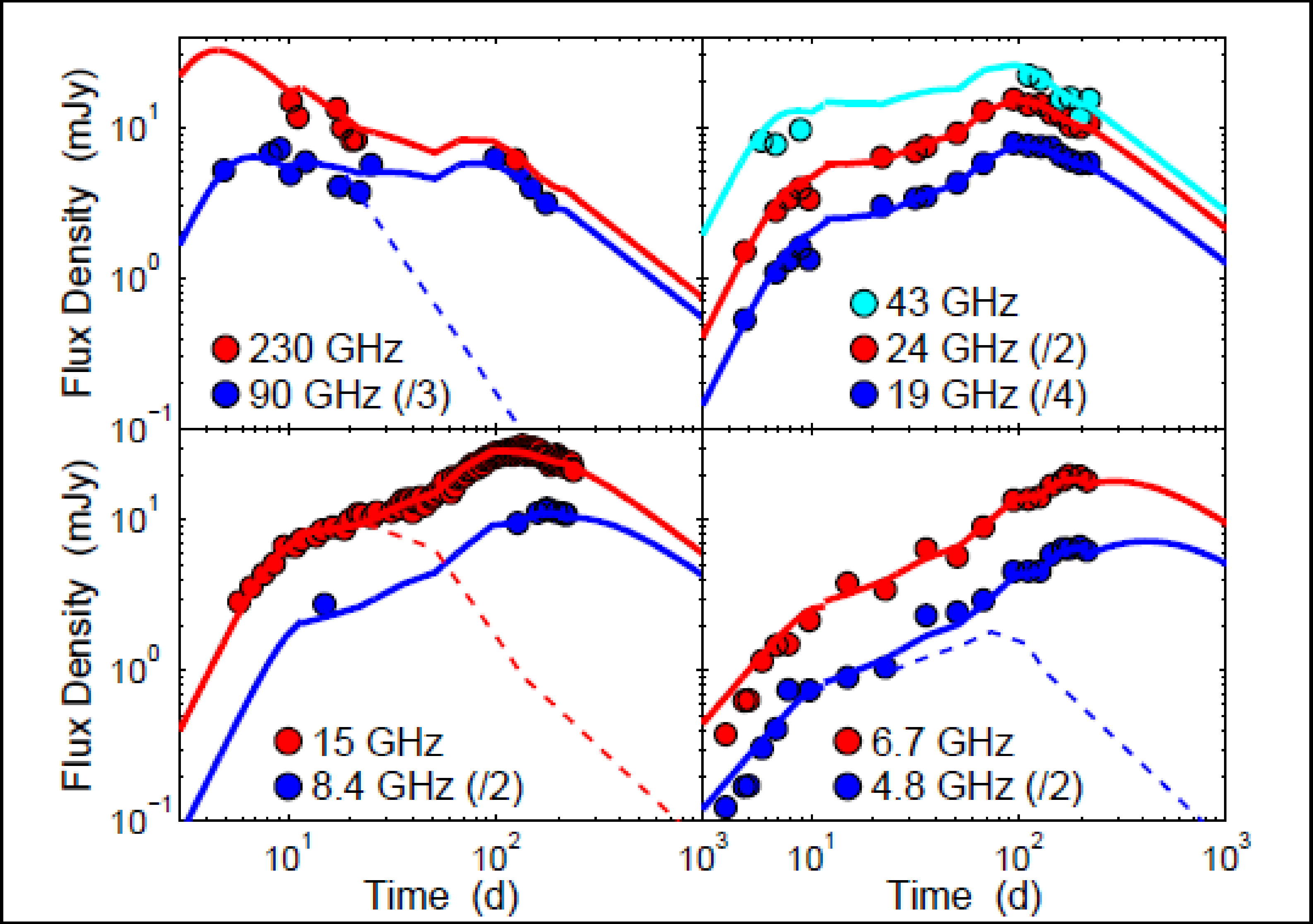
Strubbe & Quataert; Giannios & Metzger

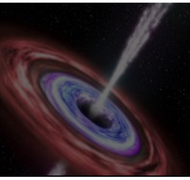
- -UV/optical/soft X-rays with $L \sim L_{edd}$
- -finite duration, very luminous, reside in non-active galaxy (Komossa 2002)

Gezari et al.



Sw1644+57 Radio Light Curves



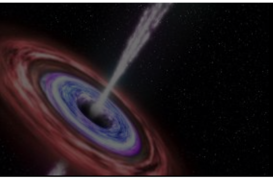


A Radio View of the Birth of a Relativistic Jet

- TDE Background
- Brief overview of discovery of GRB110328A / Sw 1644+57
  - Rees
  - Krolik & Piran
  - Giannios & Metzger
  - Ramirez-Ruiz & Rosswog
  - Strubbe
  - Lodato, Rossi

theoretical predictions
of jets / super Eddington
outflows from tidal disruption

- Summary of Radio Observations
- Modeling the data
  - Evidence for Relativistic Outflow
  - Energy injection?
  - Density structure around (previously quiescent) black hole
- Conclusions: Future/ongoing work



Relativistic Expansion

2. Interstellar scintillation:

$$\nu_o \approx 10\ \mathrm{GHz}$$

$$\theta_{F,0} \approx 1\ \mu\mathrm{as}$$

for $\nu > \nu_0$

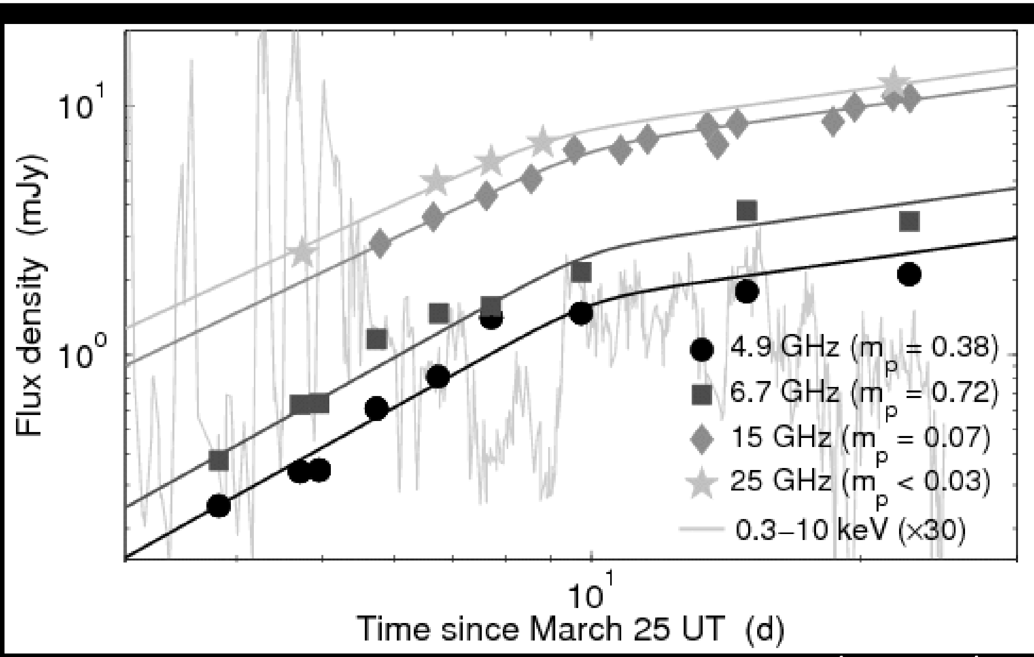
$$m_p \propto (\nu/\nu_0)^{-17/12}\,(\theta_s/\theta_{F,0})^{-7/6}$$

for $\nu < \nu_0$

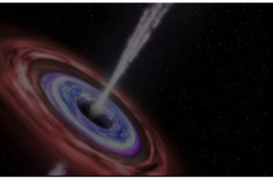
$$m_p \propto (\nu/\nu_0)^{17/30}\,(\theta_s/\theta_r)^{-7/6}$$

$$\theta_s \approx 5\ \mu\mathrm{as}$$

$$\Gamma \approx \mathrm{few}$$



Zauderer et al.



Radio Monitoring of Sw1644+57

- Summarize radio monitoring efforts of Sw1644+57
- Contribution to overall understanding of high energy transients with radio observations / context
- (1) Our monitoring program:
    - 2.5 years of radio observations (500 MHz – 345 GHz)
- (2) Radio observations complement optical, IR, NIR and high energy (X-ray/gamma-ray observations) and provide unique insights
-
- (3) What we have learned and open questions…

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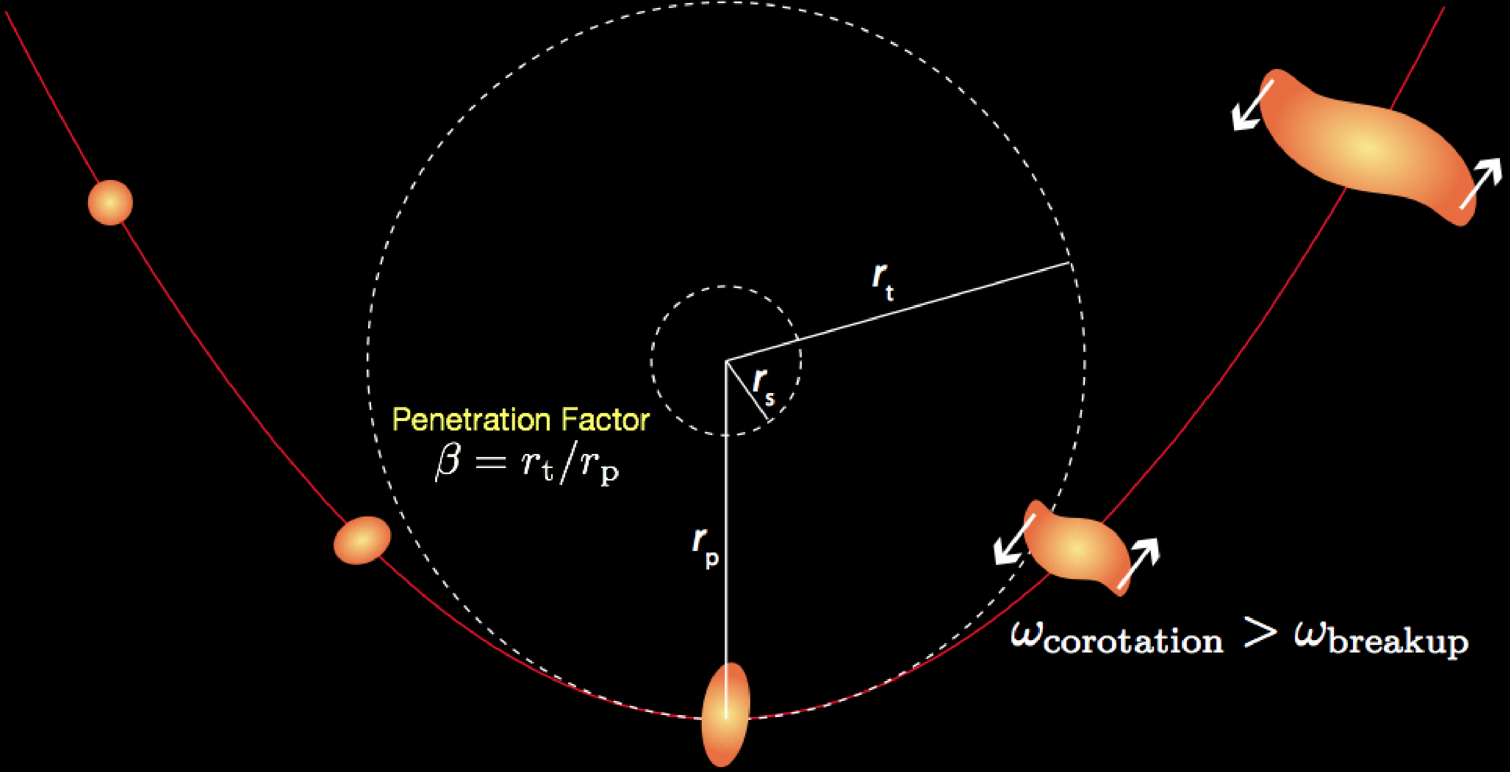

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$\beta = r_{\rm t}/r_{\rm p}$

$\omega_{\rm corotation} > \omega_{\rm breakup}$