The Fate of Debris for a Tidally Disrupted Star

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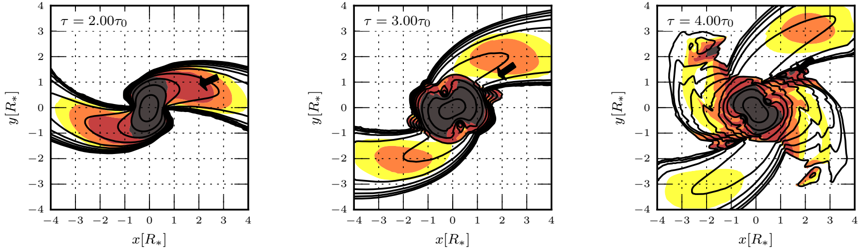
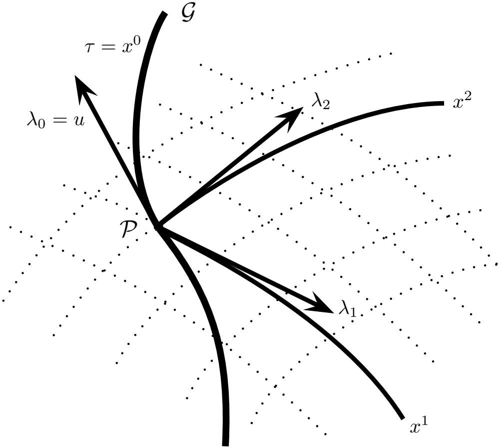


Tidal disruption in Fermi normal coordinates

Metric expansion along a geodesic in the black hole spacetime



FNC method

- local moving frame centered on the star
- relativistic tidal interaction calculation
- uses 3D Newtonian hydrodynamics (PPMLR)
- 3D self-gravity (FFTs)

Parametrizing the tidal interaction

Disruption parameter

$$\eta = \frac{t_{\rm orb}}{t_*} = \left(\frac{R_p^3}{GM} \frac{GM_*}{R_*^3} \right)^{1/2}$$

Disruption: $\eta = 1$

$$R_T \equiv R_p \simeq R_* \left(\frac{M_{bh}}{M_*} \right)^{1/3} \quad \text{(Tidal radius)}$$

Partial: $1 < \eta \lesssim 3$
Weak: $\eta \gtrsim 3$

Mass ratio: $\mu = \frac{M_*}{M_{bh}}$

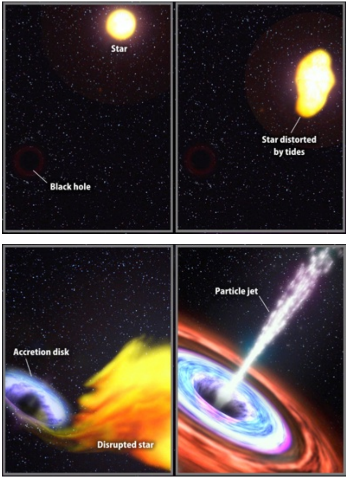
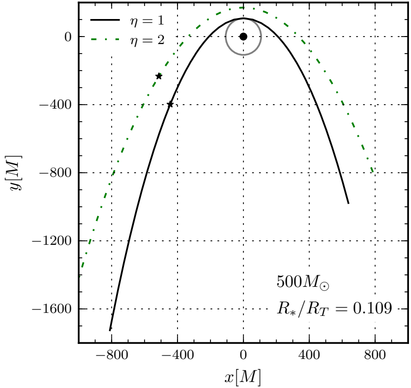
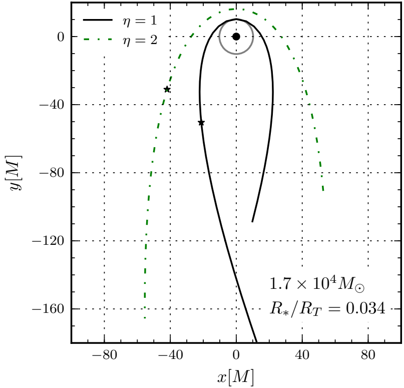


Image: NASA/Goddard Space Flight Center/Swift

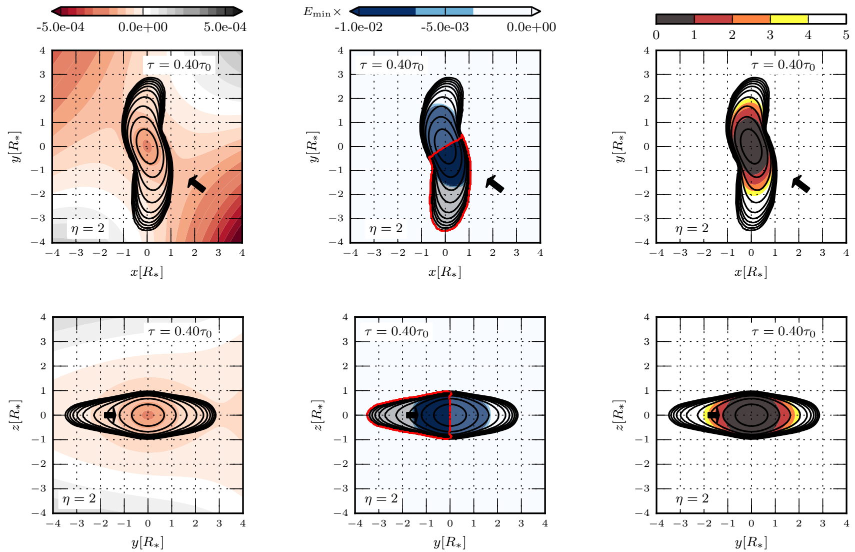
White dwarf – Intermediate mass black hole encounters

Timescale: $\tau_0 = 10.1s$
Mass: $M_* = 0.64 M_\odot$, **Radius:** $R_* = 8.6 \times 10^8$ cm

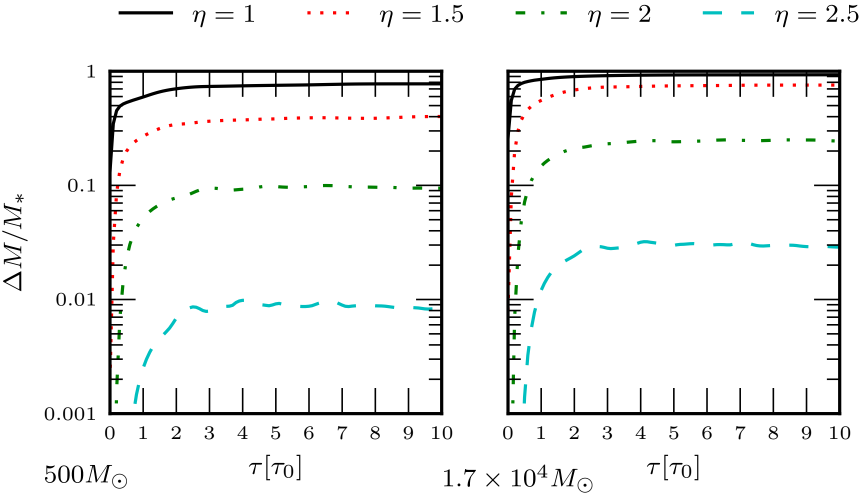




FNC domain: combined gravity field, binding/orbital energy, Mach flow

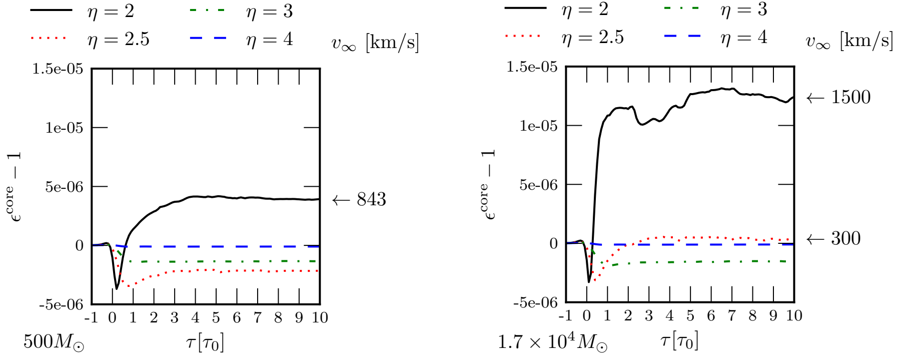


Mass loss during a tidal encounter with a black hole



Kicks at infinity: asymptotic speed v_∞ of unbound core

Largest unbound core speed in our simulations is 1500 km/s for the larger mass black hole and 850 km/s for the smaller mass black hole



Compare with Faber et al. (2005) and Manukian et al. (2013)

Estimating the rate of return

Most bound debris: change in Newtonian gravitational potential across the stellar radius at periastron

$$\Delta\epsilon_0^N = \frac{GM}{R_p}\frac{R_*}{R_p} = \epsilon_p \mu^{1/3}\eta^{-2/3} = \epsilon_* \mu^{-1/3}\eta^{-4/3}$$

Minimum semi-major axis

$$a_m = \frac{GM}{2|\Delta\epsilon_0^N|}$$

Minimum return period

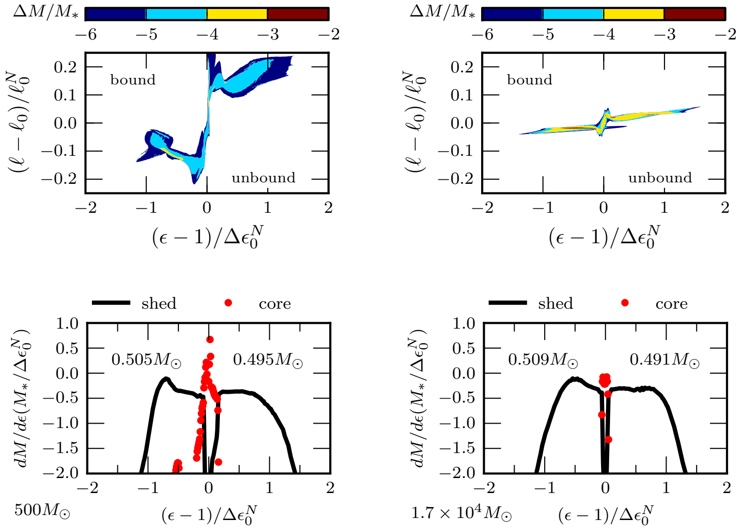
$$P_m = \frac{\pi}{\sqrt{2GM}} R_p^3 R_*^{-3/2}$$

Rate of first return of debris to periastron

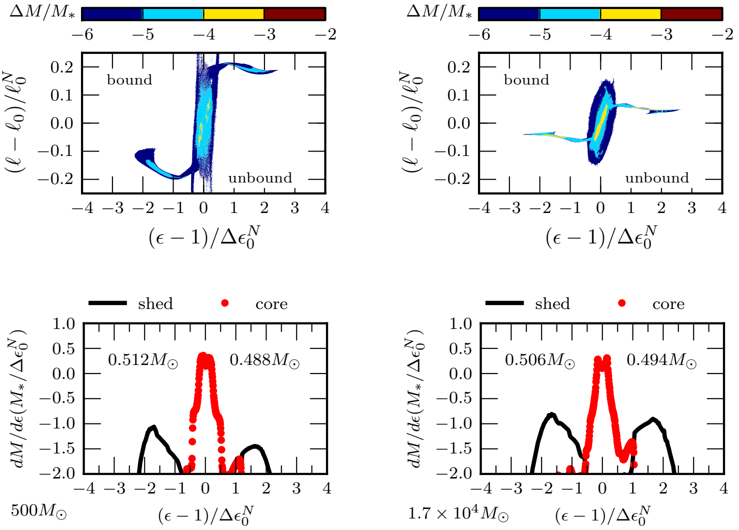
$$\frac{dM}{dt} = \frac{dM}{d\epsilon}\frac{d\epsilon}{dt} = \frac{1}{3}\frac{M_*}{P_m}\left(\frac{t}{P_m}\right)^{-5/3}$$

(Rees, 1988; Phinney, 1989; Evans & Kochanek,1989)

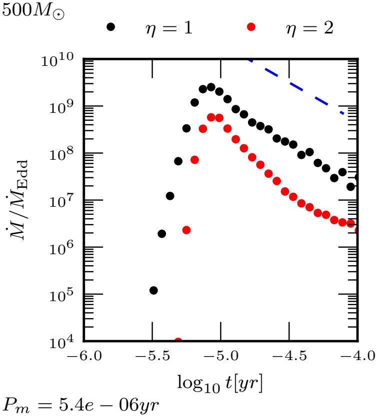
Orbital parameters of the debris: $\eta = 1$ full disruption

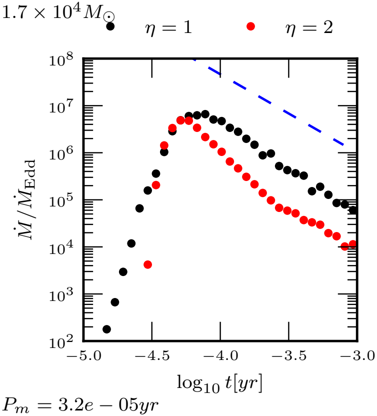


Orbital parameters of the debris: $\eta = 2$ partial disruption



Super-Eddington return rates expected for $< 10^6 M_\odot$ black holes



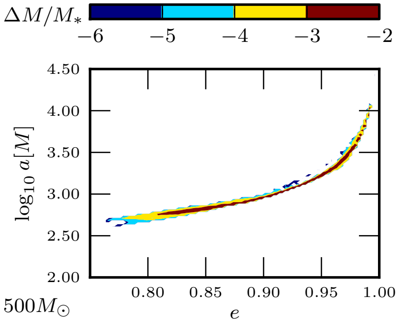


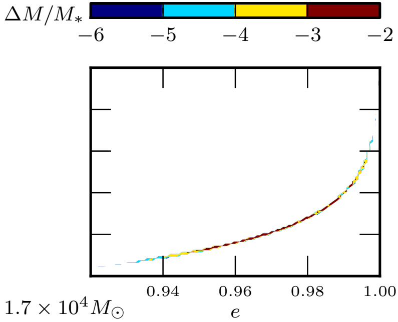
Full disruption $\eta = 1$ closely follows $t^{-5/3}$ fall-off

Extent of debris in black hole frame: $\eta = 1$ full disruption

$R_* = 11.7M$ $R_p = 107.5M$
$a_m \sim 500M$

$R_* = 0.344M$ $R_p = 10.2M$
$a_m \sim 150M$





More intersecting orbits for $500M_\odot$?

Conclusions and future work

Relativistic encounters between white dwarfs and massive non-spinning black holes at the threshold of disruption

- calculate "kick" of remnant core off of the initial orbit
- estimate orbital parameters of debris, treated as ballistic
- caveat: new calculations needed to fully address the self-gravity of the streams (Kochanek,1994; Guillochon et al., 2013)
- results highlight distinctions in encounters at different mass ratio
- provide initial conditions for follow-up fully GRMHD simulation

Future work with FNC code

- include the interaction due to a spinning black hole
- apply the same procedure to main sequence star disruptions

FNC metric series expansion in spatial distance

FNC metric: derived to quadratic order by Manasse & Misner (1963) and extended to fourth order by Ishii, Shibata, & Mino (2005)

$$
\begin{aligned}
g_{00} &= -1 - C_{ij}x^i x^j - \tfrac{1}{3} C_{ijk} x^i x^j x^k \\
&\quad - \tfrac{1}{12}\left(C_{ijkl} + 4C_{(ij}C_{kl)} - 4B_{(kl|n|}B_{ij)n}\right) x^i x^j x^k x^l + \cdots \\
g_{0m} &= \tfrac{1}{3} B_{ijm} x^i x^j + \tfrac{1}{4} R_{m(ij|0|;k)} x^i x^j x^k + \cdots \\
g_{mn} &= \delta_{mn} + \tfrac{1}{6}\left(R_{imnj} + R_{inmj}\right) x^i x^j + \cdots
\end{aligned}
$$

Transform black hole spacetime Riemann tensor using FNC tetrads

$$R_{abcd} = R_{\mu'\alpha'\nu'\beta'} \lambda_a{}^{\mu'} \lambda_b{}^{\alpha'} \lambda_c{}^{\nu'} \lambda_d{}^{\beta'}$$

Fractional error in approximation

$\mathcal{E} \sim |r_{\mathrm{FNC}}/R_0|^2 \to |R_*/R_p|^2$

Tidal tensors

$$
\begin{aligned}
C_{ij} &\equiv R_{0i0j}, \\
C_{ijk} &\equiv R_{0(i|0|j;k)}, \\
C_{ijkl} &\equiv R_{0(i|0|j;kl)}, \\
B_{ijk} &\equiv R_{k(ij)0}
\end{aligned}
$$

Superposition of tidal gravity and stellar self-gravity fields

White dwarf: $\Phi_* = \frac{GM_*}{R_*} = 1.1 \times 10^{-4} \ll 1$

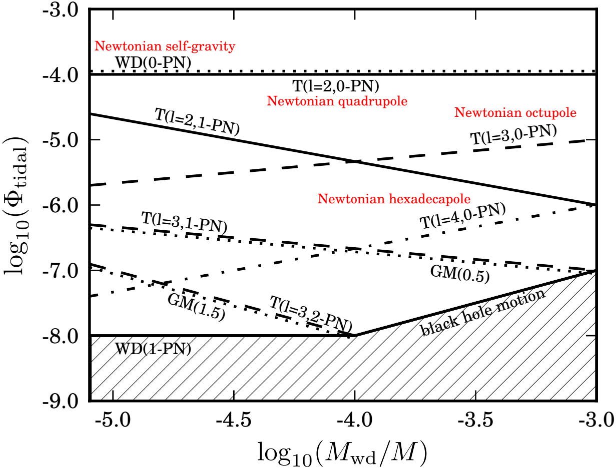
Neglect terms on the level of stellar post-Newtonian correction

$$
\begin{aligned}
g_{00} &= -1 - 2\Phi_* + h_{00}^{\rm tidal} \Big| + h_{00}^{\rm int} + \mathcal{O}\left(v_*^4\right), \\
g_{0i} &= h_{0i}^{\rm tidal} \Big| + h_{0i}^{\rm int} + \mathcal{O}\left(v_*^3\right), \\
g_{ij} &= \delta_{ij} \Big| + h_{ij}^{\rm tidal} + h_{ij}^{int} + \mathcal{O}\left(v_*^2\right),
\end{aligned}
$$

Tidal potential

$$
\begin{aligned}
\Phi_{\rm tidal} &= -\frac{1}{2}(g_{00} + 1) \\
&= \frac{1}{2} C_{ij} x^i x^j + \frac{1}{6} C_{ijk} x^i x^j x^k \\
&+ \frac{1}{24}\left[C_{ijkl} + 4 C_{(ij} C_{kl)} - 4 B_{(kl|n|} B_{ij)n}\right] x^i x^j x^k x^l
\end{aligned}
$$

Retained terms in tidal expansion of the combined metric



$$\left|C^{(0)}_{ij}\, x^i x^j\right| \lesssim \frac{M}{R_p^3}\, R_*^2 \quad \text{(\textit{Newtonian})}$$

$$\left|C^{(1)}_{ij}\, x^i x^j\right| \lesssim \frac{M}{R_p^3}\, R_*^2\, \delta^2$$

$$\left|C^{(0)}_{ijk}\, x^i x^j x^k\right| \lesssim \frac{M}{R_p^4}\, R_*^3 \quad \text{(\textit{Newtonian})}$$

$$\left|C^{(1)}_{ijk}\, x^i x^j x^k\right| \lesssim \frac{M}{R_p^4}\, R_*^3\, \delta^2$$

$$\left|C^{(2)}_{ijk}\, x^i x^j x^k\right| \lesssim \frac{M}{R_p^4}\, R_*^3\, \delta^4$$

$$\left|C^{(0)}_{ijkl} x^i x^j x^k x^l\right| \lesssim \frac{M}{R_p^5}\, R_*^4 \quad \text{(\textit{Newtonian})}$$

Accuracy of the FNC method

Quiet equilibrium model, calculates energy deposited onto star, relates spin-up of star to change in orbital angular momentum