Efficient & intuitive model building with Szekeres models

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Fast crash course on Szekeres models

Metric

$$ds^2 = -dt^2 + A^2 dr^2 + B^2(dx^2 + dy^2),$$

where $A = A(t, r, x, y), \qquad B = B(t, r, x, y)$

Usual approach

Apply field equations to the METRIC functions.

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Analytic & semi-analytic solutions from a Friedman equation.

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Compute Density & other quantities

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Build models of cosmic structure → **NUMERICALLY Solve null geodesic equations to FIT OBSERVATIONS**

Alternative approach: look at covariant objects NOT metric functions

Density $\rho = -T_{ab}u^a u^b$

Hubble scalar $\mathcal{H} = \frac{1}{3}\tilde{\nabla}_a u^a$

Shear tensor $\sigma_{ab} = \tilde{\nabla}_{(a}u_{b)} - \mathcal{H}\, h_{ab} = \Sigma\, X_{ab}$

Electric Weyl tensor $E_{ab} = u^c u^d\, C_{acbd} = \mathcal{E}\, X_{ab},$

Local covariant scalar representation

$$\{\rho,\, \mathcal{H},\, \Sigma,\, \mathcal{E},\, K\}$$

Spatial curvature

The dynamics in terms of evolution equations for these scalars (1+3 system)

Szekeres $\{\rho,\, \mathcal{H},\, K,\, \Sigma,\, \mathcal{E}\}$

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FLRW $\{\rho,\, \mathcal{H},\, K\}$ $\{\Sigma = \mathcal{E} = 0\}$

Propose a solution based on assuming "EXACT" perturbation forms:

$$\rho = \rho_q \left[1 + \delta^{(\rho)}\right], \qquad \mathcal{H} = \mathcal{H}_q \left[1 + \delta^{(\mathcal{H})}\right]$$

where: $\{\, \rho_q \,,\, \mathcal{H}_q \,\}$ **are SZEKERES scalars that satisfy FLRW dynamics**

→ **"background" variables**

and: $\{\, \delta^{(\rho)} \,,\, \delta^{(H)} \,\}$ **are obtained from the 1+3 system**

→ **exact "perturbations"**

We transform Szekeres dynamics into evolution equations for EXACT & COVARIANT perturbations on FLRW:

$$\dot{\rho}_q = -3\,\rho_q\,H_q,$$

$$\dot{H}_q = -H^2 - \frac{4\pi}{3}\rho_q,$$

background variables

$$\dot{\delta}^{(\rho)} = -3\,\left(1+\delta^{(\rho)}\right)\,H_q\,\delta^{(H)}$$

$$\dot{\delta}^{(H)} = -\left[\left(1+3\delta^{(H)}\right)\,\delta^{(H)} - \frac{\Omega_q}{2}\,\left(\delta^{(H)}-\delta^{(\rho)}\right)\right]\,H_q,$$

exact perturbations

$$H_q^2 = \frac{8\pi}{3}\rho_q - K_q, \qquad \Omega_q = \frac{8\pi\rho_q}{3H_q^2}$$

$$2\,\delta^{(H)} = \Omega_q\,\delta^{(\rho)} + (1-\Omega_q)\,\delta^{(K)}$$

$$\delta^{(\Omega)} = \delta^{(\rho)} - 2\delta^{(H)}$$

constraints

Algebraic constraints: → **Autonomous ODE's:**

DYNAMICAL SYSTEM !!

Obtain relevant scalars as exact perturbations.

$$\rho = \rho_q \left[1 + \delta^{(\rho)}\right], \qquad \mathcal{H} = \mathcal{H}_q \left[1 + \delta^{(\mathcal{H})}\right]$$

→ $\rho_q,\ \mathcal{H}_q,\ \mathcal{K}_q,\ \Omega_q$ depend ONLY on (t, r)

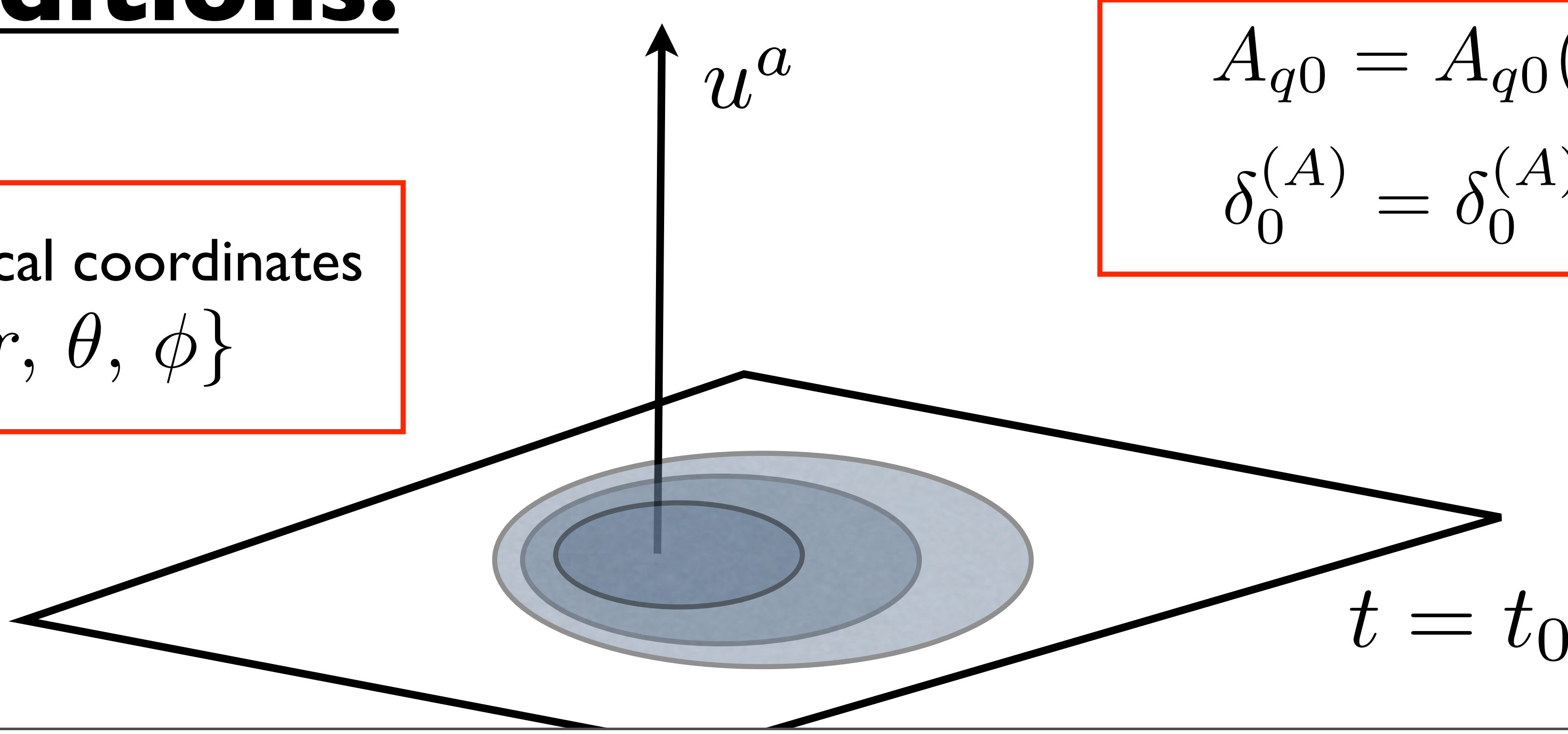
→ $\delta^{(\rho)},\ \delta^{(\mathcal{H})},\ \delta^{(K)},\ \delta^{(\Omega)}$ depend on (t, r, x, y)

Initial conditions:

$$A_{q0} = A_{q0}(r)$$

$$\delta_0^{(A)} = \delta_0^{(A)}(r, \theta, \phi)$$

Spherical coordinates $\{r,\ \theta,\ \phi\}$



The EXACT perturbations provide an invariant measure of inhomogeneity

$$\delta^{(\rho)} = \frac{\xi}{1-\xi}, \qquad \xi \equiv \frac{\psi_2}{\mathcal{R}}$$

Ratio of Weyl to Ricci curvature.

$$\delta^{(H)} = -\frac{\zeta}{1-\zeta}, \qquad \zeta \equiv \frac{\Sigma}{H}$$

where: Σ is the eigenvalue of σ_{ab}

Ratio of anisotropic to isotropic expansion.

EXACT Density Modes:

EXACT Density perturbation:

$$\delta^{(\rho)} = \frac{\mathcal{J}_{(g)} + \mathcal{J}_{(d)}}{1 - \mathcal{J}_{(g)} - \mathcal{J}_{(d)}}$$

EXACT growing mode

$$\mathcal{J}_{(g)} = 3\Delta_0^{(g)} \left[\mathcal{H}_q \left(t - t_{\rm bb} \right) - \frac{2}{3} \right]$$

Exact Decaying mode

$$\mathcal{J}_{(d)} = 3\Delta_0^{(d)} \, \mathcal{H}_q$$

Amplitudes:
Initial Conditions

$$\Delta_0^{(g)} = \Delta_0^{(g)}(r, x, y), \quad \Delta_0^{(d)} = \Delta_0^{(d)}(r, x, y)$$

Near big bang $t \to t_{\mathrm{bb}}$

$$\mathcal{J}_{(g)} \to 0, \quad \mathcal{J}_{(d)} \to \infty$$

Near max expansion or asymptotic times

$$\mathcal{J}_{(g)} \to \mathcal{J}_{\mathrm{as}}, \quad \mathcal{J}_{(d)} \to 0$$

The "Goode-Wainwright" variables can be derived from the exact forms of $\mathcal{J}_{(g)}, \mathcal{J}_{(d)}$

Analytic work: initial value formulation

$$ds^2 = -dt^2 + a^2 \left[\Gamma^2 dr^2 + r^2(dx^2 + dy^2)\right],$$

$$a = a(t,r), \qquad \Gamma = \Gamma(t,r,x,y)$$

FLRW-like scaling laws for the (t,r) variables

$$\rho_q = \frac{\rho_{q0}}{a^3}, \qquad K_q = \frac{K_{q0}}{a^2}, \qquad \Omega_q = \frac{\Omega_{q0}}{\Omega_{q0} - (\Omega_{q0} - 1)\, a},$$

$$\mathcal{H}_q^2 = \mathcal{H}_{q0}^2 \left[\frac{\Omega_{q0}}{a^3} - \frac{\Omega_{q0} - 1}{a^2}\right],$$

Non-sphericity of (x,y) variables through initial values of perturbations

$$1 + \delta^{(\rho)} = \frac{1 + \delta_0^{(\rho)}}{\Gamma}, \qquad \frac{2}{3} + \delta^{(K)} = \frac{2/3 + \delta_0^{(K)}}{\Gamma},$$

$$2\delta^{(\mathcal{H})} = \Omega_q\, \delta^{(\rho)} - (\Omega_q - 1)\, \delta^{(K)},$$

$$\Gamma = \Gamma(t,r,x,y) = (1 + \delta_0^{(\rho)})[1 - \mathcal{J}_{(g)} - \mathcal{J}_{(d)}],$$

Szekeres vs LTB models

All formal theoretical results of LTB models hold for Szekeres models (with some modifications)

Sussman & Bolejko, Class Quant Grav 2012

for example perturbations:

$$\delta_{\mathrm{sz}}^{(A)} = \frac{\delta_{\mathrm{ltb}}^{(A)}(t,r)}{1 - f(r,\theta,\phi)}$$

Dipolar deviation from sphericity specified as part of setting up initial conditions.

THANKS FOR YOUR ATTENTION