Large-Scale Growth Evolution in the Szekeres Inhomogeneous Cosmological Models with Comparison to Growth Data

Austin Peel (University of Texas at Dallas)

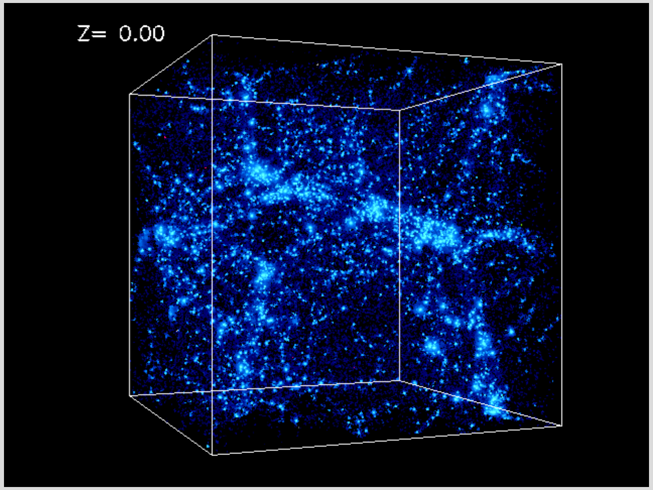
Texas Symposium 2013

Collaborators: Mustapha Ishak & Michael Troxel

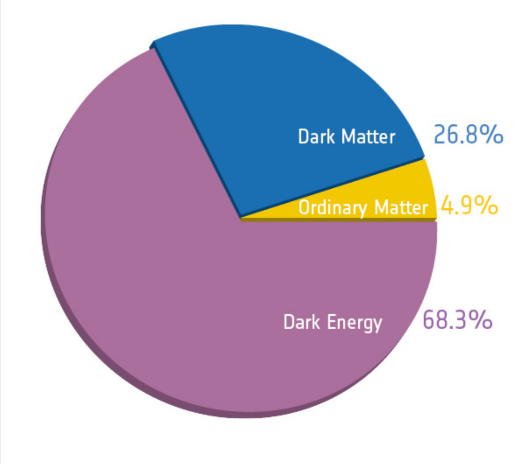
(Peel, Ishak, & Troxel, Physical Review D 2012)

Introduction/Motivation

- Universe inhomogeneous on all but largest scales
- Effect of nonlinear structure on observations and dynamics not fully understood
- Limited regime of linear perturbations
- Precise and accurate cosmology



Center for Cosmological Physics (U. Chicago)



ESA/Planck

- New insights into the contents of the universe?

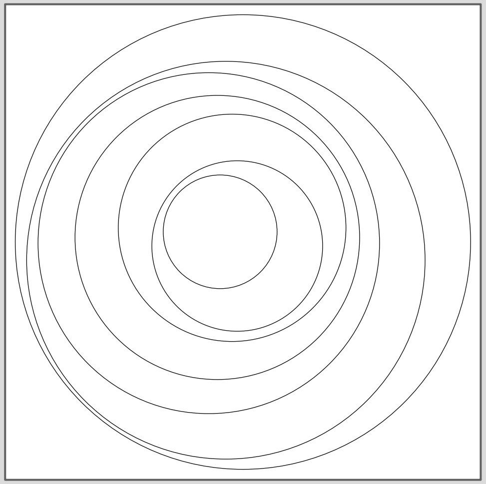
➡ Use exact solutions of EFE to study growth rate of large-scale structure

The Szekeres Solution

- Discovered by **P. Szekeres** in 1975
- Inhomogeneous and anisotropic exact solutions to EFE for dust
- Contain LTB and FLRW metrics as special cases
- Different formulations including one by **C. Hellaby** and another by **S. Goode** and **J. Wainwright**

Class I good for modeling structures; could be used like LTB for large local underdense anisotropic void;
CMB constrains anisotropy

Class II regarded as exact perturbations of a homogeneous background



Goode and Wainwright Formulation

- Class II **metric**

$$ds^2 = -\mathrm{d}t^2 + a^2 \left(H^2 \mathrm{d}r^2 + \frac{\mathrm{d}x^2 + \mathrm{d}y^2}{[1 + \frac{k}{4}(x^2 + y^2)]^2} \right)$$

- scaling function $a(t)$ obeys Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2M}{a^3} - \frac{k}{a^2} + \frac{\Lambda}{3} \qquad \dot{} = \frac{\partial}{\partial t}$$

- M constant and $k \in \{-1, 0, +1\}$

Growth Equations

- $$H(t,r,x,y) = A(r,x,y) - F(t,r)$$
- and $F(t,r)$ satisfies the linear differential equation in time
$$\ddot{F} + 2\frac{\dot{a}}{a}\dot{F} - \frac{3M}{a^3}F = 0$$
- density
$$\rho(t,r,x,y) = \frac{6MA}{a^3 H} = \frac{6M}{a^3}\left(1 + \frac{F}{H}\right)$$
$$= \bar{\rho}(t)\,(\,1\,+\,\delta\,)$$

$$\Longrightarrow \quad \ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - \frac{3M}{a^3}\delta - \frac{2}{1+\delta}\dot{\delta}^2 - \frac{3M}{a^3}\delta^2 = 0$$

linear part like FLRW — nonlinear part

1. Define by analogy with FLRW:

$$\left(\frac{\dot{a}}{a}\right)^2 := \mathbb{H}^2$$

$$\Omega_m(t) := \frac{2M}{a^3(t)\mathbb{H}^2(t)}$$

$$\Omega_\Lambda(t) := \frac{\Lambda}{3\mathbb{H}^2(t)}$$

$$\Omega_k(t) := \frac{-k}{a^2(t)\mathbb{H}^2(t)}$$

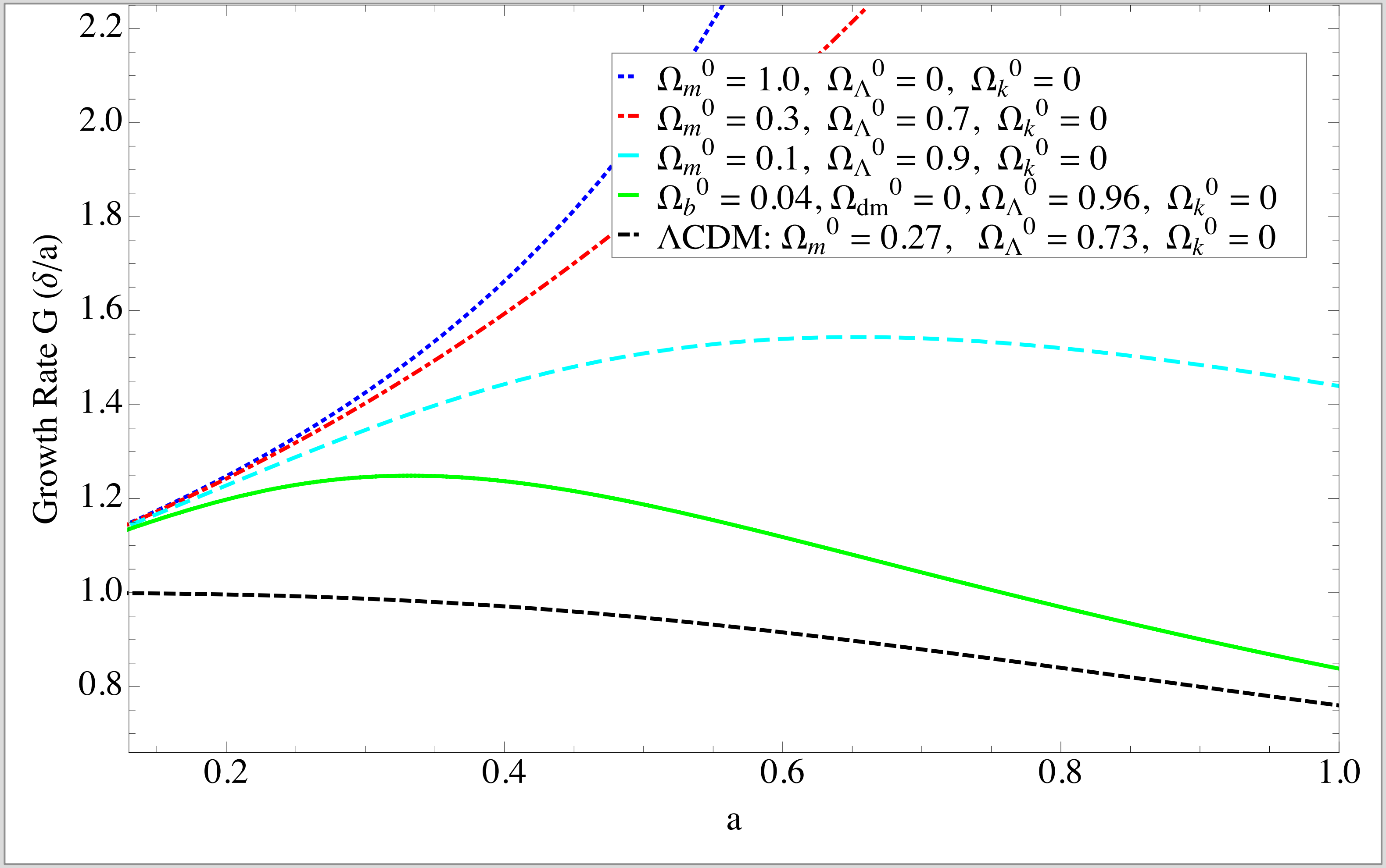
2. Recast equation for δ in terms of density parameters.

$$\delta'' + \left(\frac{4 + 2\Omega_\Lambda - \Omega_m}{2a}\right)\delta' - \frac{3}{2}\frac{\Omega_m}{a^2}\delta - \frac{2}{1+\delta}\delta'^2 - \frac{3}{2}\frac{\Omega_m}{a^2}\delta^2 = 0$$

$$' = d/da$$

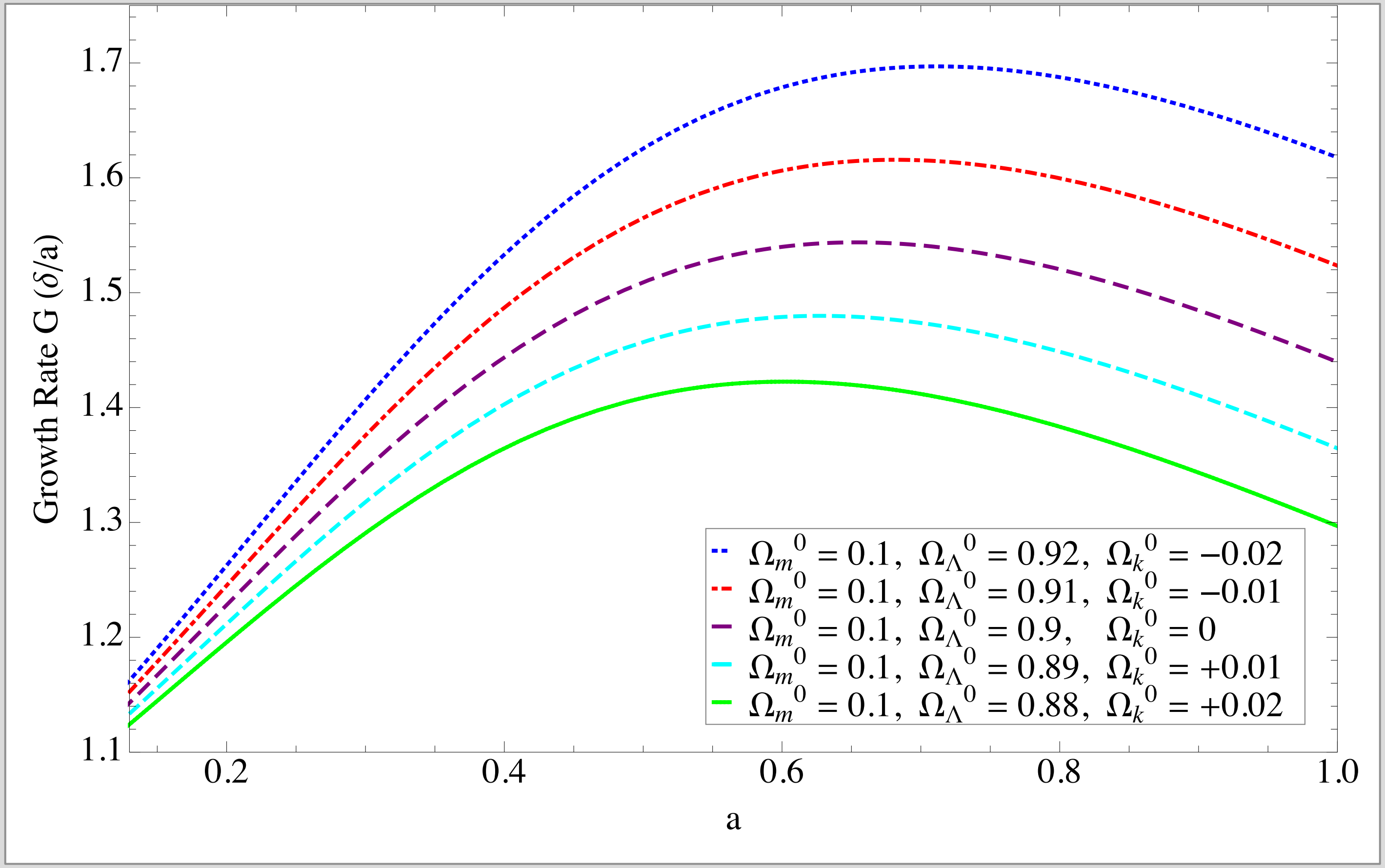
3. Solve.

Results I (flat)



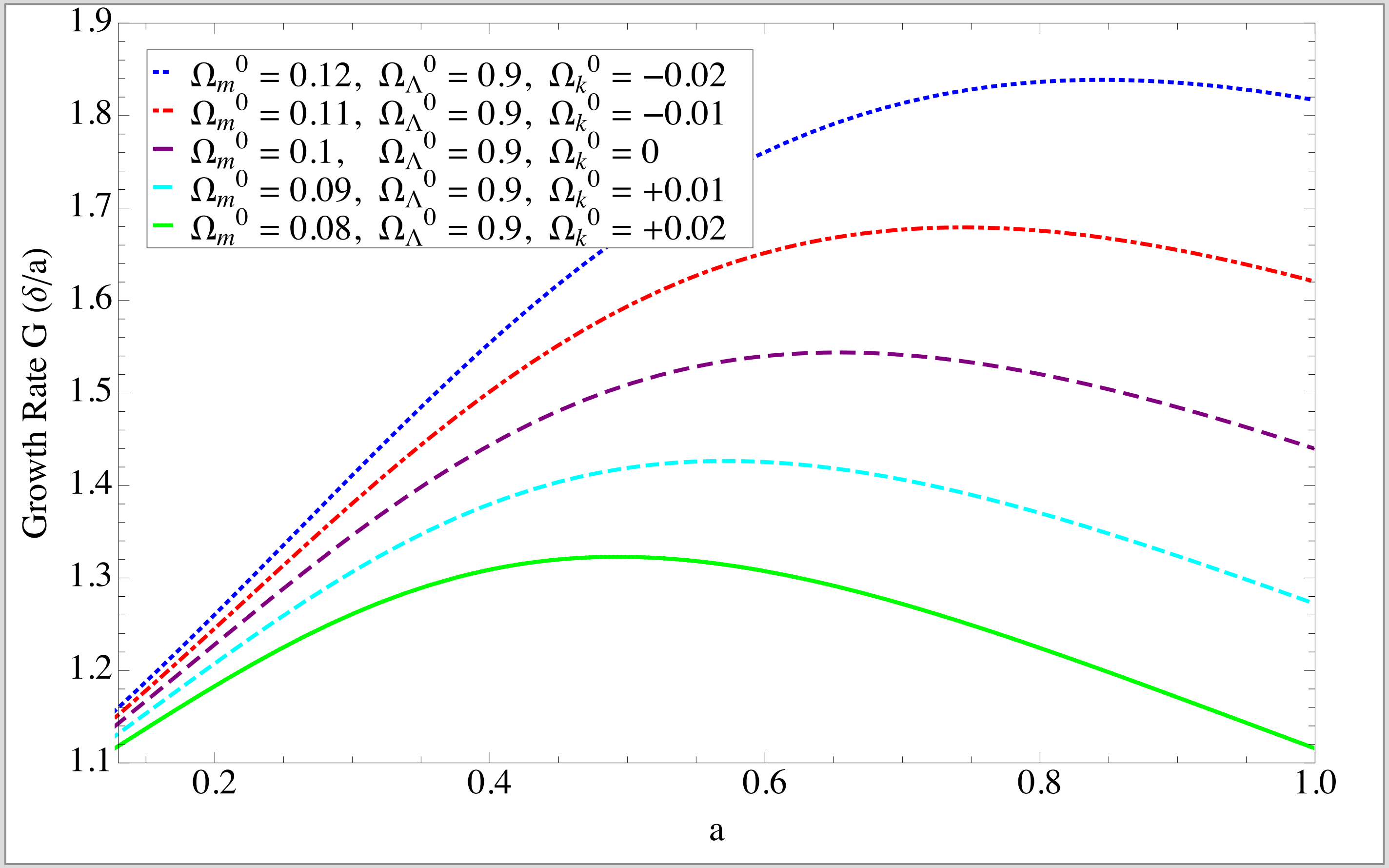
set $(\Omega_m^0, \Omega_\Lambda^0, \Omega_k^0) = (0.11, 0.71, 0.18)$ almost exactly reproduces $\Lambda\mathrm{CDM}$

Results II (curved, $\Omega_m^0 = 0.1$)



higher growth rate for positively curved models (larger Lambda contribution) than for negatively curves ones with the same matter content

Results III (curved, $\Omega_\Lambda^0 = 0.9$)



higher growth rate for positively curved models (larger matter content) than for negatively curves ones with the same Lambda contribution

Growth Factor in Szekeres

- growth factor

$$f = \frac{d \ln \delta}{d \ln a}$$

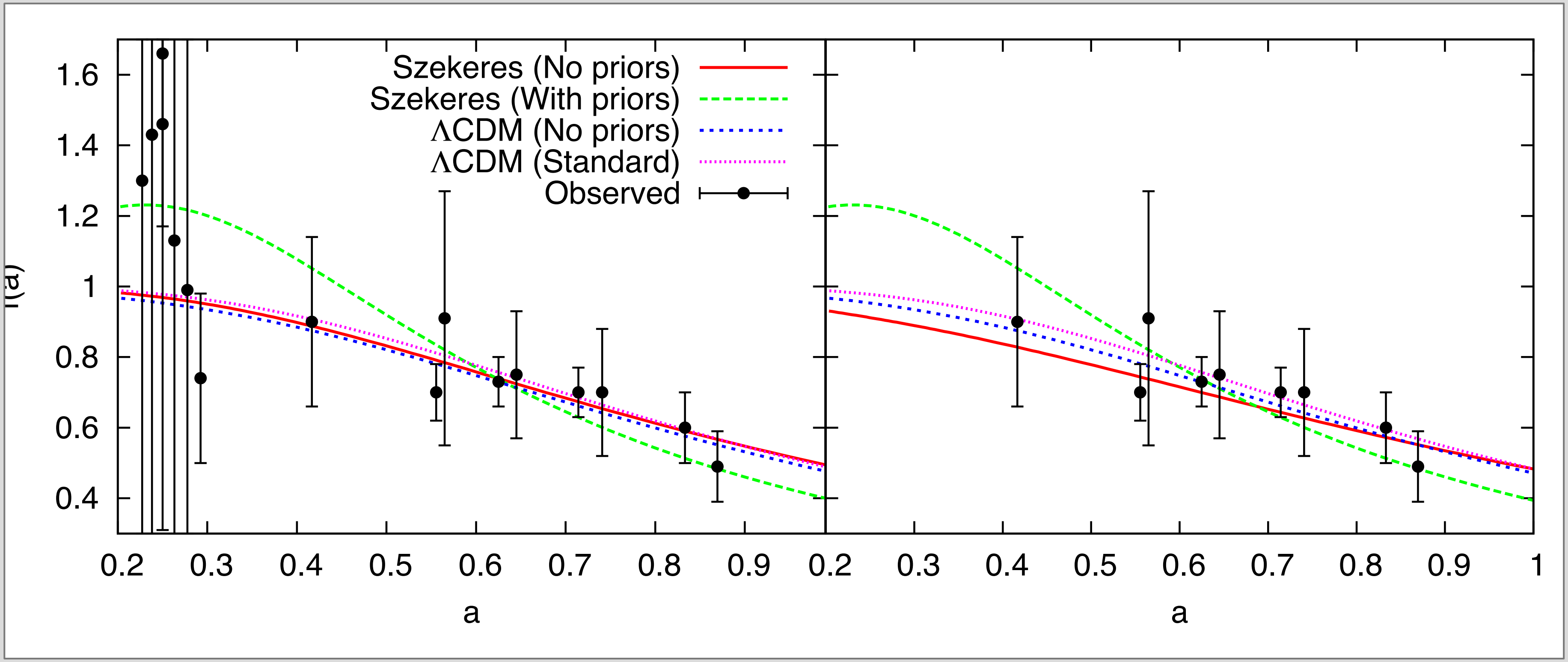
FLRW $$f' + \left(2 + \frac{\dot{H}}{H^2}\right) f + f^2 - \frac{3}{2}\Omega_m = 0$$

	$L\alpha$?	Without priors Ω_m^0	Ω_Λ^0	Ω_k^0	(χ^2)	With priors Ω_m^0	Ω_Λ^0	Ω_k^0	(χ^2)
Szekeres	No	0.12	0.59	0.29	(0.22)	0.05	0.98	-0.03	(0.62)
	Yes	0.11	0.69	0.20	(0.39)	0.05	0.98	-0.03	(0.63)
ΛCDM	No	0.29	0.56	0.15	(0.21)	0.27	0.73	0.00	(0.32)
	Yes	0.26	0.69	0.05	(0.39)	0.27	0.73	0.00	(0.43)

Szekeres $$f' + \left(1 + \Omega_\Lambda - \frac{1}{2}\Omega_m\right) f + \left(1 - \frac{2}{1+\delta^{-1}}\right) f^2 - \frac{3}{2}(1+\delta)\,\Omega_m = 0$$

- χ^2 minimization with and without priors
- caveat: data reduced assuming ΛCDM

Fitting Results



LEFT: including Lyman-α **RIGHT**: excluding Lyman-α

Summing up

- Szekeres models have lumpiness built in (exact nonlinearities)
- Szekeres growth rate stronger than in ΛCDM
- Using inhomogeneous models strongly impacts determination of cosmological parameters
  best fit: $(\Omega_m^0, \Omega_\Lambda^0, \Omega_k^0) = (0.12, 0.59, 0.29)$
- Negligible difference in fitting power between best fit Szekeres parameters and ΛCDM
- Best fit Szekeres requires less matter and more spatial curvature
- Nonzero curvature consistent with other studies of inhomogeneous models and averaging
- Hints that Lambda needed for suppression
  (Ishak, Peel, and Troxel accepted PRL)

References

[1] A. Peel, M. Ishak, and M. A. Troxel, Phys. Rev. D **86**, 123508 (2012)

[2] M. Ishak and A. Peel, Phys. Rev. D **85**, 083502 (2012)

[3] M. Ishak, A. Peel, and M. A. Troxel, accepted PRL (2013)

[4] P. Szekeres, Commun. Math. Phys. **41**, 55 (1975)

[5] P. Szekeres, Phys. Rev. D **12**, 2941 (1975)

[6] S. Goode and J. Wainwright, Mon. Not. R. Astron. Soc. **198**, 83 (1982)

[7] S. Goode and J. Wainwright, Phys. Rev. D **26**, 3315 (1982)