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Gravitation of discrete inhomogeneities for different topologies of Universe

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Outline

The following spatial topologies are studied:

- **flat (R^3 = R x R x R, K = 0);**
- **spherical (S^3, K = +1);**
- **hyperbolic (H^3, K = –1);**
- **toroidal ("lattice" Universe, T x T x T, K = 0);**
- **~~mixed (R x R x T, K = 0);~~**
- **~~mixed (R x T x T, K = 0).~~**

LANDAU

PEEBLES

SPRINGEL

The nonrelativistic gravitational potential is defined by positions of galaxies, but not by their velocities!

Preface

mechanical approach / discrete cosmology

The late Universe inside the cell of uniformity (less than 150 Mpc) is described by the slightly perturbed "ΛCDM" model:

– **the metrics** $ds^2 \approx a^2\left\{(1+2\Phi)d\eta^2 - (1-2\Phi)\gamma_{\alpha\beta}dx^\alpha dx^\beta\right\}$

– **the gravitational potential** $\Phi(\eta,\mathbf{r}) = \dfrac{\varphi(\mathbf{r})}{c^2 a(\eta)}$

$$\Delta\varphi + 3K\varphi = 4\pi G_N(\rho - \overline{\rho})$$

$$\delta\varepsilon_{\mathrm{rad}} = -\frac{3\overline{\rho}\varphi}{a^4}$$

I. Flat ($R^3 = R \times R \times R$, $K = 0$) type

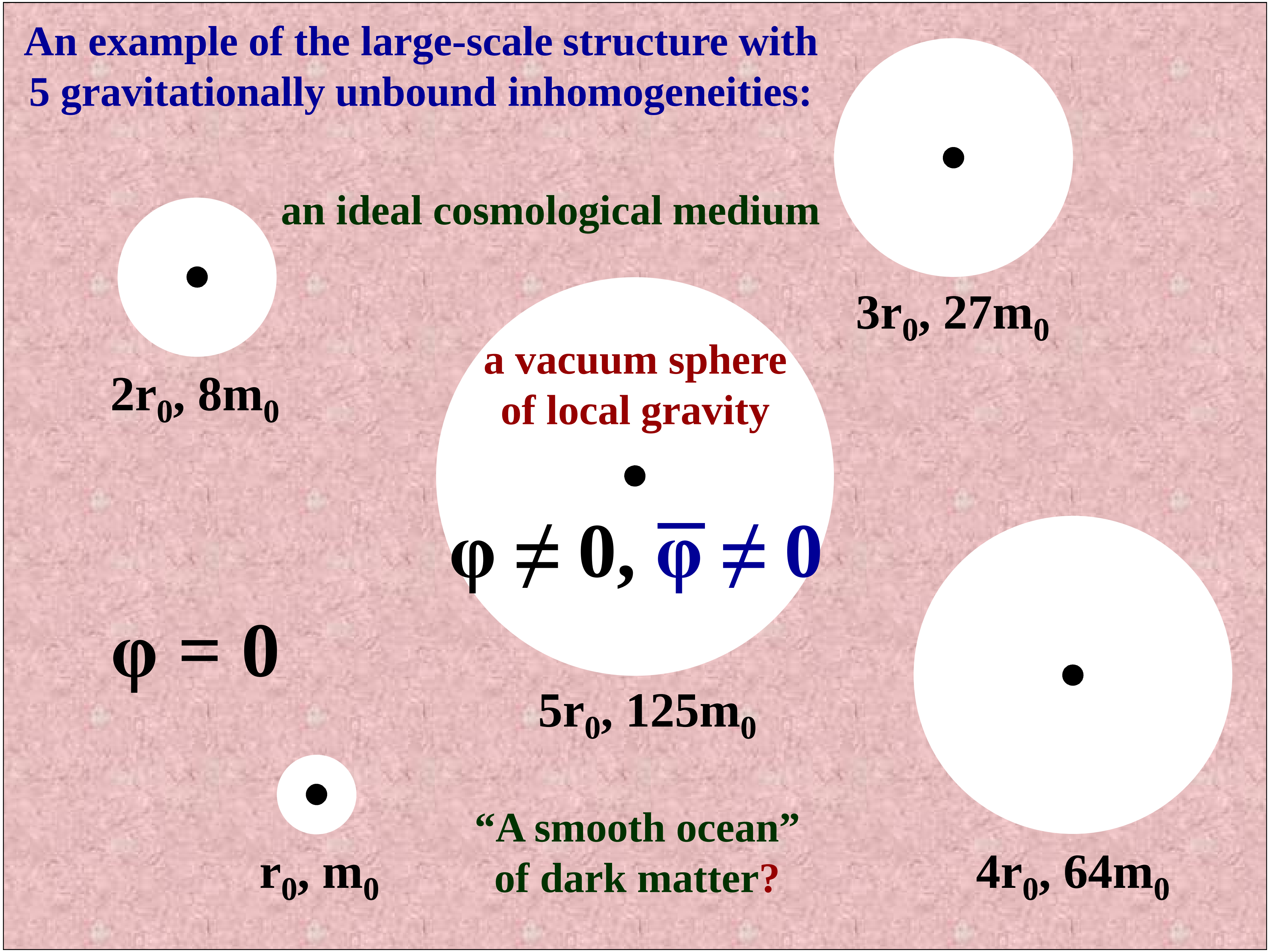
$$\Delta\varphi = 4\pi G_N(\rho - \bar{\rho})$$

The role of the term $-\bar{\rho}$: the gravitational (Neumann-Seeliger) paradox is absent but spatial distribution of inhomogeneities… is not arbitrary!

A sphere of local gravity

$$\varphi = -\frac{G_N m_0}{r} - \frac{G_N m_0}{2r_0^3} r^2 + \frac{3G_N m_0}{2r_0}, \quad r \le r_0$$

$$\varphi = 0, \quad r \ge r_0 \qquad\qquad r_0 = \left(\frac{3m_0}{4\pi\bar{\rho}}\right)^{1/3}$$


An example of the large-scale structure with
5 gravitationally unbound inhomogeneities:
an ideal cosmological medium
$3r_0$, $27m_0$
$2r_0$, $8m_0$
a vacuum sphere
of local gravity
$\varphi \neq 0$, $\overline{\varphi} \neq 0$
$\varphi = 0$
$5r_0$, $125m_0$
r_0, m_0
"A smooth ocean"
of dark matter?
$4r_0$, $64m_0$

The nonzero spatial curvature case

$$dl^2 = \gamma_{\alpha\beta} dx^\alpha dx^\beta = d\chi^2 + \Sigma^2(\chi) d\Omega_2^2$$

$K \neq 0$

$$\Sigma(\chi) = \begin{cases} \sin\chi, & \chi \in [0, \pi], \quad K = +1 \\ \chi, & \chi \in [0, +\infty), \quad K = 0 \\ \sinh\chi, & \chi \in [0, +\infty), \quad K = -1 \end{cases}$$

The Helmholtz equation:

$$\phi = \varphi + \frac{4\pi G_N \bar{\rho}}{3K}$$

$$\Delta\phi + 3K\phi = 4\pi G_N \rho$$

Spatial distribution of inhomogeneities

is absolutely arbitrary!

One gravitating mass in the origin of coordinates:

$$\frac{1}{\Sigma^2(\chi)}\frac{d}{d\chi}\left(\Sigma^2(\chi)\frac{d\phi}{d\chi}\right)+3K\phi=0$$

II. Spherical (S³, K = +1) type ("closed" Universe)

$$\Sigma(\chi)=\sin\chi$$

$$\underbrace{\phi=2C_1\cos\chi-G_N m_0\left(\frac{1}{\sin\chi}-2\sin\chi\right)}_{\text{divergent at } \chi\to\pi}\ \underset{\chi\to 0}{\longrightarrow}\ \underbrace{-\frac{G_N m_0}{\chi}}_{\text{the Newtonian limit at } \chi\to 0}$$

divergent at $\chi\to\pi$

the Newtonian limit at $\chi\to 0$

III. Hyperbolic (H^3, $K = -1$) type ("open" Universe)

$$\Sigma(\chi) = \sinh \chi$$

No gravitational paradox!

$$\phi = -G_N m_0 \frac{\exp(-2\chi)}{\sinh \chi} \xrightarrow[\chi \to +\infty]{} 0$$

The average gravitational potential

$$\bar{\phi}_{\text{total}} = -\frac{4\pi G_N \bar{\rho}}{3} \longrightarrow \bar{\varphi} = 0$$

The average gravitational potential is zero, as it should be!

IV. Toroidal (T x T x T, K = 0) type
("lattice" Universe, periodical distribution)

$$\Delta\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 4\pi G_N \left(m\delta(\mathbf{r}) - \frac{m}{l_1 l_2 l_3} \right)$$

$$x \in [0, l_1), \quad y \in [0, l_2), \quad z \in [0, l_3)$$

$$\varphi = -\frac{G_N m}{\pi l_1 l_2 l_3} \sum_{k_1=-\infty}^{+\infty} \sum_{k_2=-\infty}^{+\infty} \sum_{\substack{k_3=-\infty \\ k_1^2+k_2^2+k_3^2 \neq 0}}^{+\infty} \frac{\cos\left(\frac{2\pi k_1 x}{l_1}\right)\cos\left(\frac{2\pi k_2 y}{l_2}\right)\cos\left(\frac{2\pi k_3 z}{l_3}\right)}{\frac{k_1^2}{l_1^2} + \frac{k_2^2}{l_2^2} + \frac{k_3^2}{l_3^2}}$$

Unfortunately, this gravitational potential is divergent outside the matter sources.


THANK YOU FOR ATTENTION !