Redshift drift and supernovae in inhomogeneous pressure cosmology

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Based upon:

- A. Balcerzak and MPD, Phys. Rev. D**87**, 063506 (2013).
- A. Balcerzak, MPD, T. Denkiewicz, ArXiv: 1312.1567.
- A. Balcerzak, MPD, T. Denkiewicz, and D. Polarski - in preparation.
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- **see also old refs:**
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- MPD, Journ. Math. Phys. **34**, 1447 (1993)
- MPD, Astroph. Journ. **447**, 43 (1995) (0902.2899)
- MPD and M.A. Hendry, Astroph. Journ., **498**, 67 (1998)
- MPD, Phys. Rev. D**71**, 103505 (2005) (gr-qc/0410033)

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1. Introduction.

- Growing interest in **spherically symmetric** Lemaître-Tolman-Bondi (LTB) inhomogeneous density void models with an alternative explanation of the acceleration of the universe became kind of a new paradigm in cosmology (Célérier 2000; Marra et al. 2007; Uzan et al. 2008 and many others) – commented e.g. by Krasiński, Hellaby, Bolejko, and Célérier (2010) that not only LTB are worth investigating.
- Let me keep the side of the latter people and consider **spherically symmetric** Stephani models of the pressure gradient.
- In fact, LTB and SS Stephani are **complementary models** of the universe and both can mimic homogeneous dark energy models.
- The only **common limit** of both is the isotropic Friedmann. Both are the **simplest** inhomogeneous models (SS).
- And there are lots of less symmetric or **purely inhomogeneous** models (Goode (1986), Szafron (1977), Szekeres (1975), Wainwright-Goode (1980), Ruiz-Senovilla (1992) etc.) to investigate - cf. other talks.

Complementary models of the spherically symmetric Universe

Inhomogeneous density dust shells (LTB) and **inhomogeneous pressure** (gradient of pressure shells) (SS Stephani):

	pressure	density
FRW	$p = p(t)$	$\varrho = \varrho(t)$
LTB	$p = 0 \ (p(t))$	$\varrho = \varrho(t, r)$ - nonuniform
SS Stephani	$p = p(t, r)$ - nonuniform	$\varrho = \varrho(t)$

2. What are inhomogeneous pressure (Stephani) universes?

They are the only spherically symmetric solutions of Einstein equations for **perfect-fluid** energy-momentum tensor ($T^{ab} = (\varrho + p)u^a u^b + pg^{ab}$) which are **conformally flat** and **embeddable** in a 5-dimensional flat space (H. Stephani Commun. Math. Phys. **4**, 167 (1967)). We have

$$\begin{aligned} ds^2 &= -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 \\ &+ \frac{a^2}{V^2}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] , \end{aligned} \qquad (1)$$

where

$$V(t,r) = 1 + \frac{1}{4}k(t)r^2 , \qquad (2)$$

and $(\ldots)^{\cdot} \equiv \partial/\partial t$. The function $a(t)$ plays the role of a **generalized scale factor**, $k(t)$ has the meaning of **a time-dependent "curvature index"**, and r is the radial coordinate. **Compare: LTB has spatially dependent "curvature index" $K(r)$.**

SS Stephani universes

The energy density and pressure are given by

$$\varrho(t) = 3\left[\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k(t)}{a^2(t)}\right] , \tag{3}$$

$$p(t,r) = \varrho(t)\left\{-1 + \frac{1}{3}\frac{\dot{\varrho}(t)}{\varrho(t)}\frac{\left[\frac{V(t,r)}{a(t)}\right]^{\cdot}}{\left[\frac{V(t,r)}{a(t)}\right]}\right\} \equiv w_{eff}(t,r)\varrho(t) , \tag{4}$$

and generalize the standard Einstein-Friedmann relations

$$\varrho(t) = 3\left(\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k}{a^2(t)}\right) , \tag{5}$$

$$p(t) = -\left(2\frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a^2(t)} + \frac{k}{a^2(t)}\right) \tag{6}$$

to inhomogeneous models.

SS Stephani universes

Kinematic characteristic of the model:

$$u_{a;b} = \frac{1}{3}\Theta h_{ab} - \dot{u}_a u_b \ , \qquad \dot{u} \equiv (\dot{u}_a \dot{u}^a)^{\frac{1}{2}} \ . \tag{7}$$

where $\dot{u}$ is the acceleration scalar and the **acceleration vector**

$$\dot{u}_r = \frac{\left\{ \frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right] \right\}_{,r}}{\frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right]} \tag{8}$$

while the expansion scalar is the same as in FRW model, i.e.,

$$\Theta = 3\frac{\dot{a}}{a} \ . \tag{9}$$

Compare: LTB has non-zero expansion and shear.

Inhomogeneous pressure models - null geodesics

The four-velocity and the acceleration are

$$u_\tau = -\,c\frac{1}{V}\,, \qquad \dot{u}_r = -\,c\frac{V_{,r}}{V}\,. \tag{10}$$

The components of the vector tangent to zero geodesic are

$$k^\tau = \frac{V^2}{a}\,, \quad k^r = \pm\frac{V^2}{a^2}\sqrt{1 - \frac{h^2}{r^2}}\,, \quad k^\theta = 0\,, \quad k^\varphi = h\frac{V^2}{a^2 r^2}\,, \tag{11}$$

where h = const., and the plus sign in applies to a ray moving away from the centre, while the minus sign applies to a ray moving towards the centre. The **acceleration scalar** is

$$\dot{u} \equiv (\dot{u}_\mu \dot{u}^\mu)^{\frac{1}{2}} = \frac{V_{,r}}{a} = \frac{1}{2}\frac{k(t)}{a(t)}r \tag{12}$$

The **farther away** from the center at $r = 0$, the **larger the acceleration**.

Inhomogeneous pressure models - topology & singularities

- Global topology still $S^3 \times R$. The models are just **specific deformations of the de Sitter** hyperboloid near the "neck circle", **but with local topology of the constant time hypersurfaces (index $k(t)$) changing in time.**
- Usually we cut hyperboloid by either $k = 1$ (S^3 topology), $k = 0$ (R^3) or $k = -1$ (H^3) – here we have "3-in-1" – the Universe may either **"open up" or "close down"**.
- standard **Big-Bang** singularities $a \to 0$, $\varrho \to \infty$, $p \to \infty$ are possible (FRW limit).
- **Finite Density (FD)** singularities of pressure appear at some particular value of a radial coordinate r – in standard FRW cosmology there exist exotic (sudden future) singularities of pressure (SFS) with finite scale factor and energy density – they differ (Dąbrowski 2005).
- There is **no global equation of state** - it changes from shell to shell and on the hypersurfaces $t =$ const.

Exact classes of inhomogeneous pressure models ...

... have been found in Dąbrowski (1993). In particular, for the so-called **Model II** one has

$$\left(\frac{k}{a}\right)^{\cdot} = 0 \quad \text{i.e.} \quad k(\tau) = -\beta a(\tau) \tag{13}$$

with the unit $[\beta] = Mpc^{-1}$.

A subcase of model II (from now on **IIA**) was proposed by Stelmach and Jakacka (2001)) – it assumes that the **standard barotropic** equation of state

$$\frac{p(\tau)}{c^2} = w\varrho(\tau) \tag{14}$$

at the center of symmetry and **no exact form** of the scale factor. This assumption gives that

$$\frac{8\pi G}{3c^2}\varrho(\tau) = C^2(\tau) = \frac{A^2}{a^{3(w+1)}(\tau)} \quad (A = \text{const.}) \tag{15}$$

and allows to write a generalized Friedmann equation as

Exact inhomogeneous pressure models

and

$$\frac{p(\tau)}{c^2} = \left[w + \frac{\beta}{4}(w+1)a(\tau)r^2 \right] \varrho(\tau) = w_{eff}\,\varrho(\tau) \quad . \tag{17}$$

Similarly as in the Friedmann model, we can define critical density as

$$\varrho_{cr}(\tau) = \frac{3c^2}{8\pi G}\left(\frac{a_{,\tau}}{a(\tau)}\right)^2 \tag{18}$$

and the density parameter $\Omega(\tau) = \varrho(\tau)/\varrho_{cr}(\tau)$ which after taking $\tau = \tau_0$ gives

$$1 = \frac{A^2}{H_0^2 a^{3(w+1)}(\tau_0)} - \frac{\beta c^2}{H_0^2 a_0} \equiv \Omega_0 + \Omega_{inh} \quad , \tag{19}$$

and so

$$\beta = \frac{a_0 H_0^2}{c^2}(\Omega_0 - 1) \quad . \tag{20}$$

Exact inhomogeneous pressure models

In Model **IIB** the scale factor is of the **dust-like type**

$$a(t) = \sigma t^{2/3},\ k(t) = -\alpha\sigma a(t),\ \ , \tag{21}$$

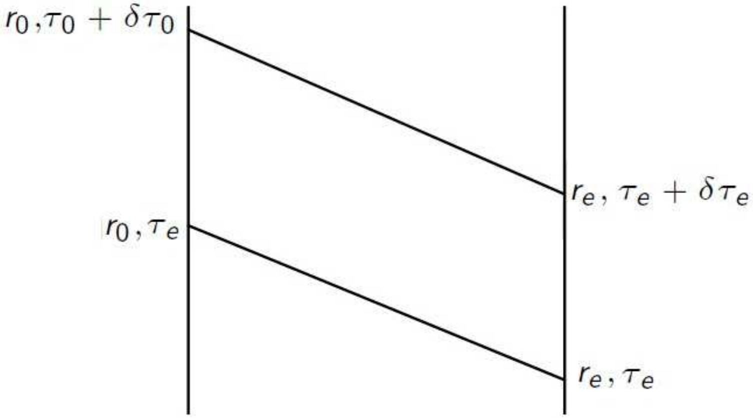
($[\alpha] = (s/km)^{2/3} Mpc^{-4/3}$, $[\sigma] = (km/s)^{2/3} Mpc^{1/3}$, $[t] = sMpc/km$) but the equation of state at the center of symmetry is no longer barotropic:

$$\rho = p \left(\frac{32\pi^2 G^2}{3\alpha^3 c^8} p^2 - \frac{3}{2} \right) \ . \tag{22}$$

In the limit of the **inhomogeneity parameter** $\alpha \to 0$ one obtains the Friedmann universe. FD singularity of pressure is at $r \to \infty$.

3. Redshift drift test to discriminate between Stephani and LTB.

Redshift drift (Sandage 1962) - the idea is to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.



There is a relation between the times of emission of light by the source τ_e and $\tau_e + \delta\tau_e$ and times of their observation at τ_o and $\tau_o + \delta\tau_o$:

$$\int_{\tau_e}^{\tau_o} \frac{d\tau}{a(\tau)} = \int_{\tau_e+\delta\tau_e}^{\tau_o+\delta\tau_o} \frac{d\tau}{a(\tau)} , \tag{23}$$

which for small $\delta\tau_e$ and $\delta\tau_o$ reads as $\frac{\delta\tau_e}{a(\tau_e)} = \frac{\delta\tau_o}{a(\tau_o)}$.

Redshift drift in inhomogeneous pressure models.

For small $\delta\tau_e$ and $\delta\tau_o$ we expand in Taylor series

$$
\begin{aligned}
(u_a k^a)_o &= (u_a k^a)(r_0, \tau_0 + \delta\tau_0) = (u_a k^a)(r_0, \tau_0) + \left[\frac{\partial(u_a k^a)}{\partial\tau}\right]_{(r_0,\tau_0)} \delta\tau_0 \\
(u_a k^a)_e &= (u_a k^a)(r_e, \tau_e + \delta\tau_e) = (u_a k^a)(r_e, \tau_e) + \left[\frac{\partial(u_a k^a)}{\partial\tau}\right]_{(r_e,\tau_e)} \delta\tau_e \ ,
\end{aligned}
$$

where for inhomogeneous pressure models the readshift reads as

$$
1 + z = \frac{(u_a k^a)_e}{(u_a k^a)_O} = \frac{\frac{V(t_e,r_e)}{R(t_e)}}{\frac{V(t_0,r_0)}{R(t_0)}} \tag{24}
$$

From the definition of the redshift drift by Sandage (1962):

$$
\delta z = z_e - z_0 = \frac{(u_a k^a)(r_e, \tau_e + \delta\tau_e)}{(u_a k^a)(r_0, \tau_0 + \delta\tau_0)} - \frac{(u_a k^a)(r_e, \tau_e)}{(u_a k^a)(r_0, \tau_0)} \ , \tag{25}
$$

Redshift drift in inhomogeneous pressure models.

For general SS Stephani metric we obtain

$$\frac{\partial}{\partial \tau} \left(u_a k^a \right) = - \left(\frac{1}{a} \right)^{\cdot} - \frac{1}{4} \left(\frac{k}{a} \right)^{\cdot} r^2 , \qquad (26)$$

and

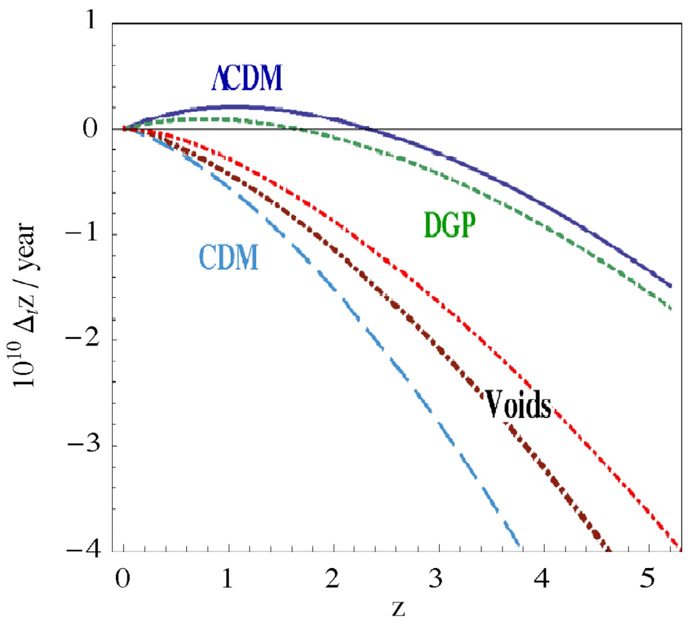
$$\frac{\delta z}{\delta \tau_0} = \frac{\left[\left(\frac{1}{a} \right)^{\cdot} - \frac{1}{4} \left(\frac{k}{a} \right)^{\cdot} r^2 \right]_e}{\left[1 + \frac{1}{4} k r^2 \right]_e} a(\tau_e) - \frac{\left[\left(\frac{1}{a} \right)^{\cdot} + \frac{1}{4} \left(\frac{k}{a} \right)^{\cdot} r^2 \right]_o}{\left[1 + \frac{1}{4} k r^2 \right]_o} a(\tau_0)(1+z) \qquad (27)$$

For the model with $(k/a)^{\cdot} = 0$ we have

$$\frac{\delta z}{\delta \tau} = - \frac{H_0}{1 + \frac{1}{4} k(\tau_0) r_0^2} \left[\frac{H_e(z)}{H_0} - (1+z) \right] . \qquad (28)$$

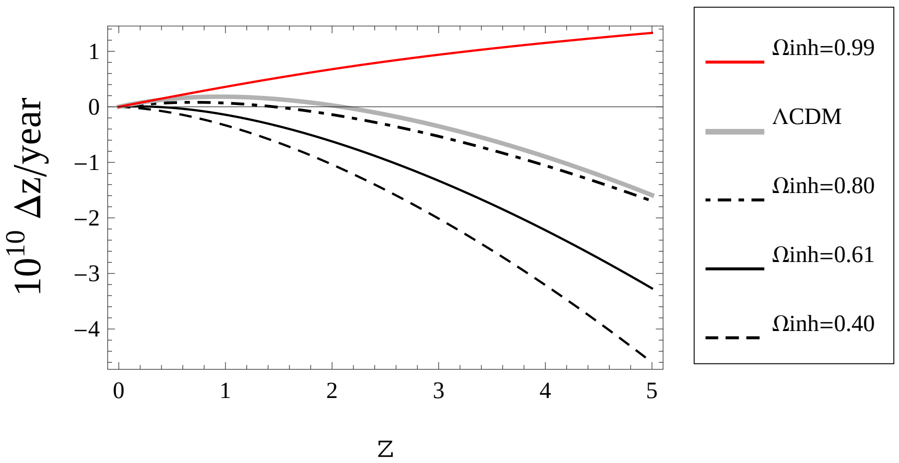
Sandage-Loeb CDM formula for $\Omega_{inh} \to 0$; $H_e(z) = H_0 (1+z)^{3/2}$, $r_0 \to 0$.

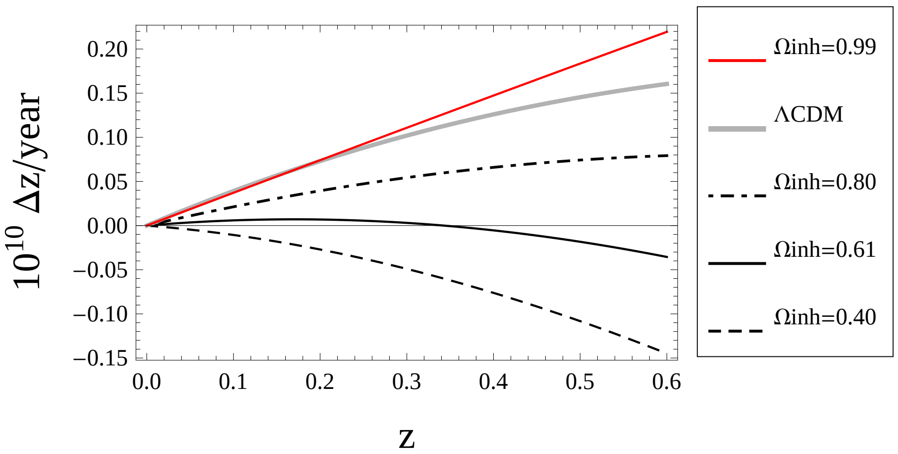
Redshift drift - LTB voids.



- Plots for **3 different LTB** void models, ΛCDM, brane DGP, Cold Dark Matter (CMD) (Quercellini et. al, 2012).
- ΛCDM – the drift is **positive at small redshift**, but becomes negative for $z \gtrsim 2$.
- Giant void (LTB) model mimicking dark energy - the drift is **always negative**.

Redshift drift - inhomogeneous pressure models ($r_0 = 0$, $w = 0$).





- Ω_{inh} **small** - drift as in LTB and CDM models
- Ω_{inh} **larger** - drift as in ΛCDM models (first positive, then negative), e.g. for $\Omega_{inh} = 0.61$ drift is positive for $z \in (0, 0.34)$.
- Ω_{inh} **very large** - drift positive ($\Omega_{inh} = 0.99$ up to $z = 17$; $\Omega_{inh} = 1$ (inhomogeneity-domination) $z > 0$) and $\frac{\delta z}{\delta t} = H_0 \frac{z}{2}$ which means that the drift **grows linearly** with redshift.

Redshift drift - observational perspective.

- One is able to **differentiate** between the drift in ΛCDM models, in LTB models, and in Stephani models - this can be done in future experiments.
- At larger $z > 1.7$ redshifts by **giant telescopes**: European Extremely Large Telescope (E-ELT) with spectrograph CODEX (COsmic Dynamics EXperiment); Thirty Meter Telescope (TMT); Giant Magellan Telescope (GMT).
- At smaller (even $z \sim 0.2$) redshifts by **space-borne gravitational wave interferometers** like DECIGO/BBO (DECi-hertz Interferometer Gravitational Wave Observatory/Big Bang Observer).

4. Off-center observer's position against Union2 supernovae.

The luminosity distance is given by

$$d_L = \frac{a_0(1+z)\hat{r}'}{1 + \frac{\beta}{4} a_0 r_0^2} \ , \qquad (29)$$

with an **off-center** observer placed at r_0, θ_0, ϕ_0 as meant in the coordinate system $\{t, r, \theta, \varphi\}$ of the Stephani metric. More precisely we have

$$d_L = \frac{(1+z)}{1 - \frac{a_0 H_0^2 \Omega_{inh}}{4} r_0^2} \, \hat{r}'(\Omega_{inh}, w, r_0, \theta_0, \varphi_0, H_0, \hat{\theta}', \hat{\varphi}', z) \ , \qquad (30)$$

where

$$\hat{r}' = \hat{r}'(a) = \frac{1}{H_0} \int_{a_e}^{1} \frac{dx}{\sqrt{(1-\Omega_{inh}) x^{1-3w} + \Omega_{inh} x^3}} \ , \qquad (31)$$

and a_e is the value of the scale factor at the moment of an emission of the light ray.

Off-center observers

For the redshift one takes

$$1 + z = \frac{a_0(4 - a_e H_0^2 \Omega_{inh} r_e^2)}{a_e(4 - a_0 H_0^2 \Omega_{inh} r_0^2)} \, , \tag{32}$$

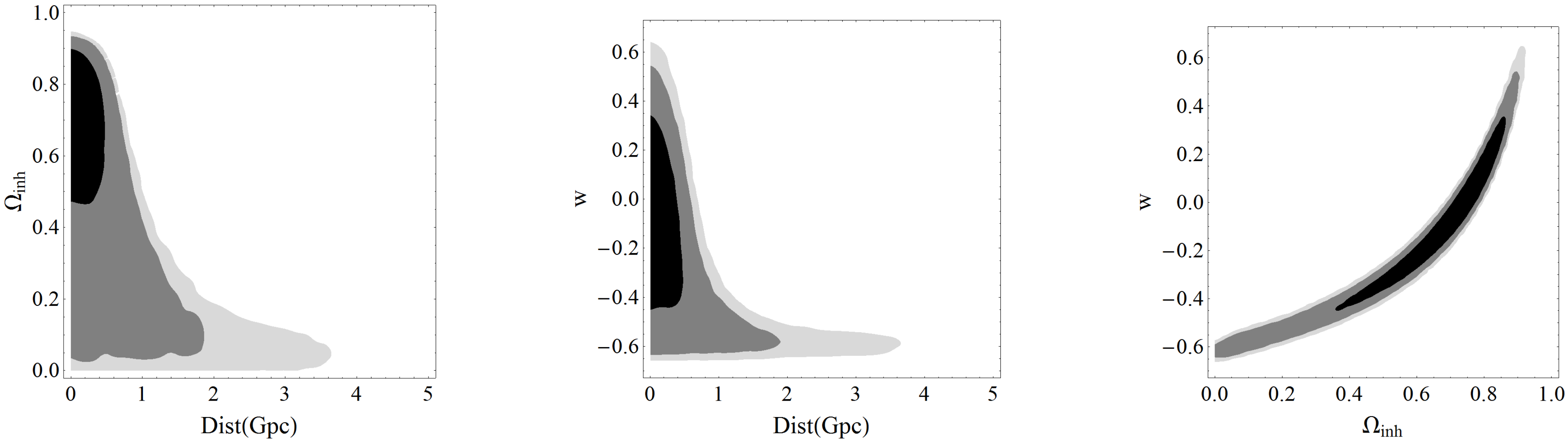
where

$$\begin{aligned} r_e^2 &= (r_0 \sin\theta_0 \cos\varphi_0 + \hat{r}'(a) \sin\hat{\theta}' \cos\hat{\varphi}')^2 \\ &+ (r_0 \sin\theta_0 \sin\varphi_0 + \hat{r}'(a) \sin\hat{\theta}' \sin\hat{\varphi}')^2 \\ &+ (r_0 \cos\theta_0 + \hat{r}'(a) \cos\hat{\theta}' \sin\hat{\varphi}')^2 \end{aligned} \tag{33}$$

and $\hat{\theta}'$ and $\hat{\varphi}'$ are the coordinates of a supernova as seen by an off-center observer in the sky.

We applied **Union2 557 supernovae data** of Amanullah et al. (2010, ApJ, 716, 712) - we note the courtesy of M. Kowalski and U. Feindt to consult the sample.

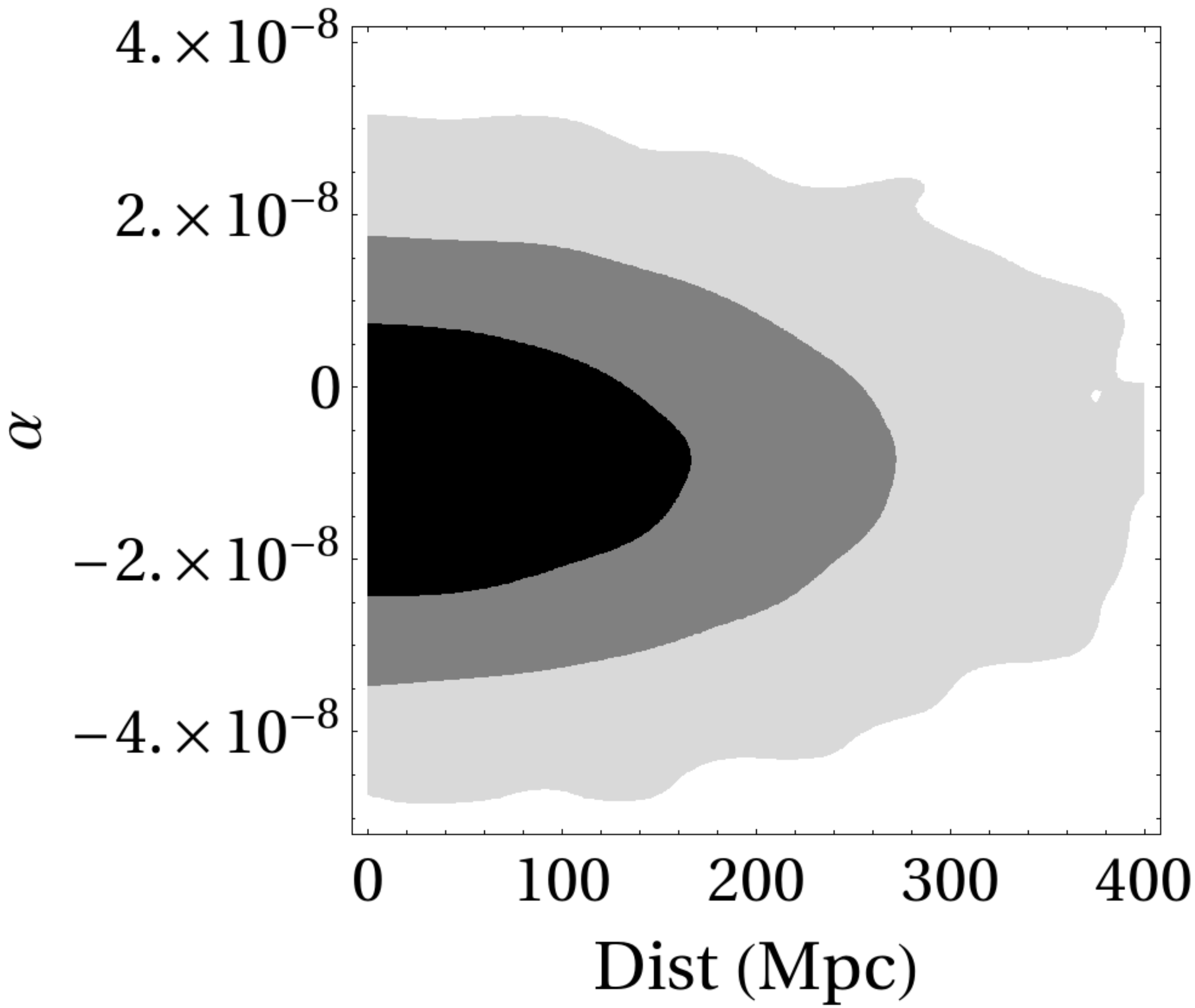
Off-center observers - model IIA



Best-fit values: inhomogeneity density $\Omega_{inh} \sim 0.74$, center of symmetry equation of state barotropic index $w \sim 0.031$, off-center observer position $Dist = 270$ Mpc ($\chi^2 = 339$).

An observer cannot be farther away than $\sim 3.5Gpc$.

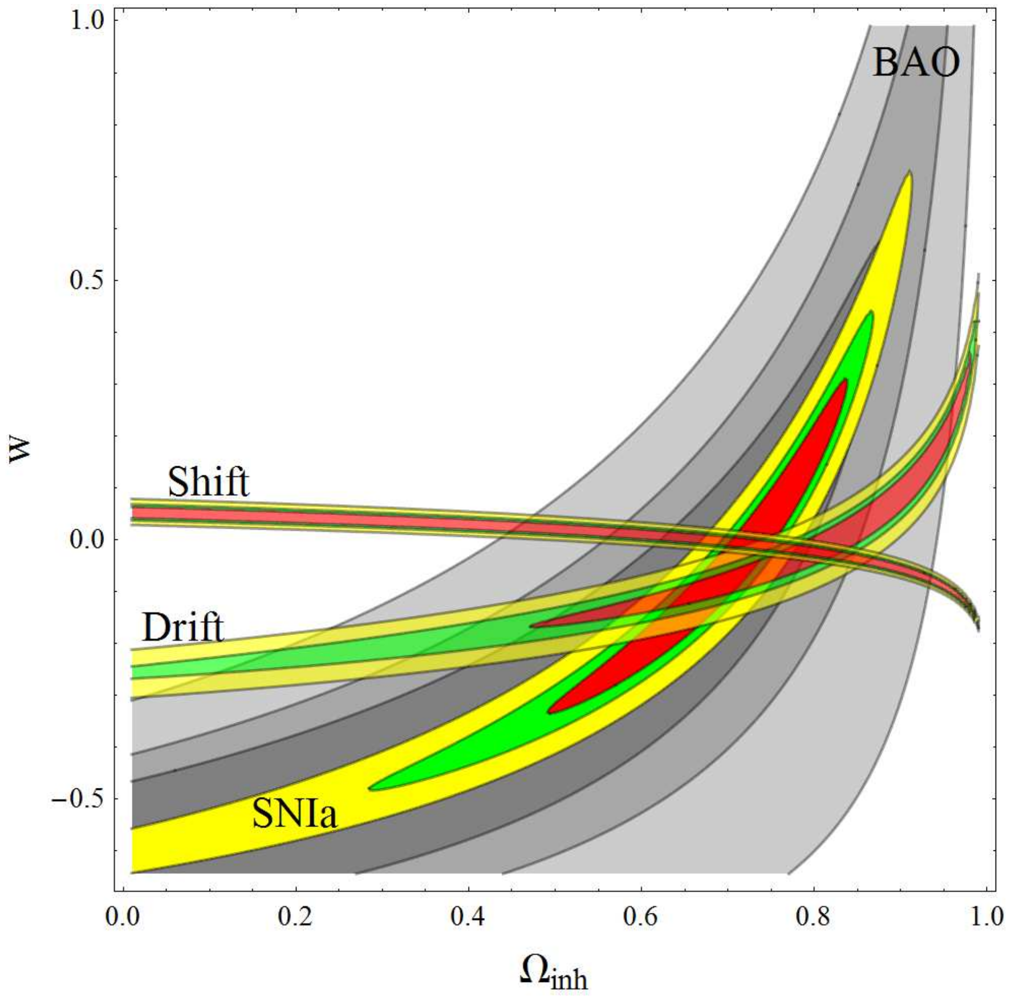
Off-center observers - model IIB



Non-barotropic EOS **limits stronger** the position of an observer.

5. Combined tests (central observer): luminosity distance, BAO, shift parameter, drift.

Preliminary plot (Balcerzak, MPD, Denkiewicz, Polarski - in progress):



Not all countours overlap.

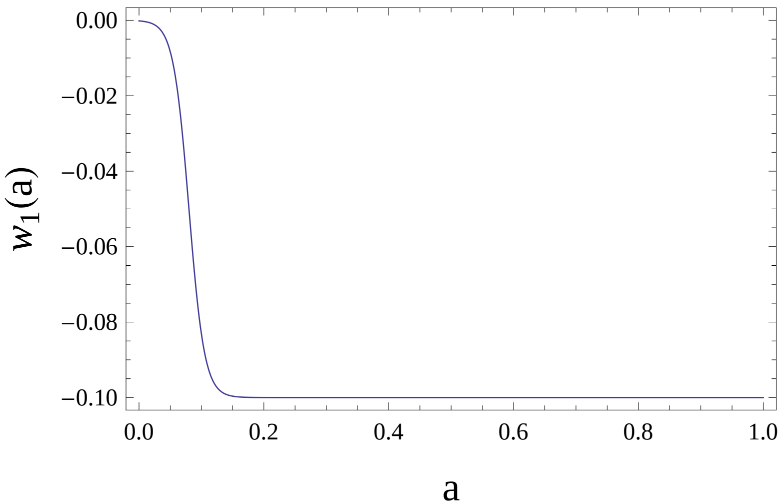
Combined tests - keeping inhomogeneity, new EOS $w_1(a)$ parametrization

We suggest a varying barotropic index $w_1(a)$ which can fit the data is:

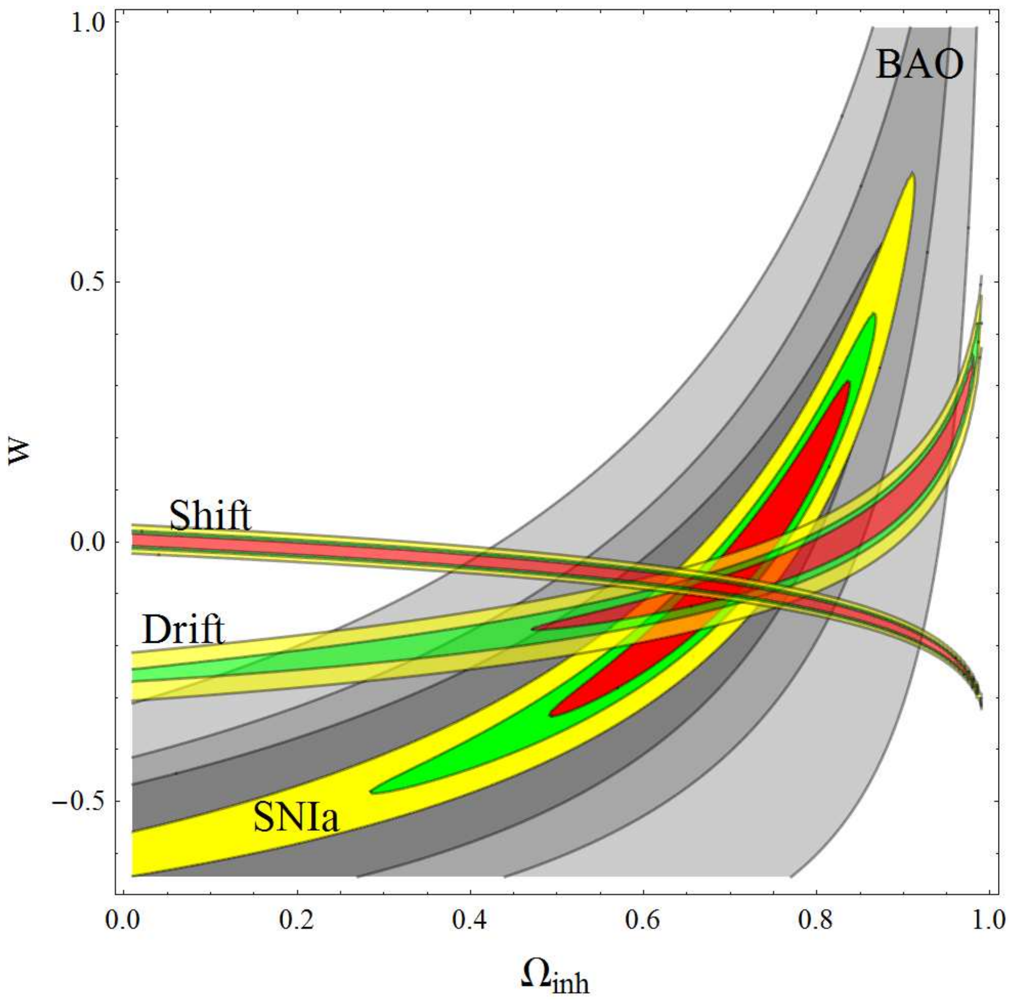
$$w_1(a) = w + \frac{w_0}{2}\left(1 + \tanh[\lambda(a_{tr} - a)]\right) \ . \tag{34}$$

where w, w_0, λ, and a_{tr} are constants. Here: $\lambda = 40$, $a_{tr} = 0.08$, $w_0 = 0.1$, $w = -0.1$ and $\Omega_{inh} = 0.68$, $z_{tr} \sim 10.66$. Also, as in the standard case:

$$\varrho(a) = \varrho_0 \ \exp\left[-3\int_{a_0}^{a} da' \ \frac{1 + w_1(a')}{a'}\right] \equiv \varrho_0 \ f(a) \ . \tag{35}$$



Inhomogeneous pressure models - combined tests for $w_1(a)$



Good news: we can still have inhomogeneity while $w \to w(a)$!

Advantages for future work:

The general Stephani metric

$$ds^2 = -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}\left[dx^2 + dy^2 + dz^2\right], \quad (36)$$

$$V(t,x,y,z) = 1 + \frac{1}{4}k(t)\left\{[x - x_0(t)]^2 + [y - y_0(t)]^2 + [z - z_0(t)]^2\right\},$$

with x_0, y_0, z_0 being arbitrary functions of time is just a generalization of both the FRW and SS Stephani metrics in isotropic coordinates.

- **It has no symmetries** acting on spacetime.
- The center of symmetry **changes its position** from slice to slice.

6. Conclusions

- Viable and complementary to LTB cosmologies which can **drive acceleration**.
- Clear **difference** against LTB for redshift drift (can be tested by very large telescopes and GW detectors).
- Comparison with the 557 Union2 supernovae data **restricts the position of non-centrally placed observers** to be not more than $\sim 3.5Gpc$ away from the center.
- Stephani model fits well the data for SNIa, redshift drift, shift parameter, and BAO provided a specific parametrization for $w = w(a)$ is applied.
- Can even model **total spacetime inhomogeneity**.