What can we learn about the neutron-star equation of state from inspiralling binary neutron stars?

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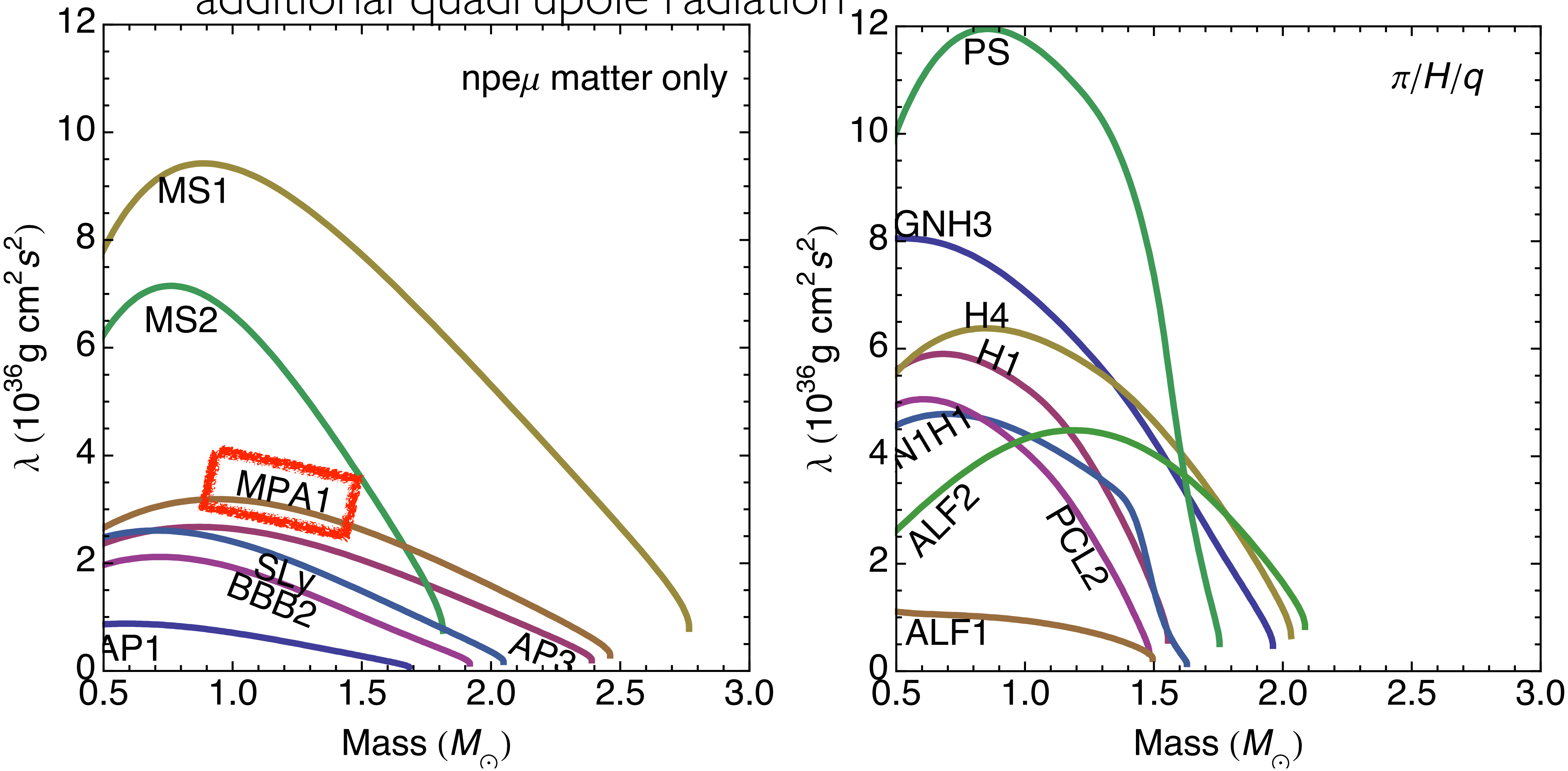
What can we measure from the waveform?

- Phase shift during the inspiral from tidal interactions
  - Tidal field $\mathcal{E}_{ij}$ of one NS induces quadrupole moment Q_{ij} in other NS

$$Q_{ij} = -\lambda(\mathrm{EOS}, M_{\mathrm{NS}})\mathcal{E}_{ij}$$
$$-\Lambda(\mathrm{EOS}, M_{\mathrm{NS}})M_{\mathrm{NS}}^5\mathcal{E}_{ij}$$

  - Increased quadrupole moment leads to more tightly bound system and additional quadrupole radiation



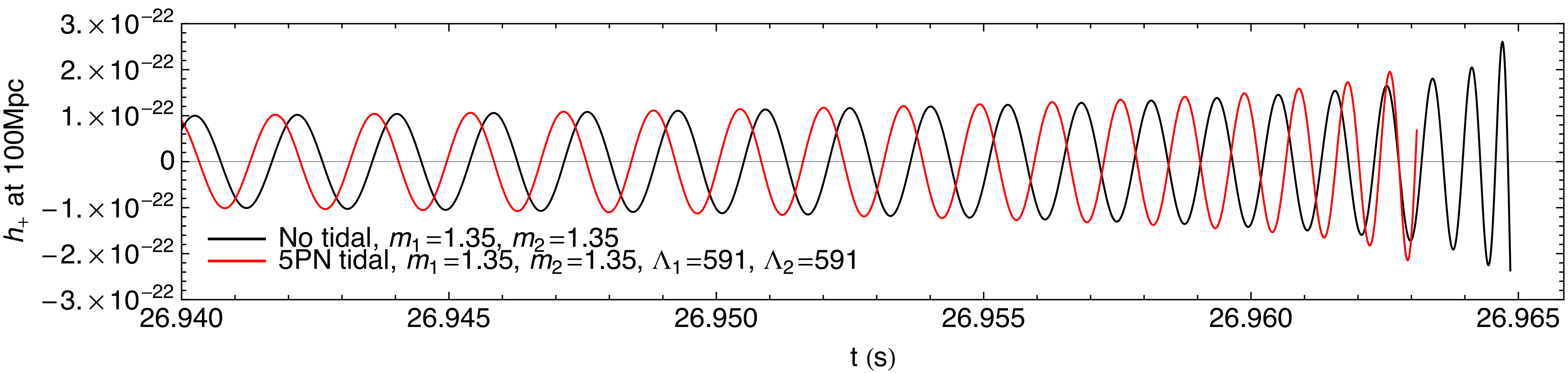
What can we measure from the waveform?

- Tidal effects first appear at same order as 5PN point-particle terms
- Leading term is a linear combination of the tidal deformabilities for each NS
- Results in a phase shift of ~1 cycle up to ISCO depending on the EOS

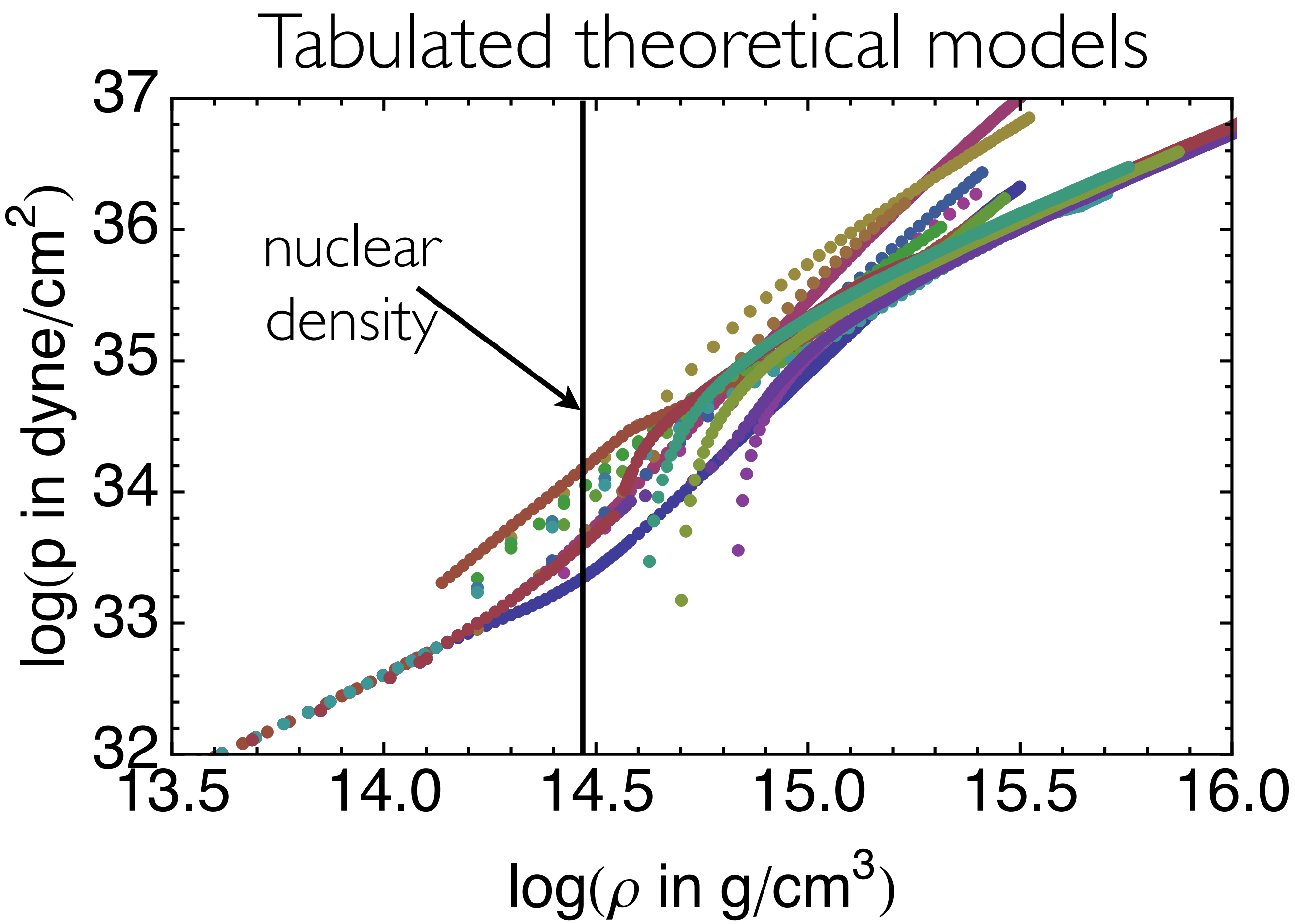
$$\tilde{h}(f) \propto \frac{\mathcal{M}^{5/6} f^{-7/6}}{D_L} e^{i\psi(f)}$$

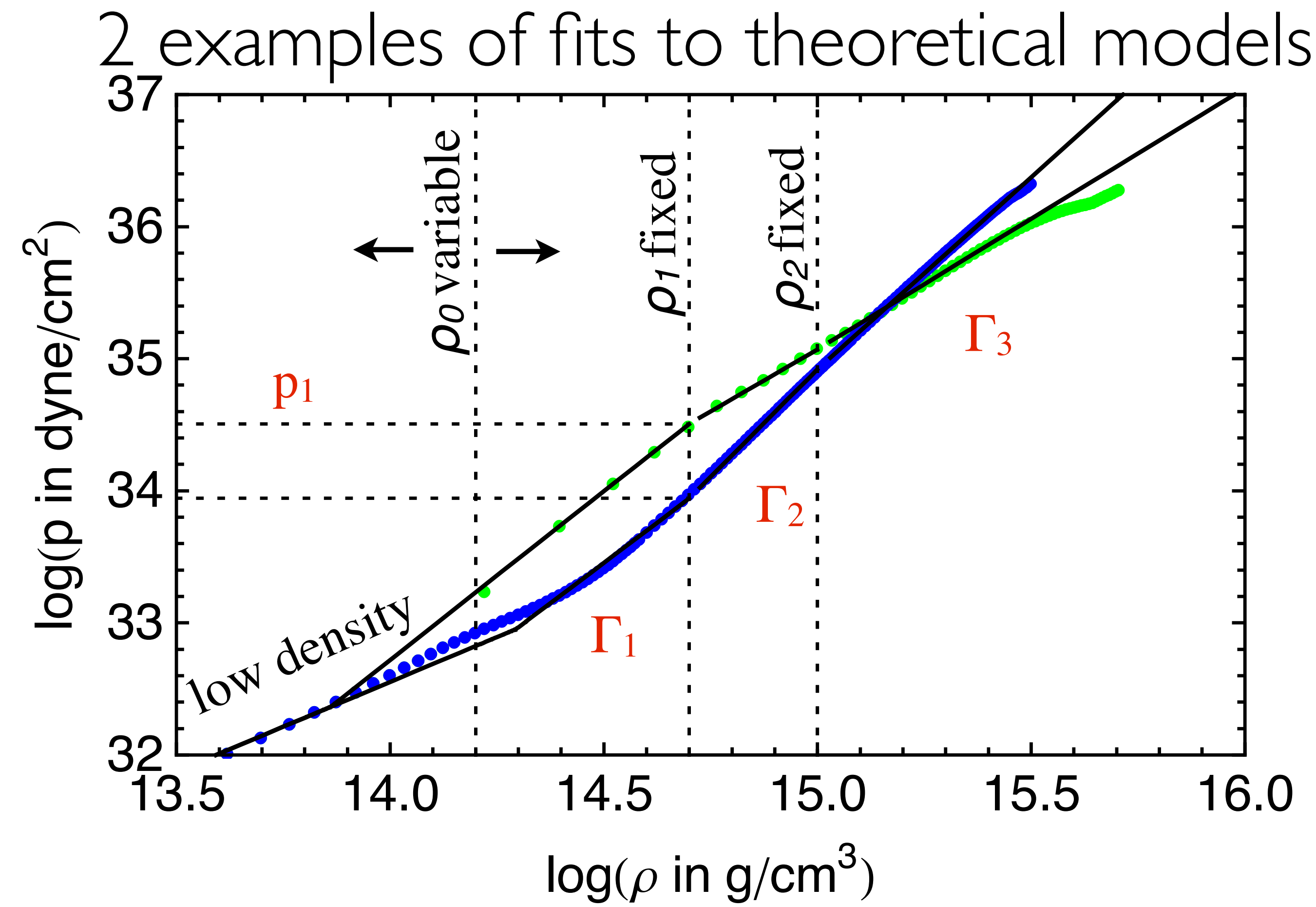
$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta v^5}\left[1 + (\text{PP-PN}) - \frac{39}{2}\tilde{\Lambda}(\mathcal{M}, \eta, \text{EOS}) v^{10}\right]$$

$$\tilde{\Lambda} = \frac{8}{13}\left[(1 + 7\eta - 31\eta^2)(\Lambda_1 + \Lambda_2) + \sqrt{1 - 4\eta}(1 + 9\eta - 11\eta^2)(\Lambda_1 - \Lambda_2)\right]$$



Parametrized EOS





$$p(\rho) = \begin{cases} K_1 \rho^{\Gamma_1}, & \rho_0 < \rho < \rho_1 \\ K_2 \rho^{\Gamma_2}, & \rho_1 < \rho < \rho_2 \\ K_3 \rho^{\Gamma_3}, & \rho > \rho_2 \end{cases}$$

- Available physical models for EOS can be accurately fit by a parametrized piecewise polytrope
- Parametrized piecewise polytrope reproduces neutron star properties to a few percent

Estimating EOS parameters from LIGO data

- Analogue of 2-step procedure described by Steiner, Lattimer, Brown (Astrophys. J. 722, 33) for mass-radius measurements
  - They combined several mass-radius measurements from accreting neutron stars to estimate EOS parameters
  - We will use estimates of $\mathcal{M}$–η–$\tilde{\Lambda}(\mathcal{M}, \eta, \mathrm{EOS})$ from several BNS inspiral events to estimate EOS parameters

Step 1: Estimate $\mathcal{M}$–η–$\tilde{\Lambda}(\mathcal{M}, \eta, \mathrm{EOS})$

- Can estimate BNS parameters from Bayes theorem:

Posterior, Prior, Likelihood, Evidence

$$p(\vec{\theta}|d_n, \mathcal{H}, \mathcal{I}) = \frac{p(\vec{\theta}|\mathcal{H}, \mathcal{I})p(d_n|\vec{\theta}, \mathcal{H}, \mathcal{I})}{p(d_n|\mathcal{H}, \mathcal{I})}$$

  - $\vec{\theta} = \{\alpha, \delta, \iota, \psi, D_L, t_c, \phi_c, \mathcal{M}, \eta, \tilde{\Lambda}\}$
  - d_n: gravitational wave data from nth BNS system
  - $\mathcal{H}$: waveform model
  - $\mathcal{I}$: prior information about the parameters
- Posterior calculated with MCMC or estimated with Fisher matrix which assumes Gaussian likelihood and prior
- Marginalized distribution trivial to compute with MCMC or Fisher

$$p(\mathcal{M}, \eta, \tilde{\Lambda}|d_n, \mathcal{H}, \mathcal{I}) = \int p(\vec{\theta}|d_n, \mathcal{H}, \mathcal{I})d\vec{\theta}_{\mathrm{marg}}$$

Step 2: Estimate EOS parameters

- Can estimate EOS parameters from Bayes theorem:

Posterior, Prior, Likelihood, Evidence

$$p(\vec{x}|d_1 \ldots d_N, \mathcal{H}, \mathcal{I}) = \frac{p(\vec{x}|\mathcal{H}, \mathcal{I})p(d_1 \ldots d_N|\vec{x}, \mathcal{H}, \mathcal{I})}{p(d_1 \ldots d_N|\mathcal{H}, \mathcal{I})}$$

- $\vec{x} = \{\log(p_1), \Gamma_1, \Gamma_2, \Gamma_3, \mathcal{M}_1, \eta_1, ..., \mathcal{M}_N, \eta_N\}$
- $d_1 \ldots d_N$: gravitational wave data from all N BNS events
- prior: Flat in EOS parameters except $v_s = \sqrt{dp/d\epsilon} \leq c$ and $M_{\rm max} \geq 1.93 M_\odot$
- Likelihood:

Posterior from single BNS event

$$p(d_1, \ldots, d_N|\vec{x}, \mathcal{H}, \mathcal{I}) = \prod_{n=1}^{N} p(\mathcal{M}_n, \eta_n, \tilde{\Lambda}_n|d_n, \mathcal{H}, \mathcal{I})|_{\tilde{\Lambda}_n = \tilde{\Lambda}(\mathcal{M}_n, \eta_n, \mathrm{EOS})}$$

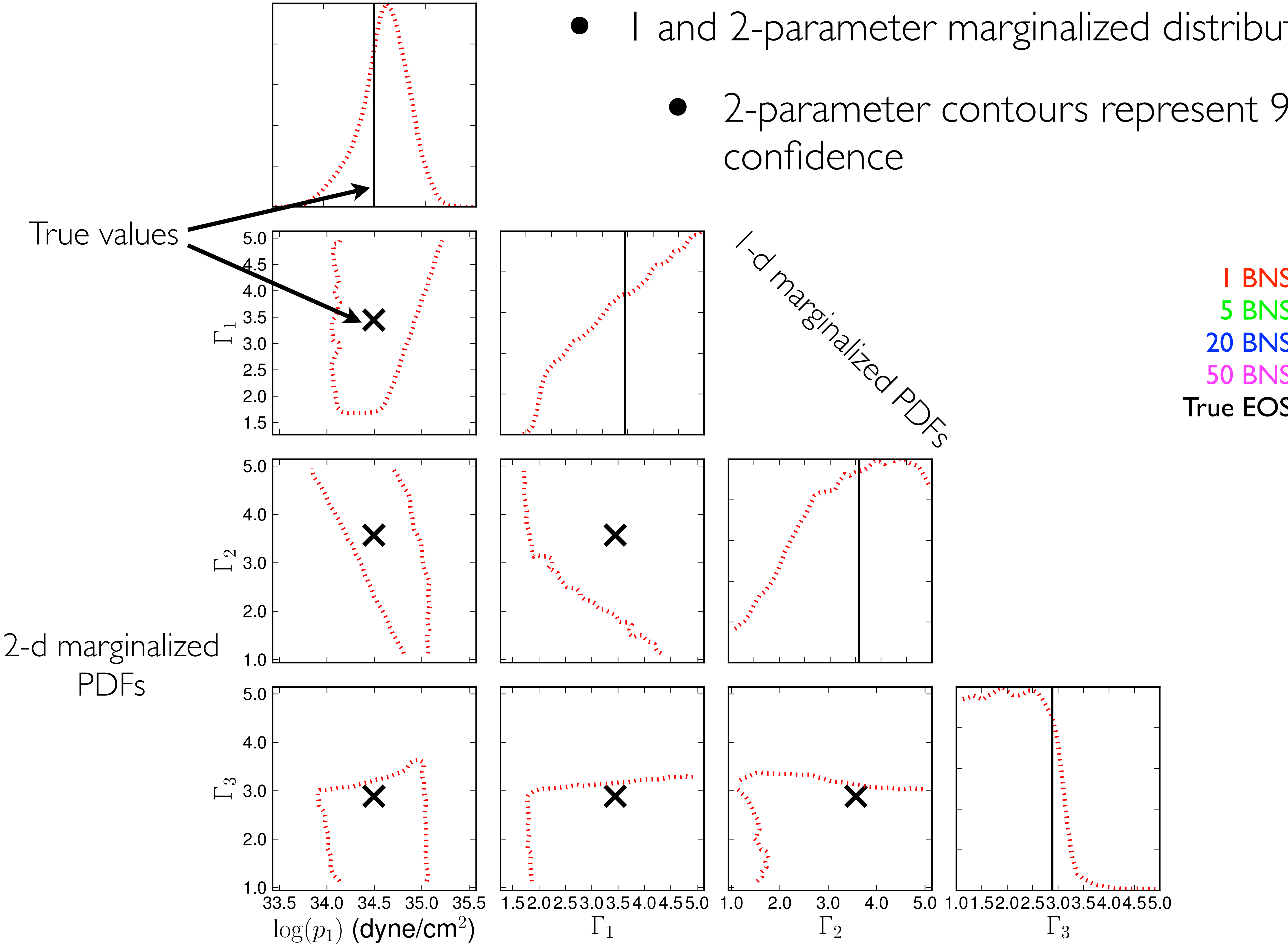
- Perform MCMC simulation over the 4+2N parameters, then marginalize over the 2N mass parameters

Simulating a population of BNS events

- We chose the "true" EOS to be MPA1
  - Moderate EOS in middle of parameter space
  - $R(1.4M_\odot) \sim 12.5\mathrm{km}$, and $M_{\max} \sim 2.5M_\odot$
- Sampled 50 BNS systems with SNR > 8
  - Individual masses distributed uniformly in $(1.2M_\odot, 1.6M_\odot)$
  - Sky position and distance distributed uniformly in volume
  - Orientation distributed uniformly on unit sphere
  - $\tilde{\Lambda}$ then calculated from masses and "true" EOS

EOS Parameters

- 1 and 2-parameter marginalized distributions
  - 2-parameter contours represent 95% confidence



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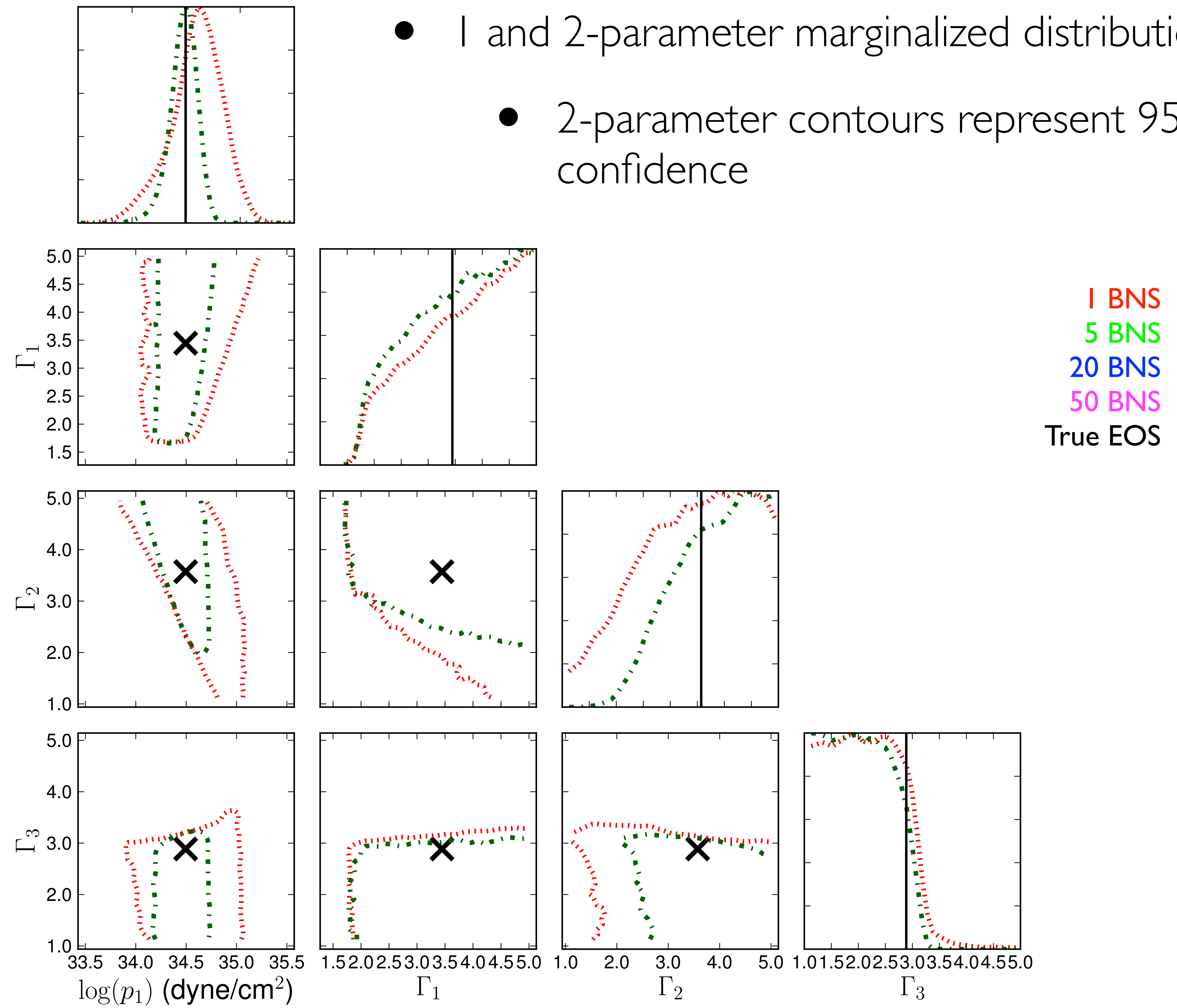

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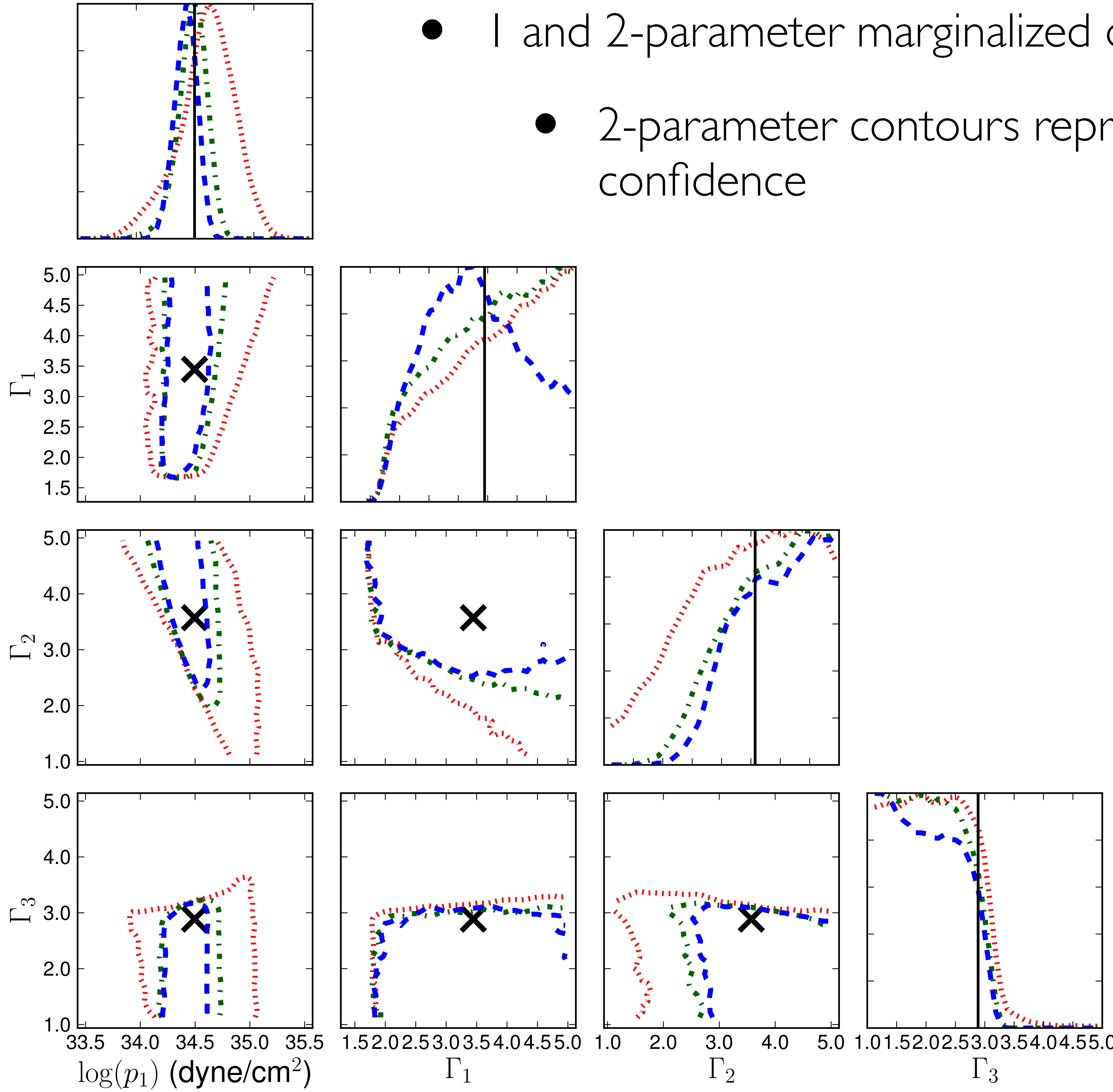

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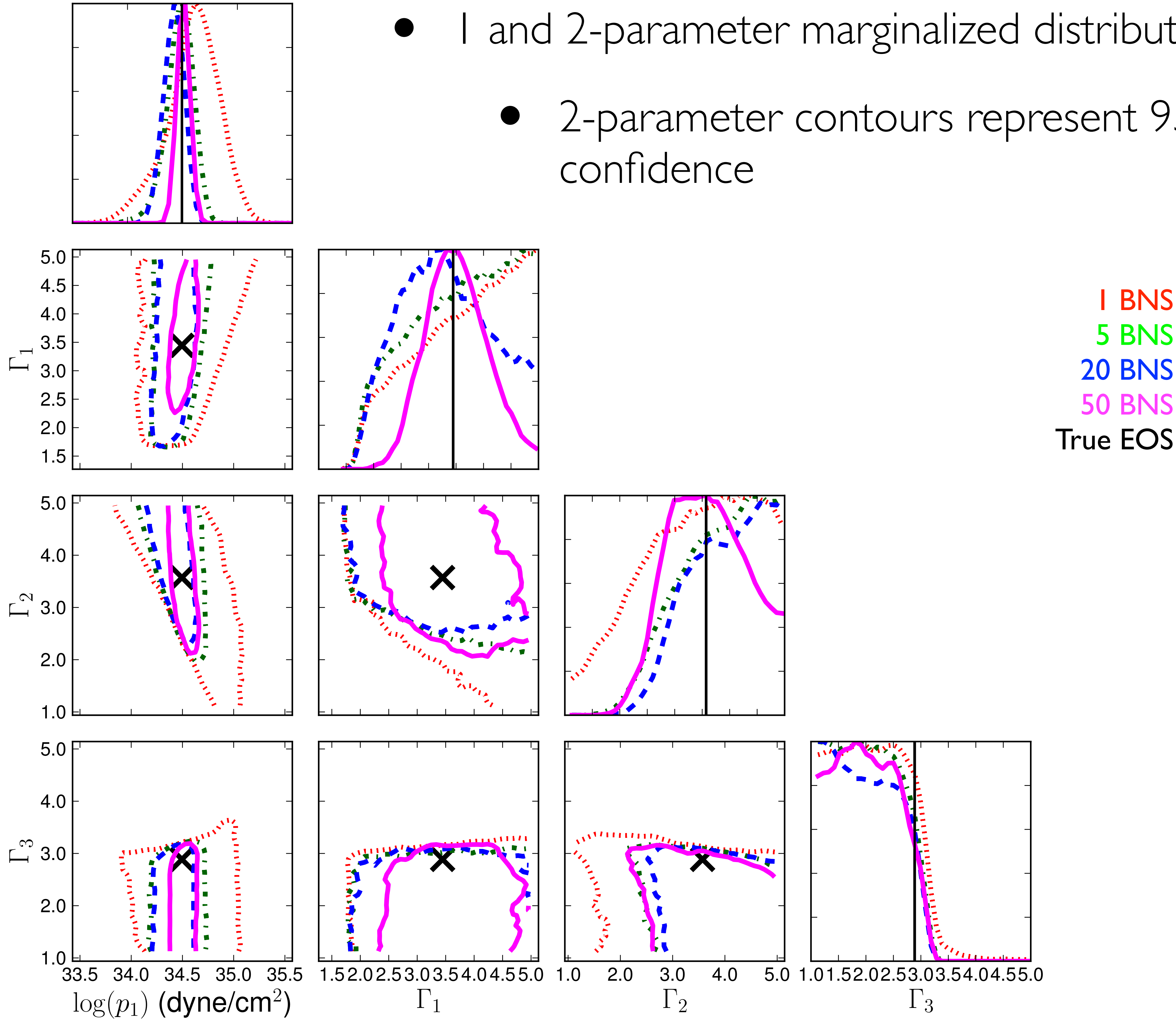
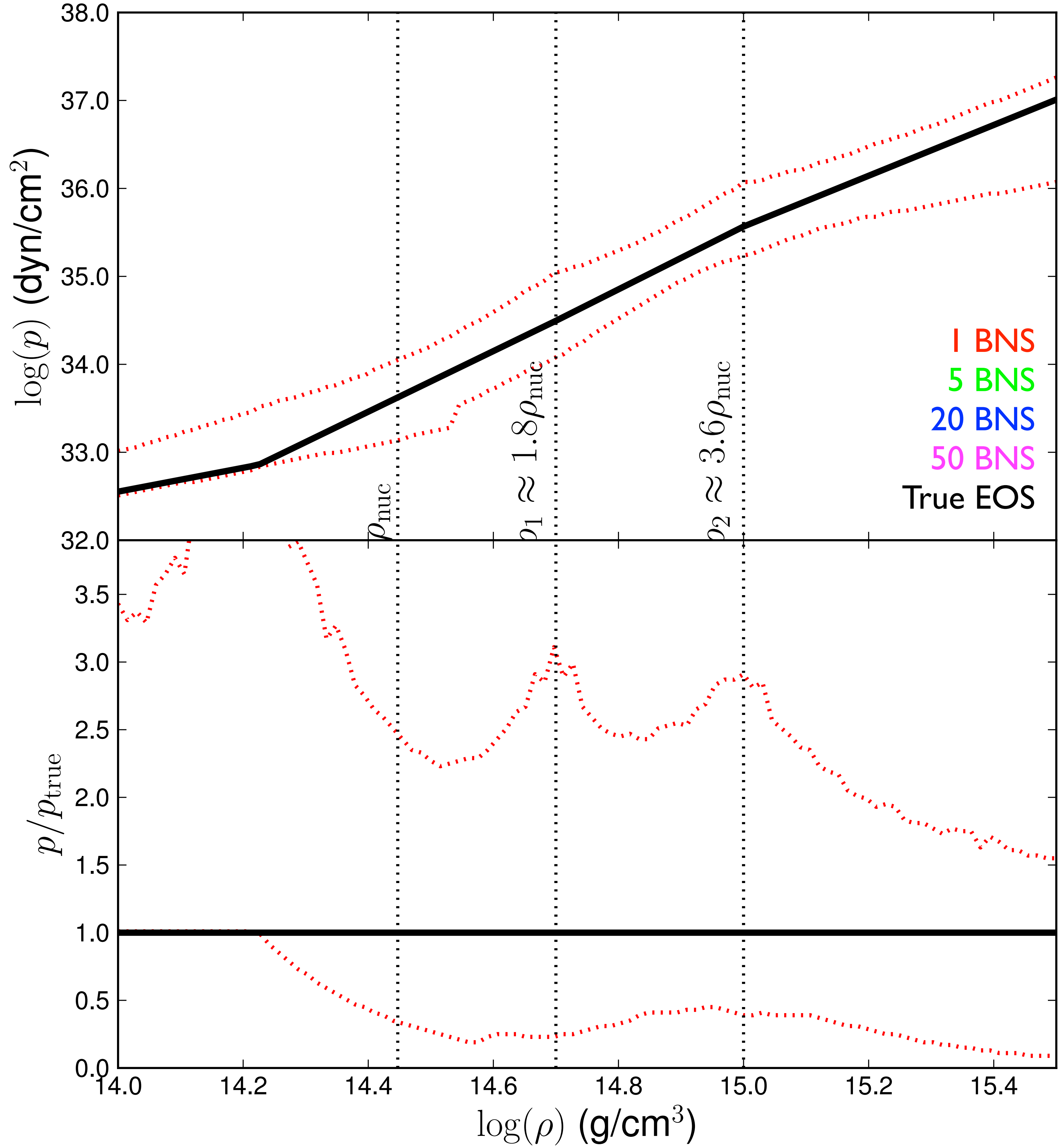


EOS function $p(\rho)$

- Chain of EOS parameters from MCMC simulation gives histogram of pressures for each density
- 95% confidence interval shown for each density



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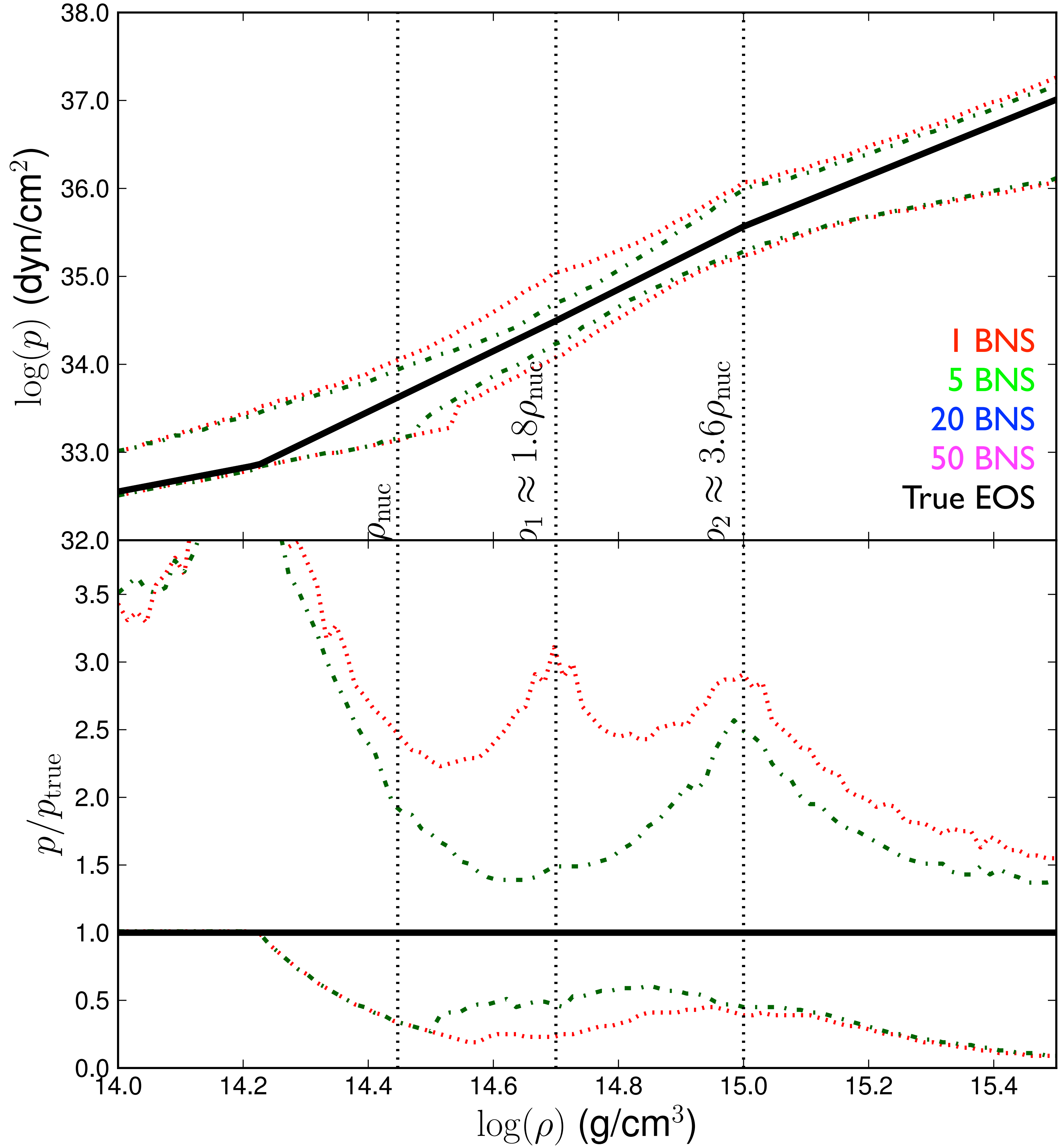

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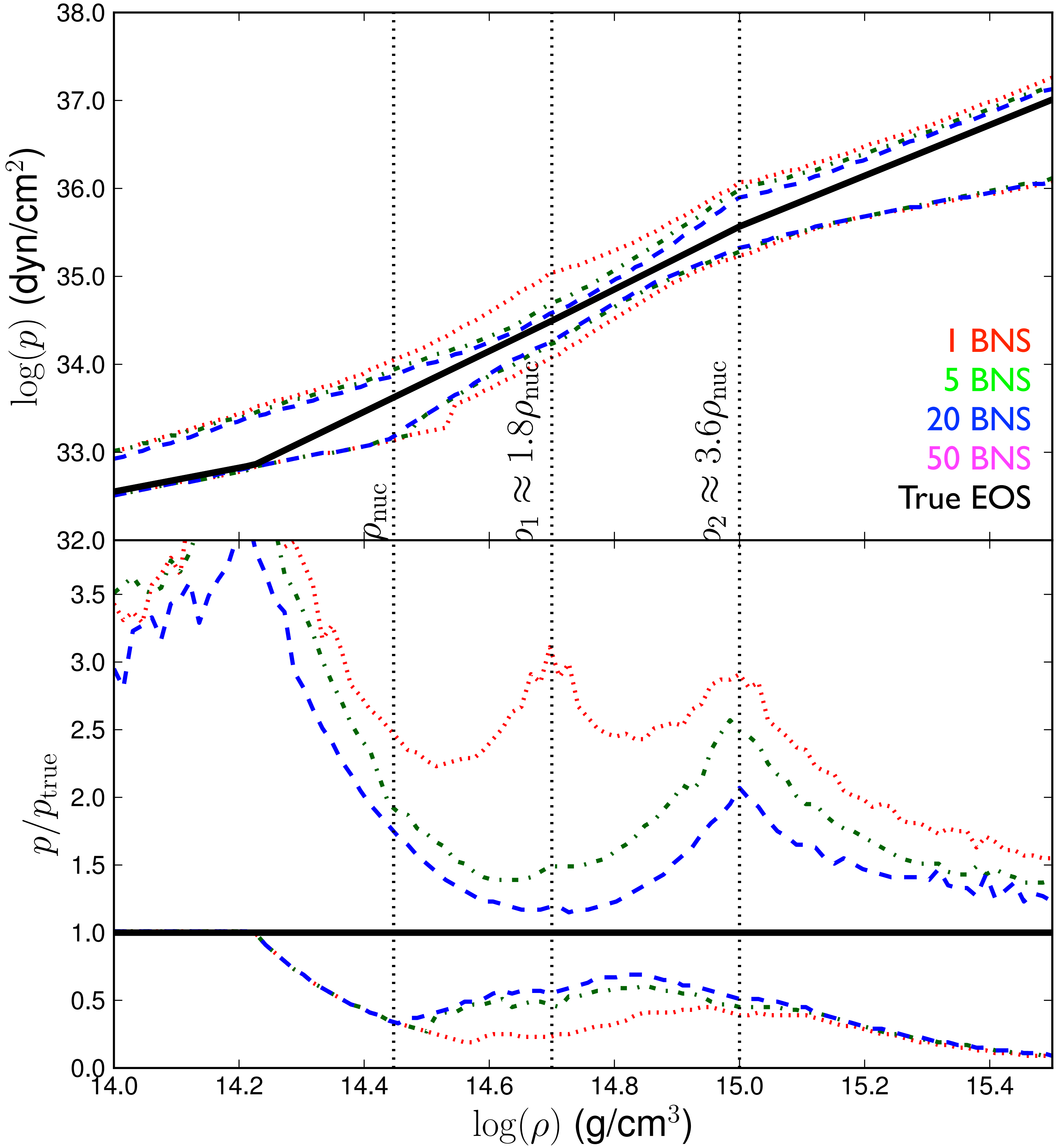

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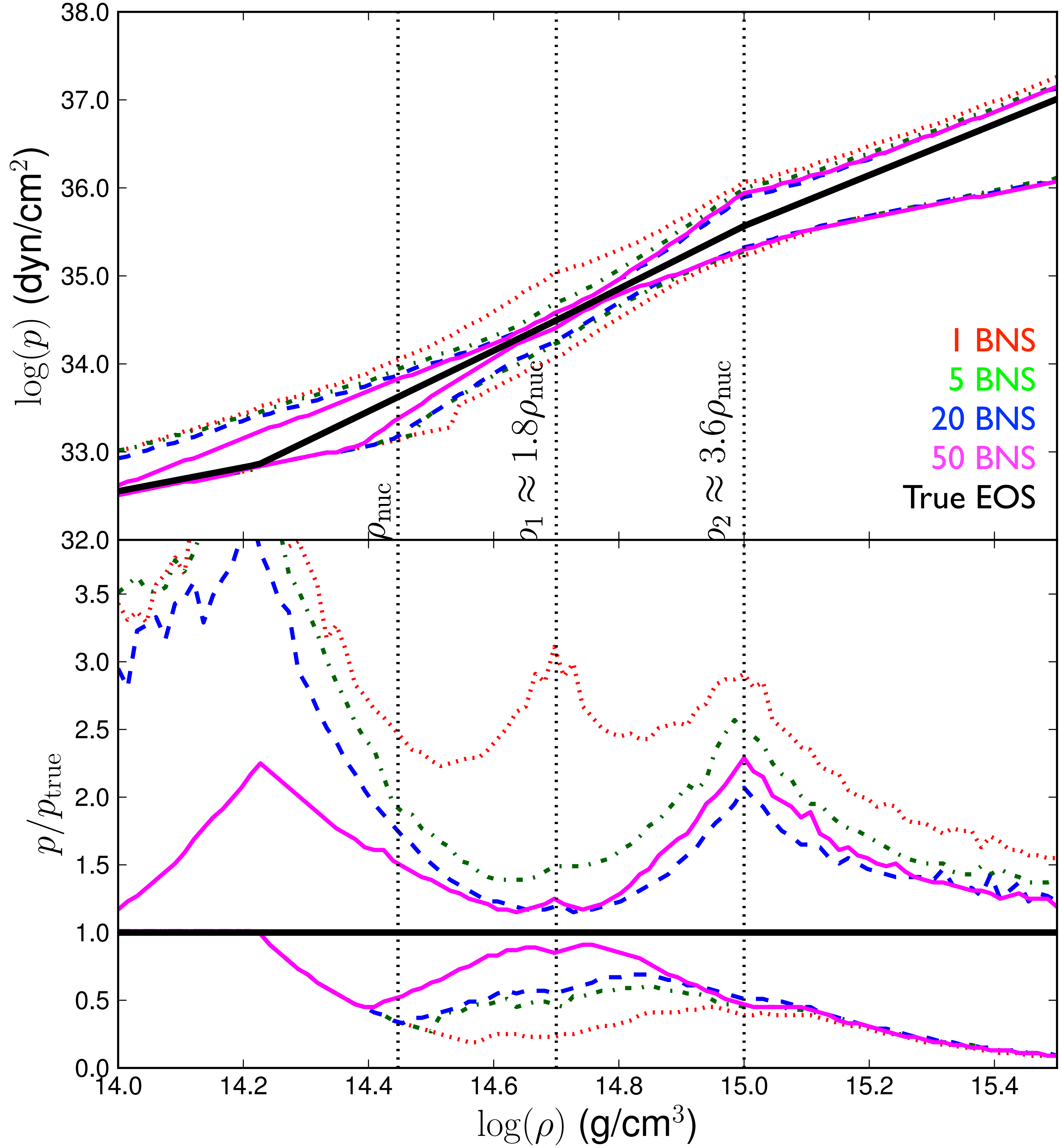
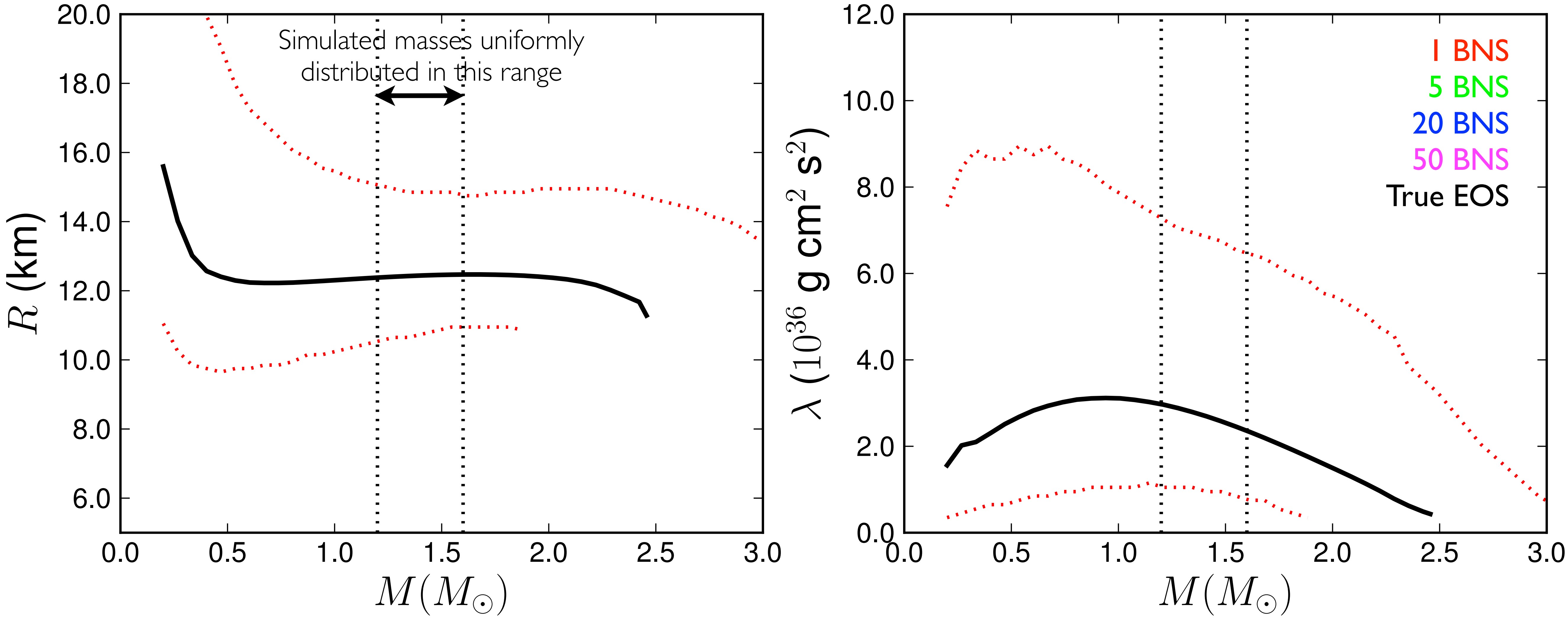


NS Radius and Tidal Deformability

- Del Pozzo et al. found $\lambda(1.4M_\odot)$ can be measured to +/-10% with 50 sources but the slope of $\lambda(M)$ cannot be measured
- Fitting the EOS function $p(\rho)$ instead of $\lambda(M)$, and using known EOS properties and mass constraints, dramatically improves the measurement of $\lambda(M)$ as well as any other quantity derived from the EOS
  - Radius, love number, moment of inertia, upper limit on spin, ...



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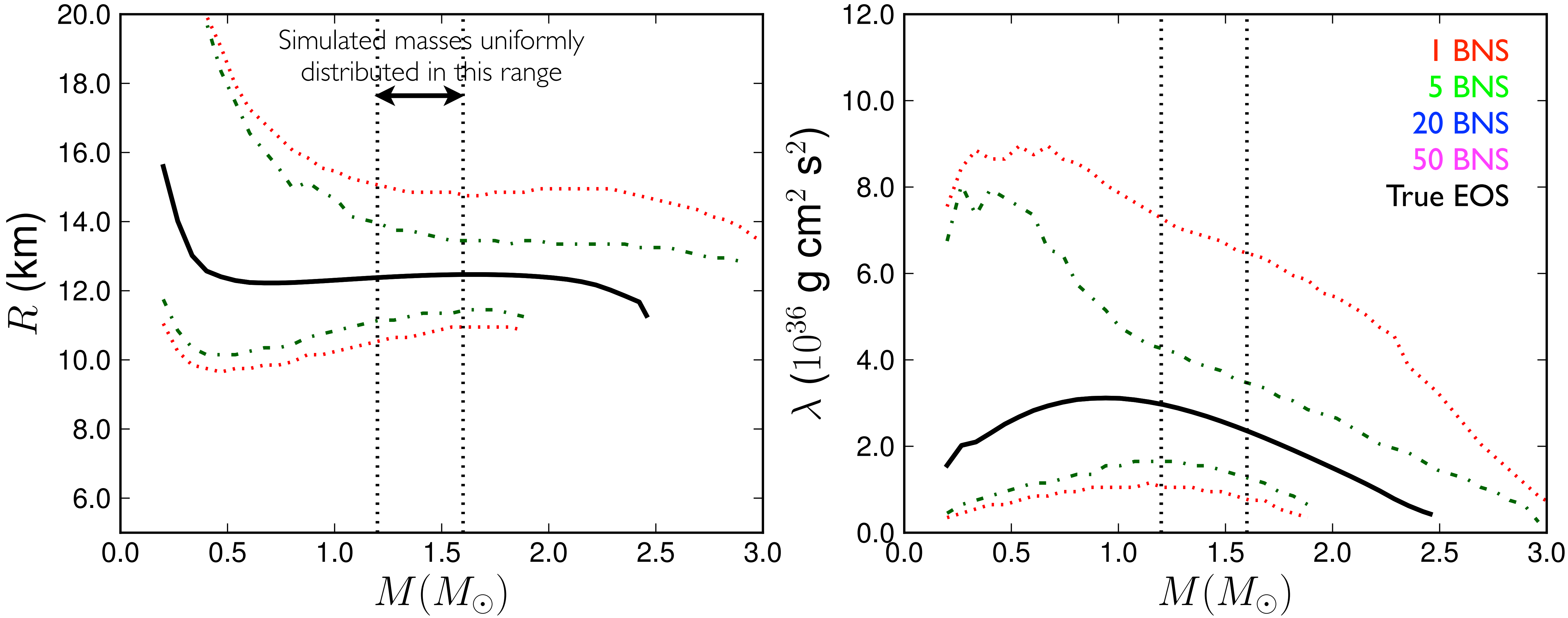

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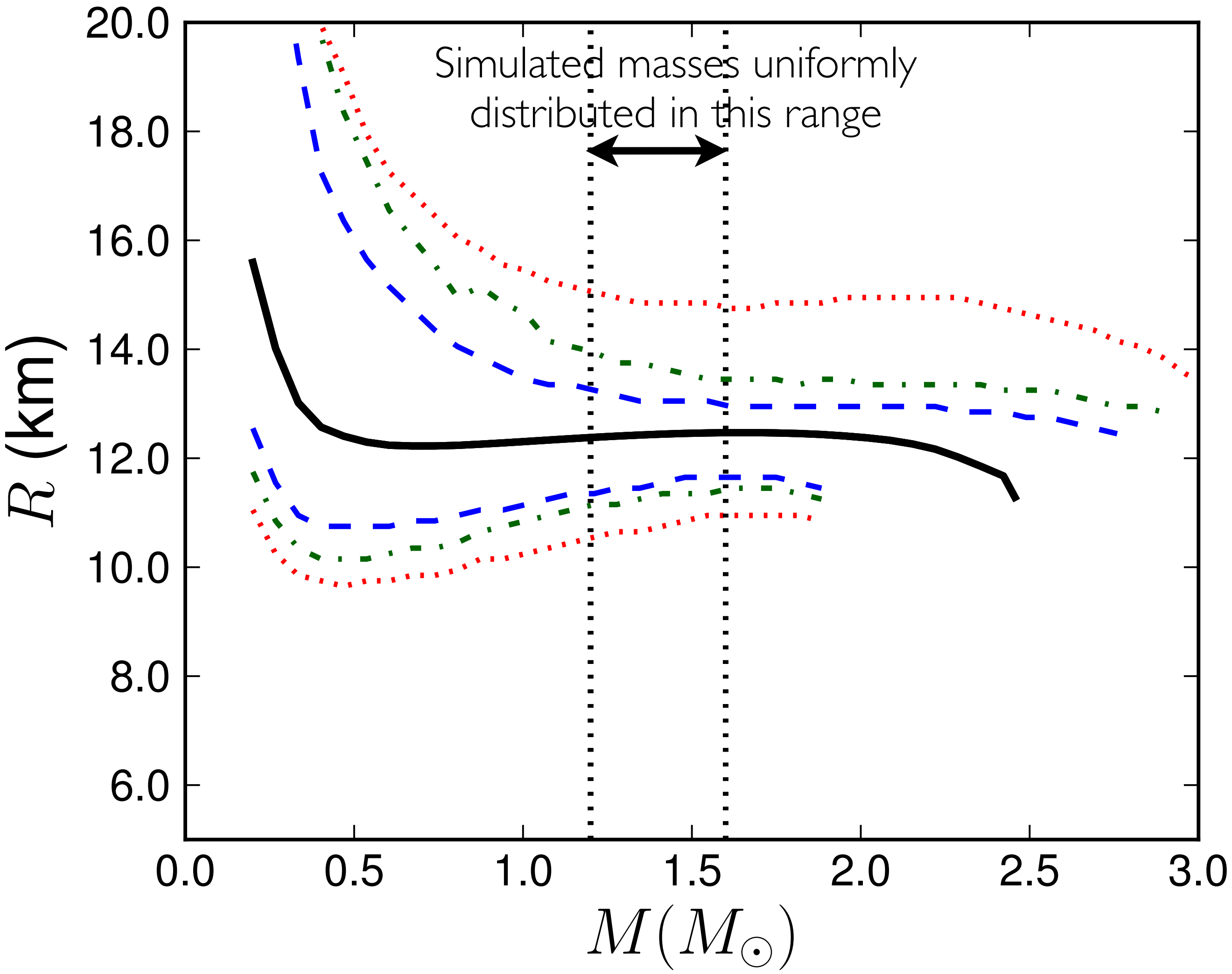

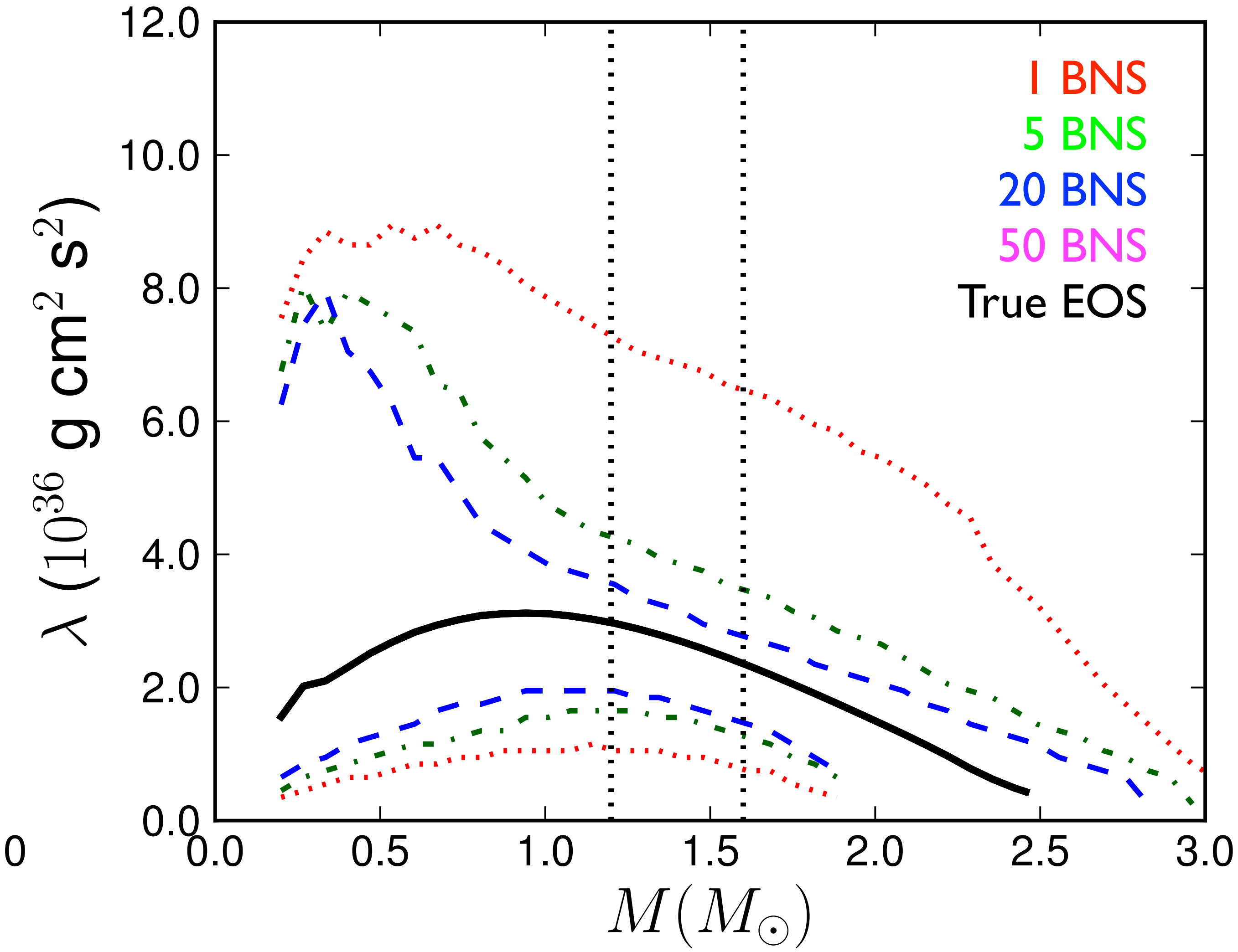

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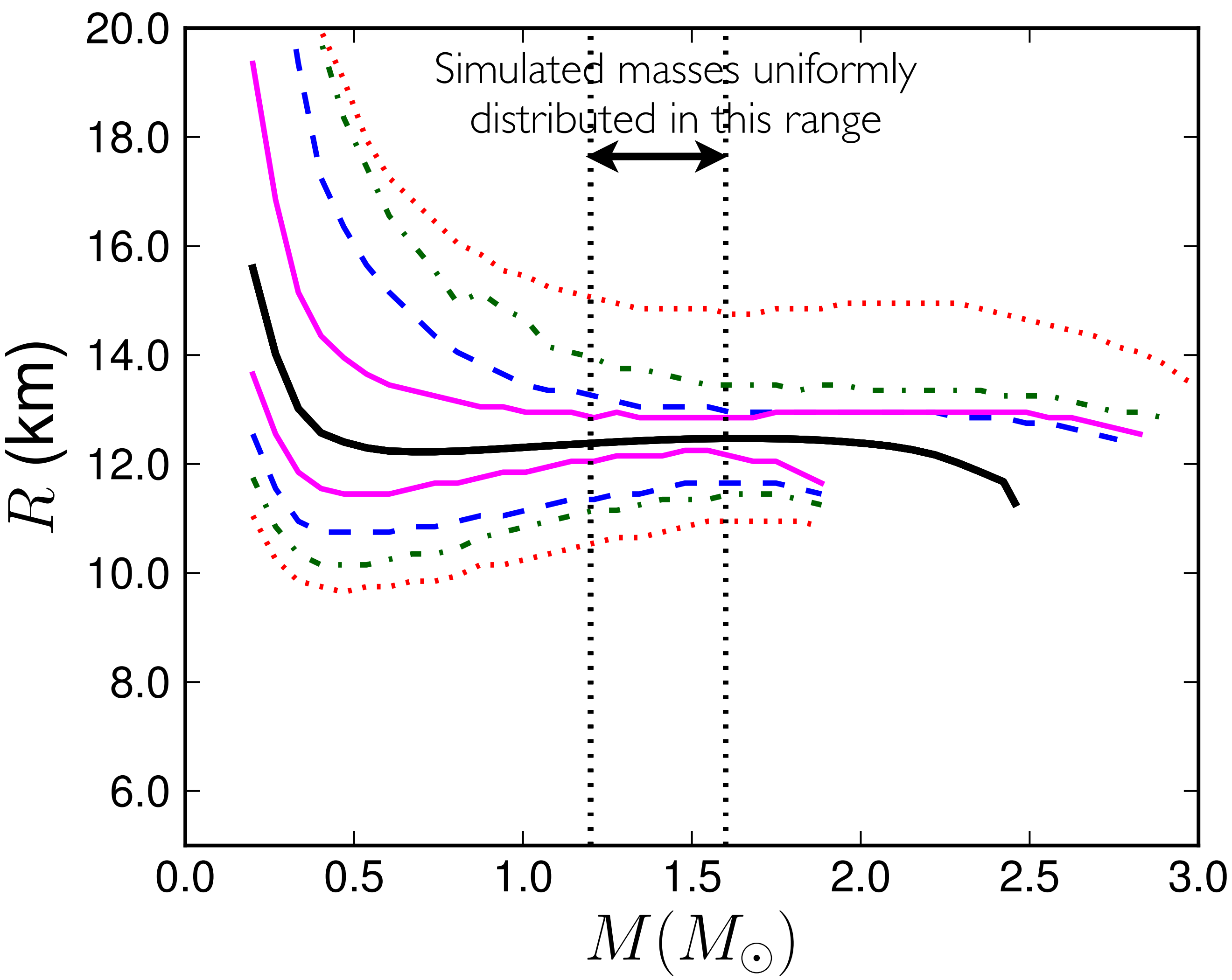
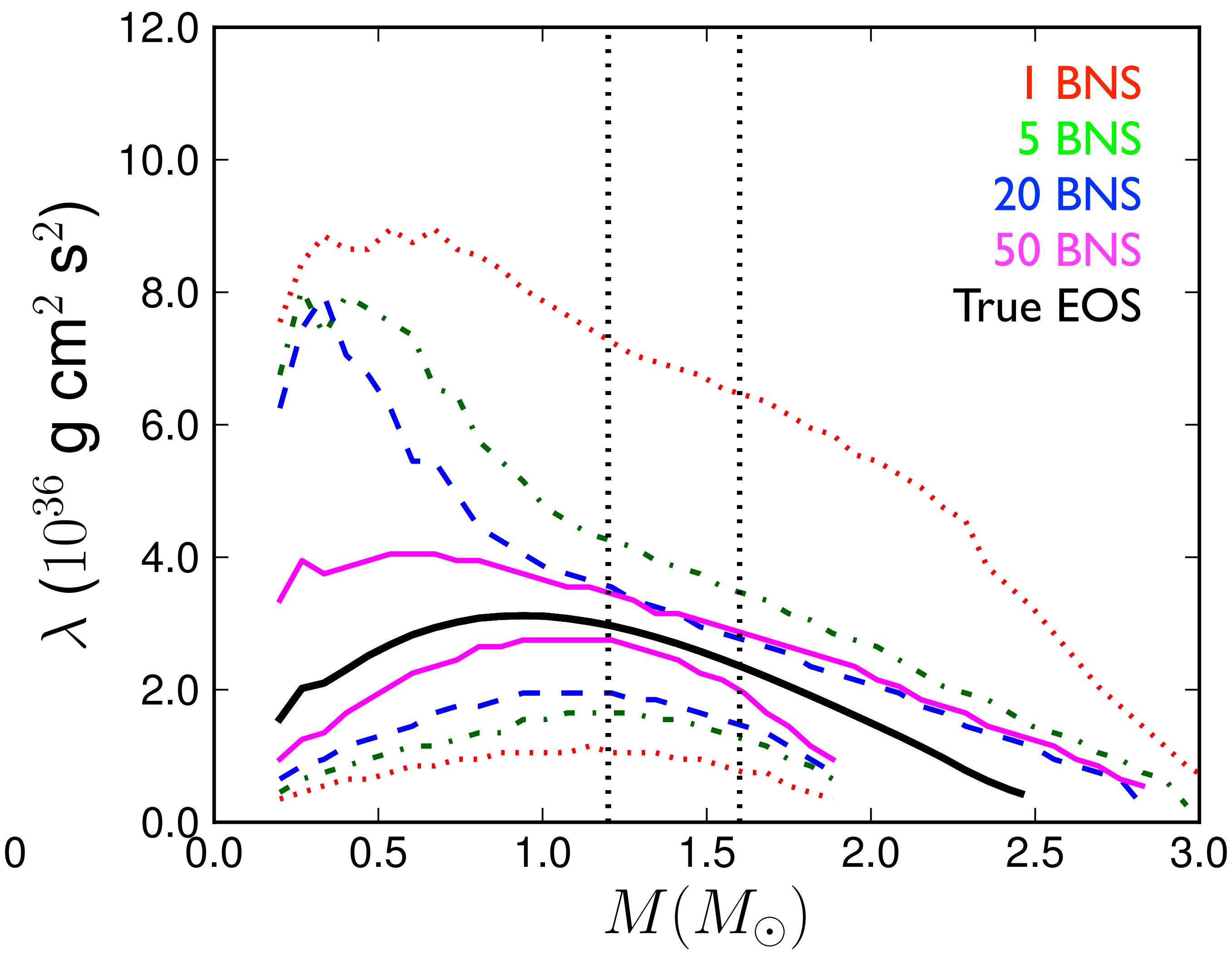
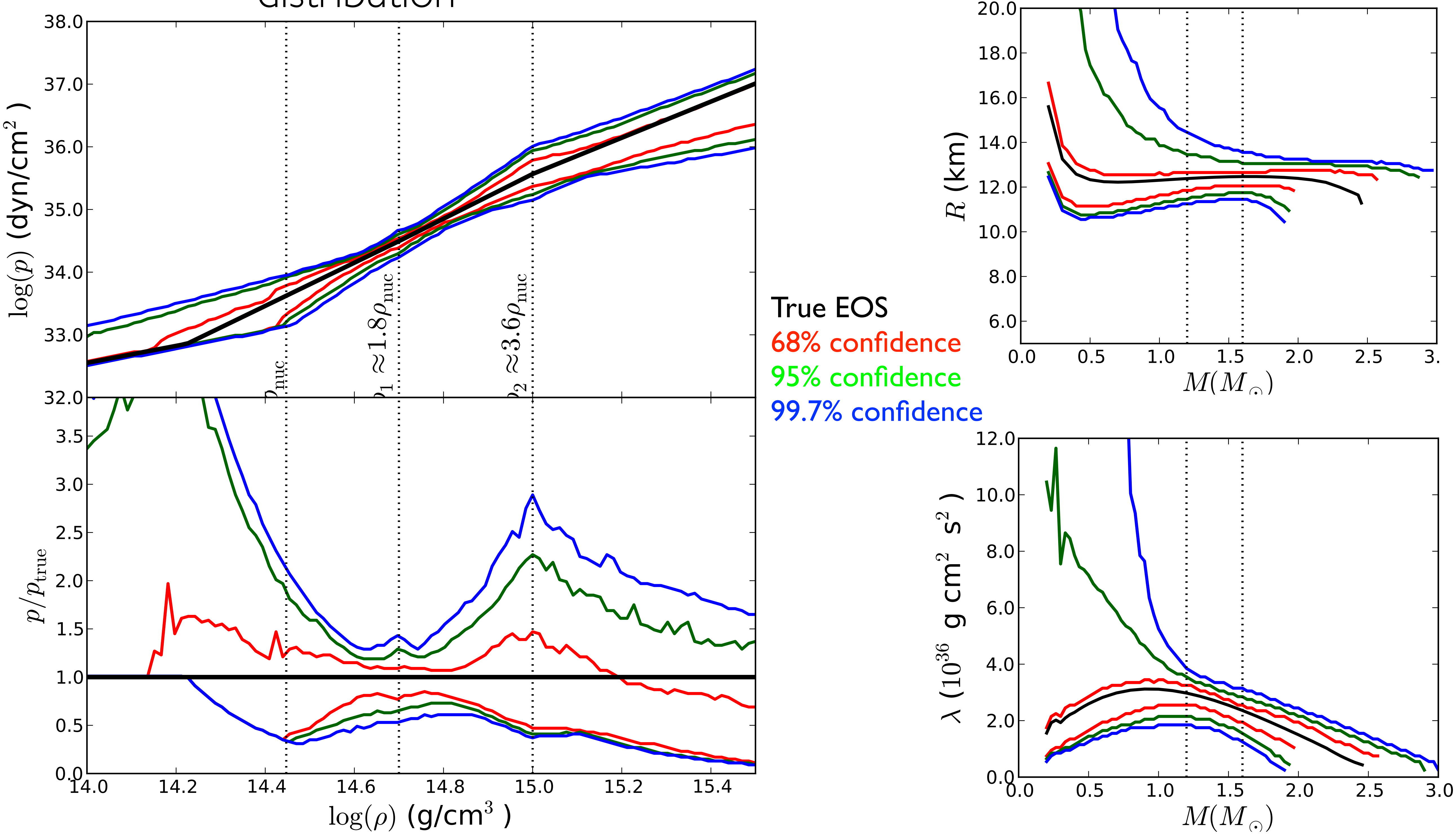




Using MCMC instead of Fisher matrix

- Currently have 18 simulations using lalinference_mcmc from Les Wade
  - Parameters sampled using MPA1 EOS and uniform 1.2--1.6$M_\odot$ mass distribution



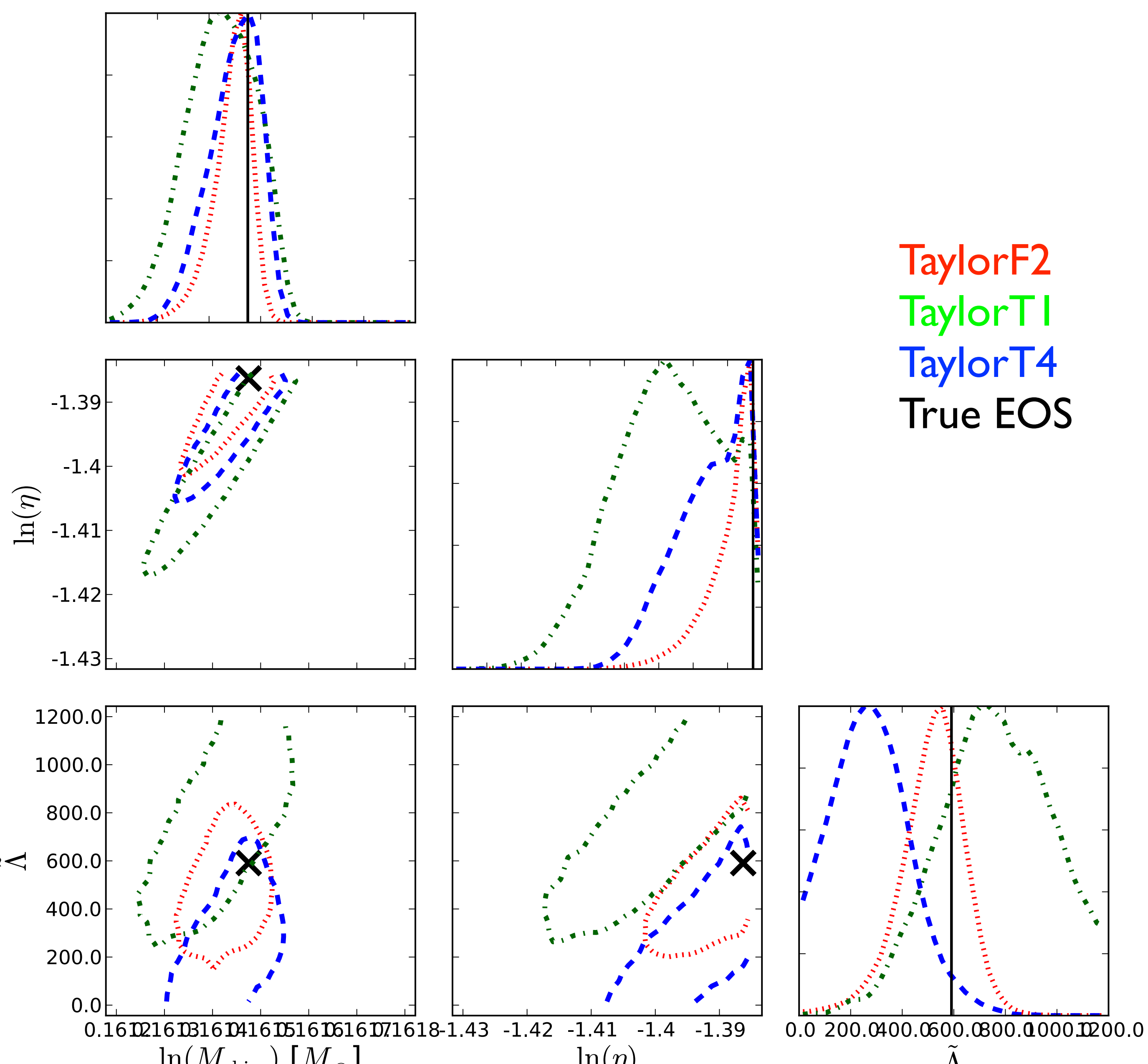
Conclusion

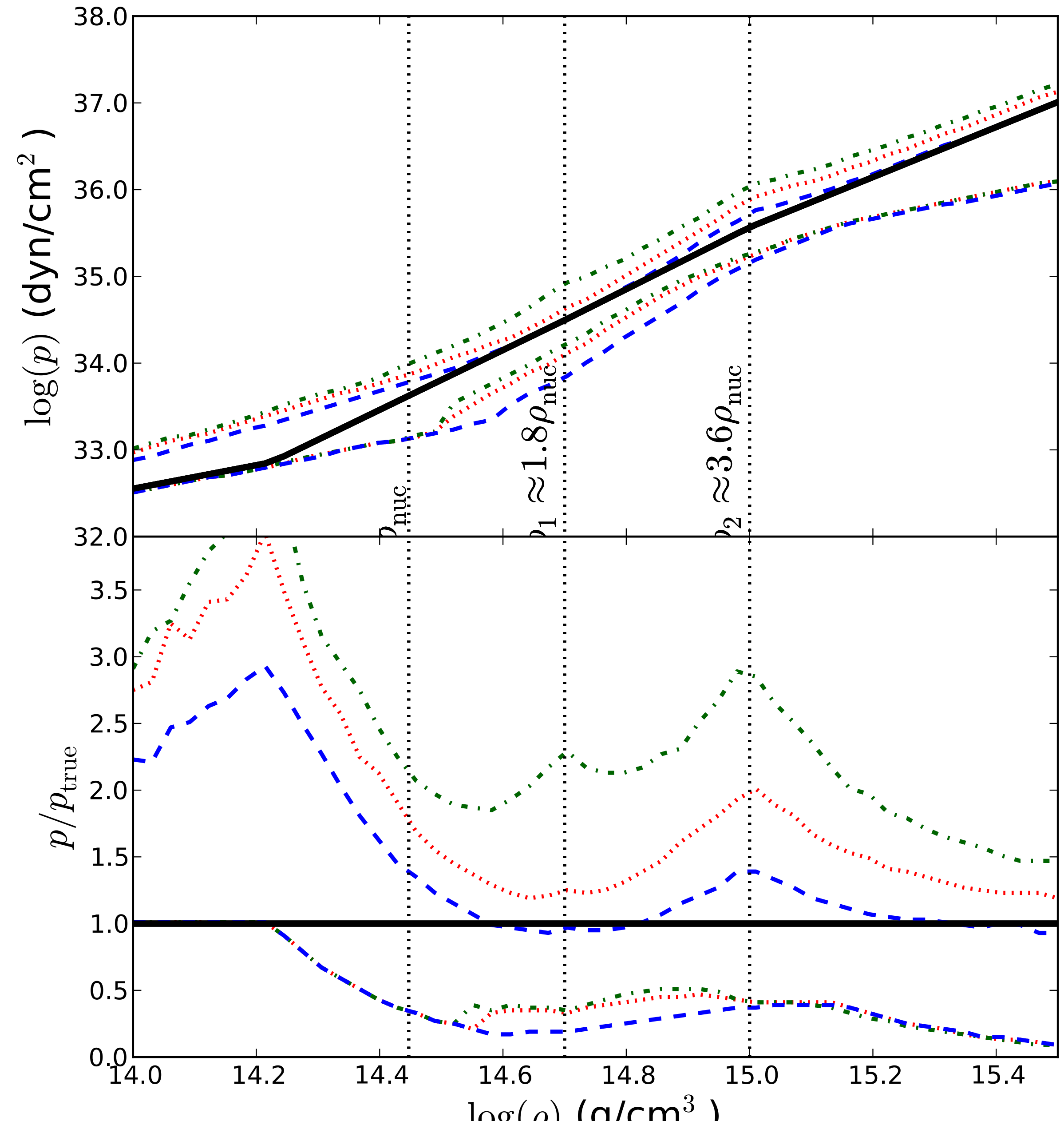
- Detailed EOS information can be found from the inspiral of BNS systems with aLIGO
    - Errors in pressure between ~10% and factor of 2 depending on density using 50 BNS systems
    - Corresponds to errors in radius of ~0.5km and error in tidal parameter of ~10%

Extra Slides

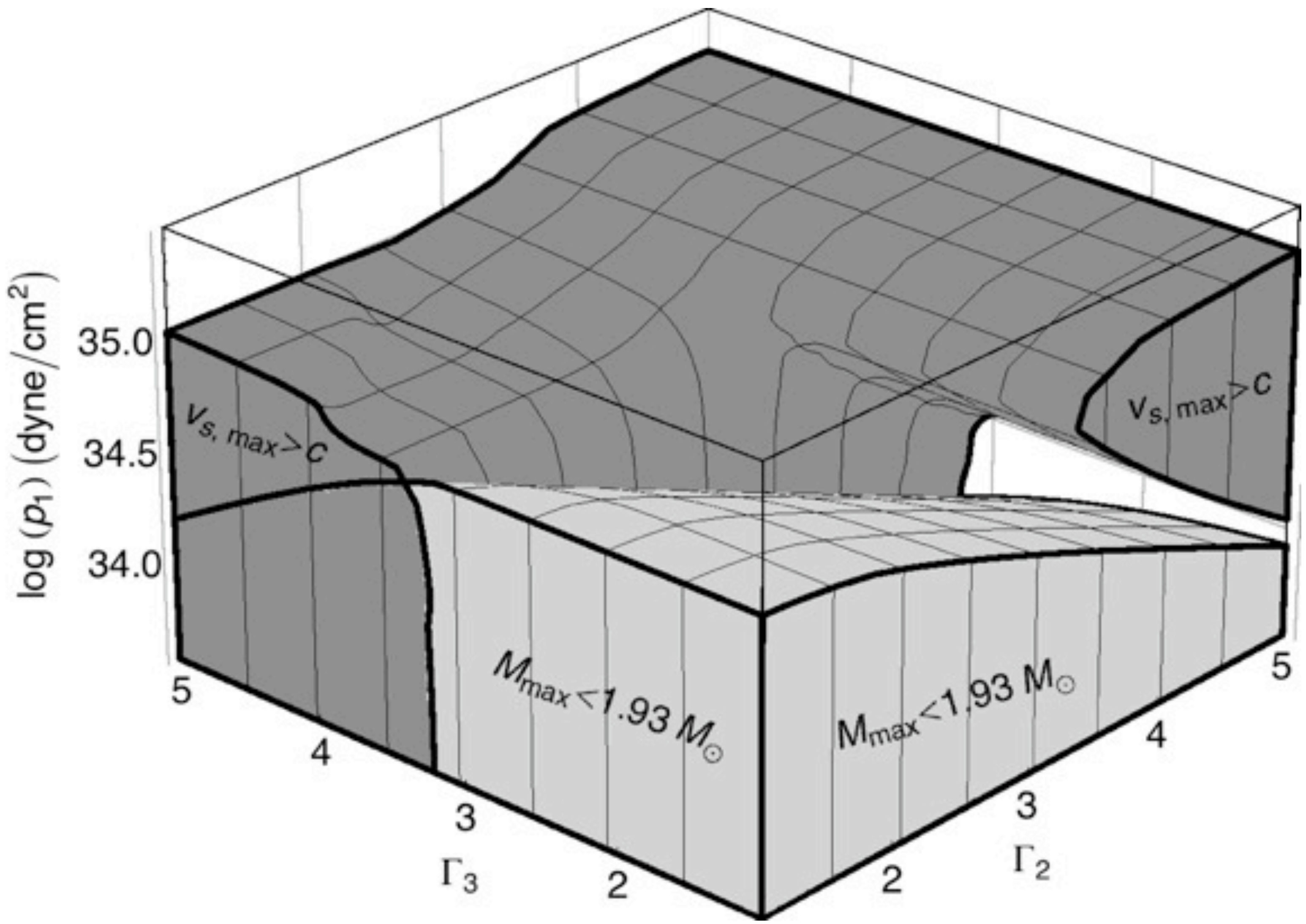
Systematic errors in point particle waveform

- Injected TaylorF2, TaylorT1, TaylorT4 waveforms and used TaylorF2 as template
    - Network SNR = 20, $m_1 = m_2 = 1.35 M_\odot$
- Systematic errors about as large as statistical errors





Current EOS constraints



- **Causality**: Speed of sound must be less than the speed of light in a stable neutron star $v_s = \sqrt{dp/d\epsilon} < c$
- **Maximum mass**: EOS must be able to support the observed star with mass greater than $1.93 M_\odot$
- Mass-radius measurements also provide strong constraints (Steiner et al.)

Parametrized EOS

- Each exact measurement of $\mathcal{M}$–η–$\tilde{\Lambda}(\mathcal{M}, \eta, \mathrm{EOS})$ restricts parameter space to N-1 dimensions
- Multiple observations with different NS masses can measure all 3 parameters if accurate enough
  - Most BNS systems will have maximum central density below $10^{15}\ \mathrm{g/cm^3}$, so Γ_3 is not constrained by inspiral observations