Probing the High-Density Behavior of Symmetry Energy with Gravitational Waves

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Outline

- Nuclear Symmetry Energy
- Equation of State (EOS) Models
- Gravitational Wave Signals:
  1. Tidal Love Number and Tidal Polarizability
  2. Elliptically Deformed Pulsars
  3. Neutron-Star Oscillations: R-modes
- Concluding Remarks

Nuclear Symmetry Energy

The EOS of asymmetric nuclear matter can be expressed in terms of the binding energy per nucleon as:

$$E(\rho,\alpha) = E(\rho,0) + S(\rho)\alpha^2 + \ldots$$

where ρ is the baryon density, and α is the so-called asymmetry parameter defined as:

$$\alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p)$$

1. The EOS of SNM E(ρ,0) is tightly constrained around nuclear saturation density.
2. Its density dependence at supra-saturation densities can be constrained combining data from collective flow and kaon production in relativistic heavy-ion collisions RHIC (Danielewicz et al. 2002) and observation of the maximum mass of a neutron star (Demorest et al. 2010; Antoniadis et al. 2013)

Nuclear Symmetry Energy

The EOS … energy per nucleon …

where ρ … ed as:

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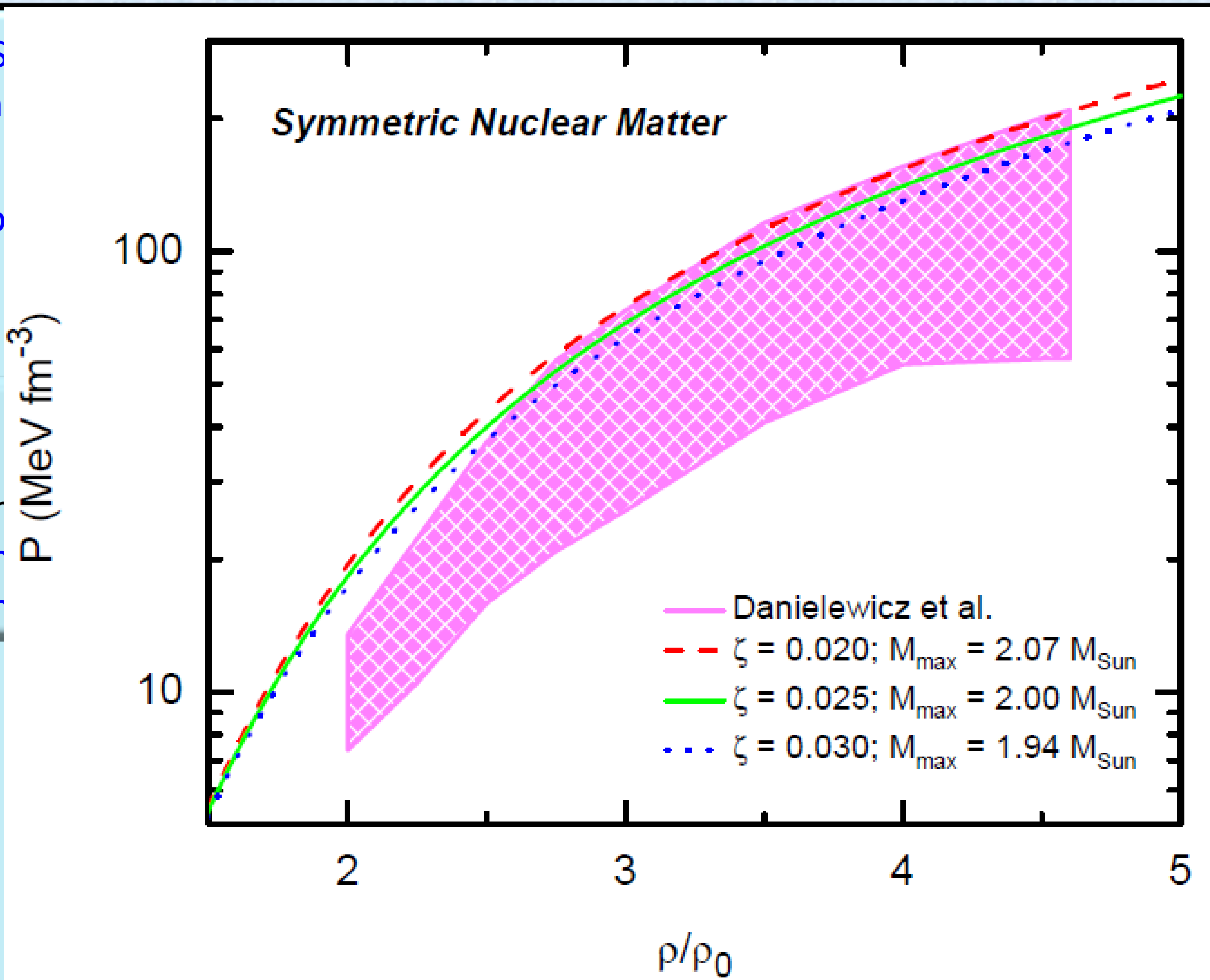

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1. Despite intensive efforts devoted, the nuclear symmetry energy S(ρ) is very loosely constrained even around the nuclear saturation density (Li et al., 2013; Fattoyev & Piekarewicz, 2013).
2. Its high-density component is totally unconstrained partially due to the lack of the experimental probes, yet its knowledge is extremely important for properties of neutron stars including their radii, moments of inertia, cooling process, etc.

Nuclear Symmetry Energy

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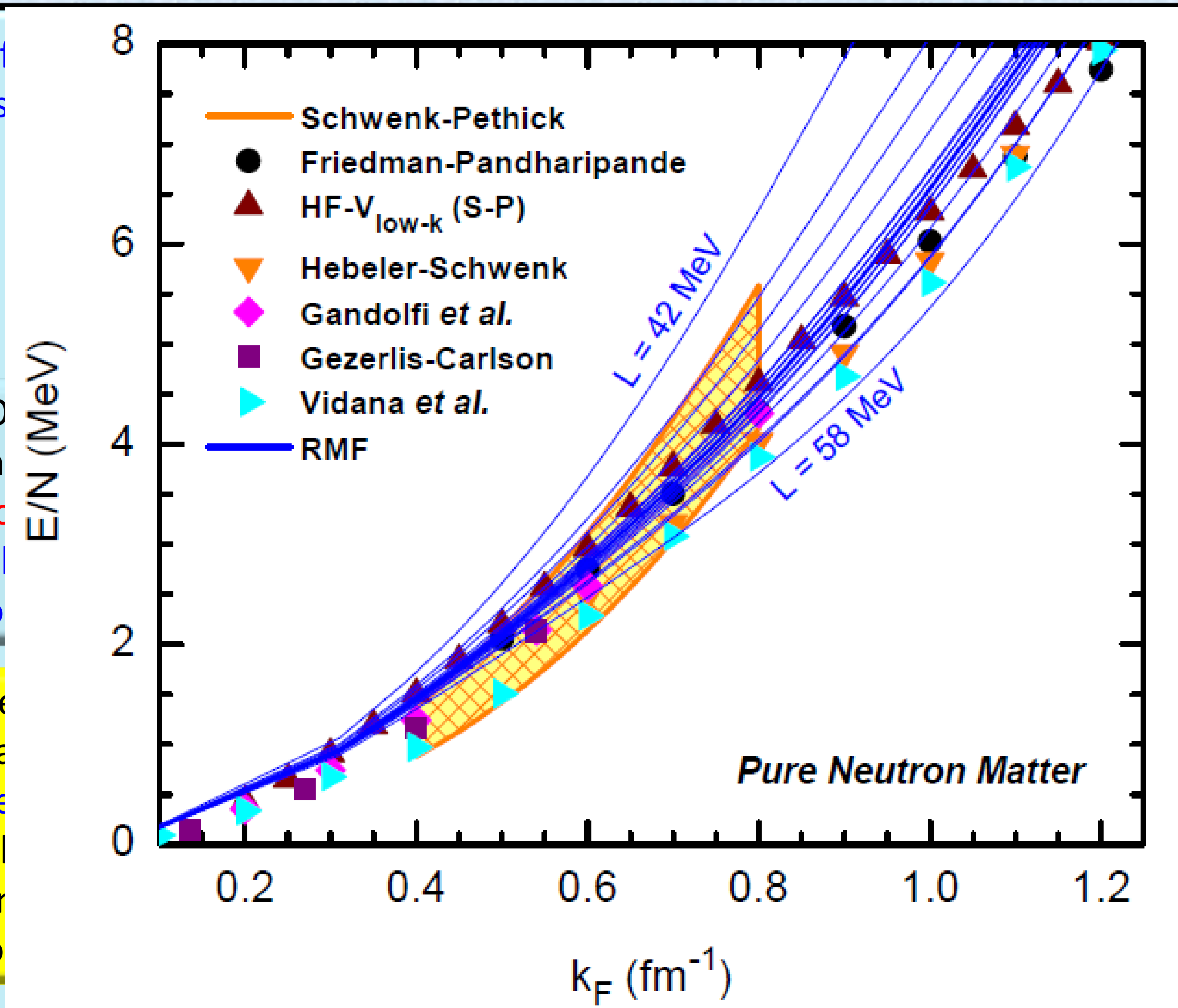

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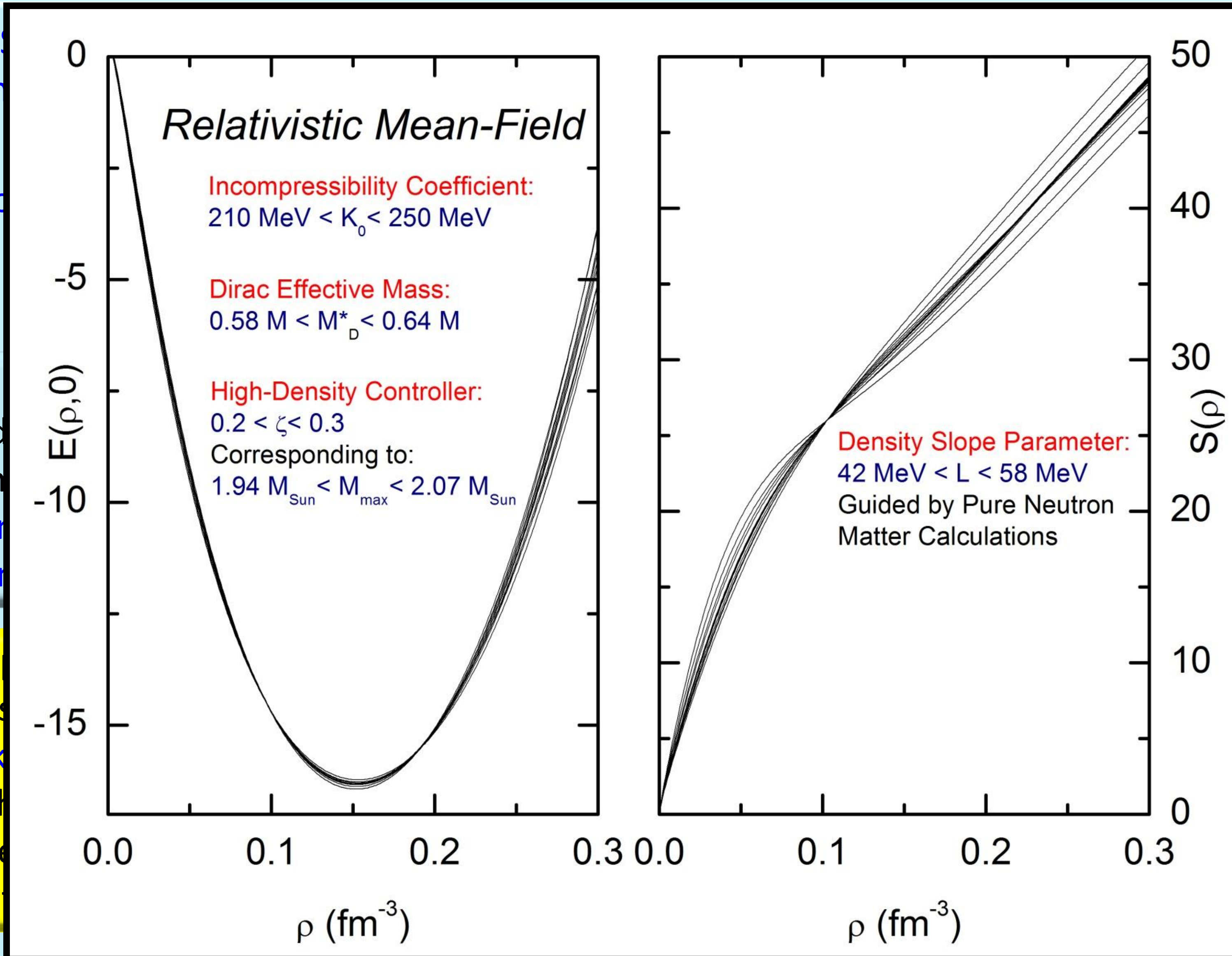

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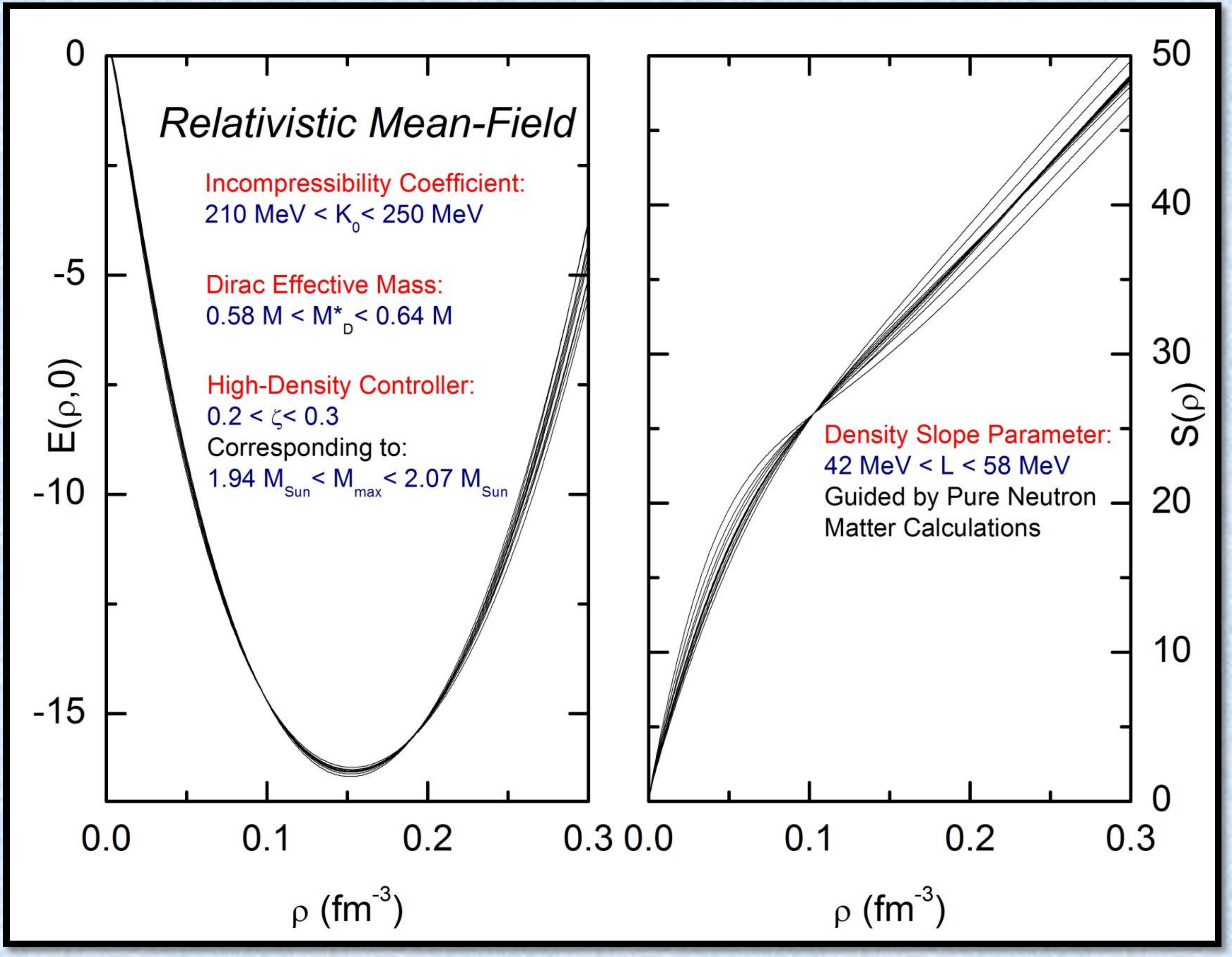
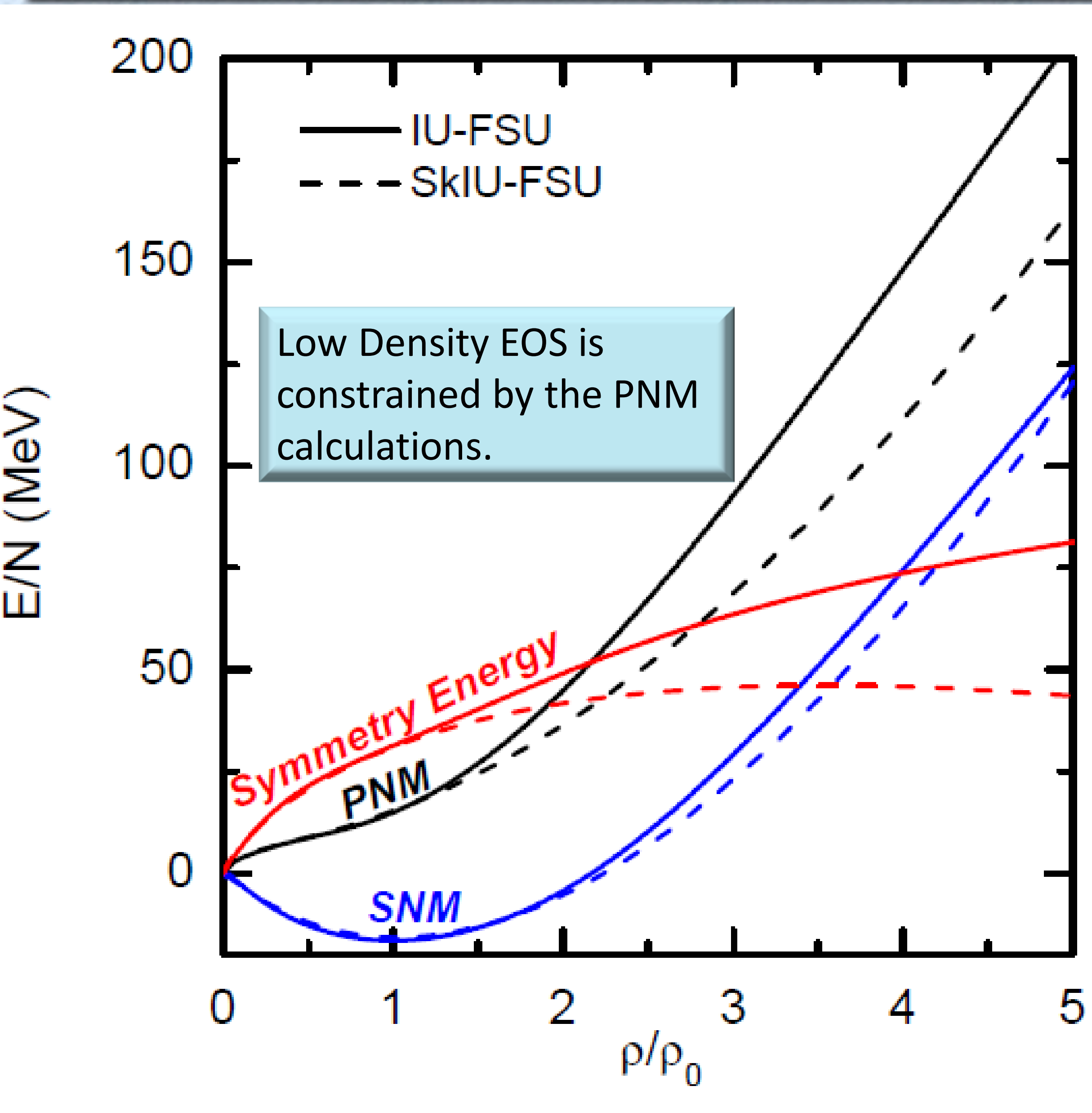
EOS Models



0
-5
-10
-15
E(ρ,0)
Relativistic Mean-Field
Incompressibility Coefficient:
210 MeV < K0 < 250 MeV
Dirac Effective Mass:
0.58 M < M*D < 0.64 M
High-Density Controller:
0.2 < ζ < 0.3
Corresponding to:
1.94 MSun < Mmax < 2.07 MSun
0.0
0.1
0.2
0.3
ρ (fm-3)
50
40
30
20
10
0
S(ρ)
Density Slope Parameter:
42 MeV < L < 58 MeV
Guided by Pure Neutron
Matter Calculations
0.0
0.1
0.2
0.3
ρ (fm-3)

EOS Models



Although a theoretical construct Pure Neutron Matter Calculations using Quantum Monte Carlo are becoming a standard and powerful tool to constrain the EOS of PNM at low densities.

High-Density Symmetry Energy remains quite uncertain!

$$S_{\rm RMF}(\rho) = A(\rho)\rho^{2/3} + B(\rho)\rho\ ,$$
$$S_{\rm SHF}(\rho) = a\rho^{2/3} - b\rho - c\rho^{5/3} - d\rho^{\sigma+1}$$

Gravitational Wave Signals:
Tidal Love number and Tidal Polarizability

At low frequency, tidal corrections to the GW waveform's phase depends on a single parameter: tidal Love number! Fattoyev et al., Phys. Rev. C, 087, 015806 (2013)

1. LIGO II detects inspirals at a rate (2-64)/year over a volume-averaged distance of 187 Mpc O'Shaughnessy et al., Astrophys. J. (2010)
2. Conventional view: the internal structure can be seen during the last several orbits f > 500 Hz;
3. Difficulties:
   (a) Highly complex behavior requires solving full non-linear GR eqns together with Relativistic Hydrodynamics.
   (b) Signal depends on unknown quantities such as spin of stars;
   (c) the signals from the hydrodynamic mergers are outside LIGO's most sensitive band (f > 1000 Hz);
4. On the contrary: At low frequency, tidal corrections to the waveforms' phase depends on a single parameter: Love number! $Q_{ij} = -\lambda E_{ij}$

Tidal polarizability is measurable at a 10% level – Flanagan and Hinderer, Phys. Rev. D, 077, 021502 (2008)

Gravitational Wave Signals: Tidal Love number and Tidal Polarizability

At low frequency, tidal corrections to the GW waveforms phase depends on single parameter: Love number! Fattoyev et al., Phys. Rev. C, 087, 015806 (2013)

$$\lambda = 2k_2 R^5/(3G)$$

$$k_2 = \frac{1}{20}\left(\frac{R_s}{R}\right)^5 \left(1-\frac{R_s}{R}\right)^2 \left[2 - y_R + (y_R - 1)\frac{R_s}{R}\right] \times \left\{ \frac{R_s}{R}\left(6 - 3y_R + \frac{3R_s}{2R}(5y_R - 8) + \frac{1}{4}\left(\frac{R_s}{R}\right)^2 \left[26 - 22y_R + \left(\frac{R_s}{R}\right)(3y_R - 2) + \left(\frac{R_s}{R}\right)^2 (1 + y_R)\right]\right) + 3\left(1 - \frac{R_s}{R}\right)^2 \left[2 - y_R + (y_R - 1)\frac{R_s}{R}\right] \times \log\left(1 - \frac{R_s}{R}\right) \right\}^{-1}, \quad (1)$$

Tidal polarizability is measurable at a 10% level (even up to merger, Damour and Nagar, 2012)

Flanagan, Hinderer, Phys. Rev. D (2008); Hinderer, Astrophys. J. (2008)
Binnington, Poisson, Phys. Rev. D (2009); Damour, Nagar, Phys. Rev. D (2009, 2010, 2012)
Hinderer et al., Phys. Rev. D (2010); Postnikov et al., Phys. Rev. D (2010)
Baiotti et al., Phys. Rev. Lett. (2010); Baiotti et al., Phys. Rev. D (2011)
Pannarale et al., Phys. Rev. D (2011); Lackey et al., Phys. Rev. D (2012)

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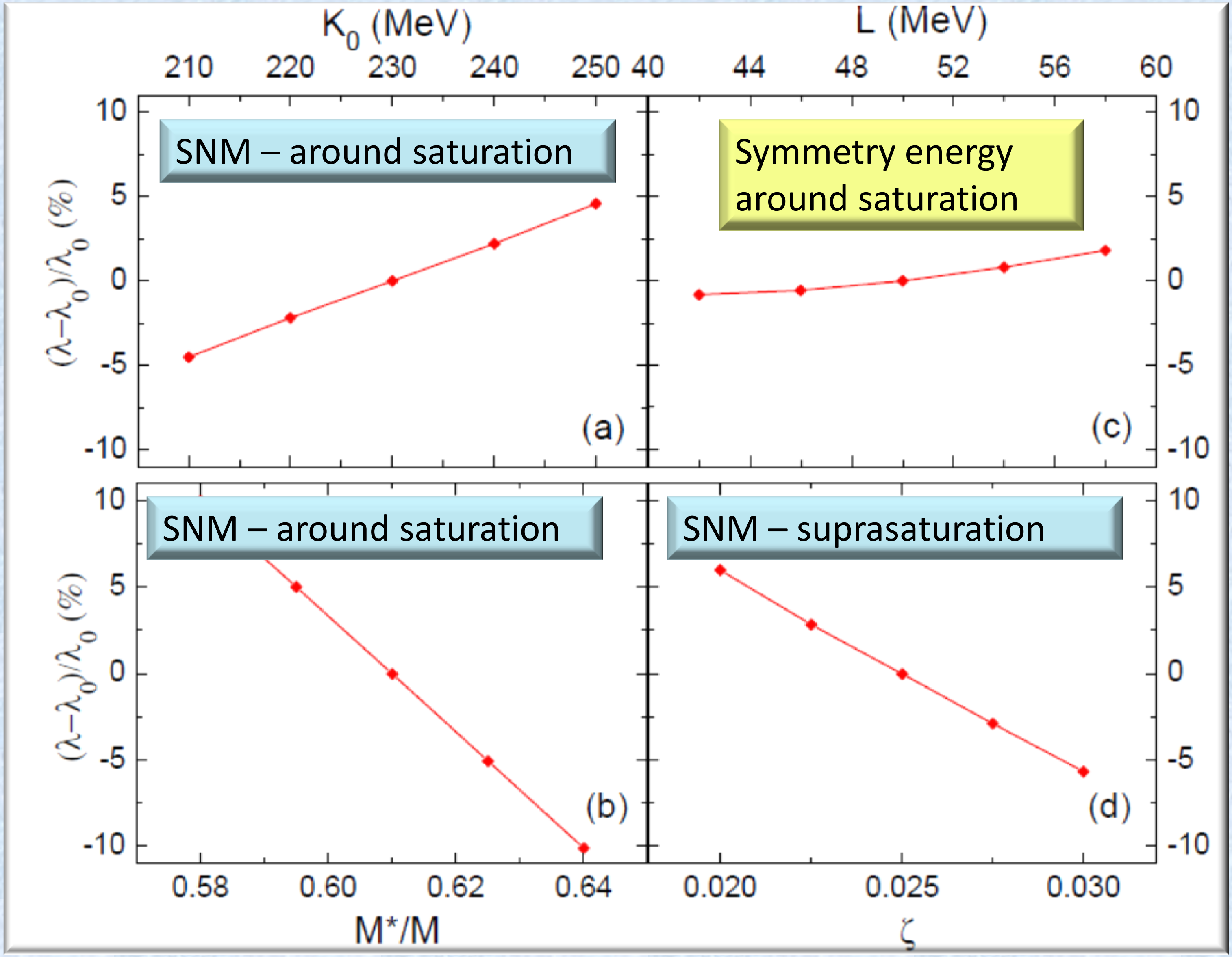

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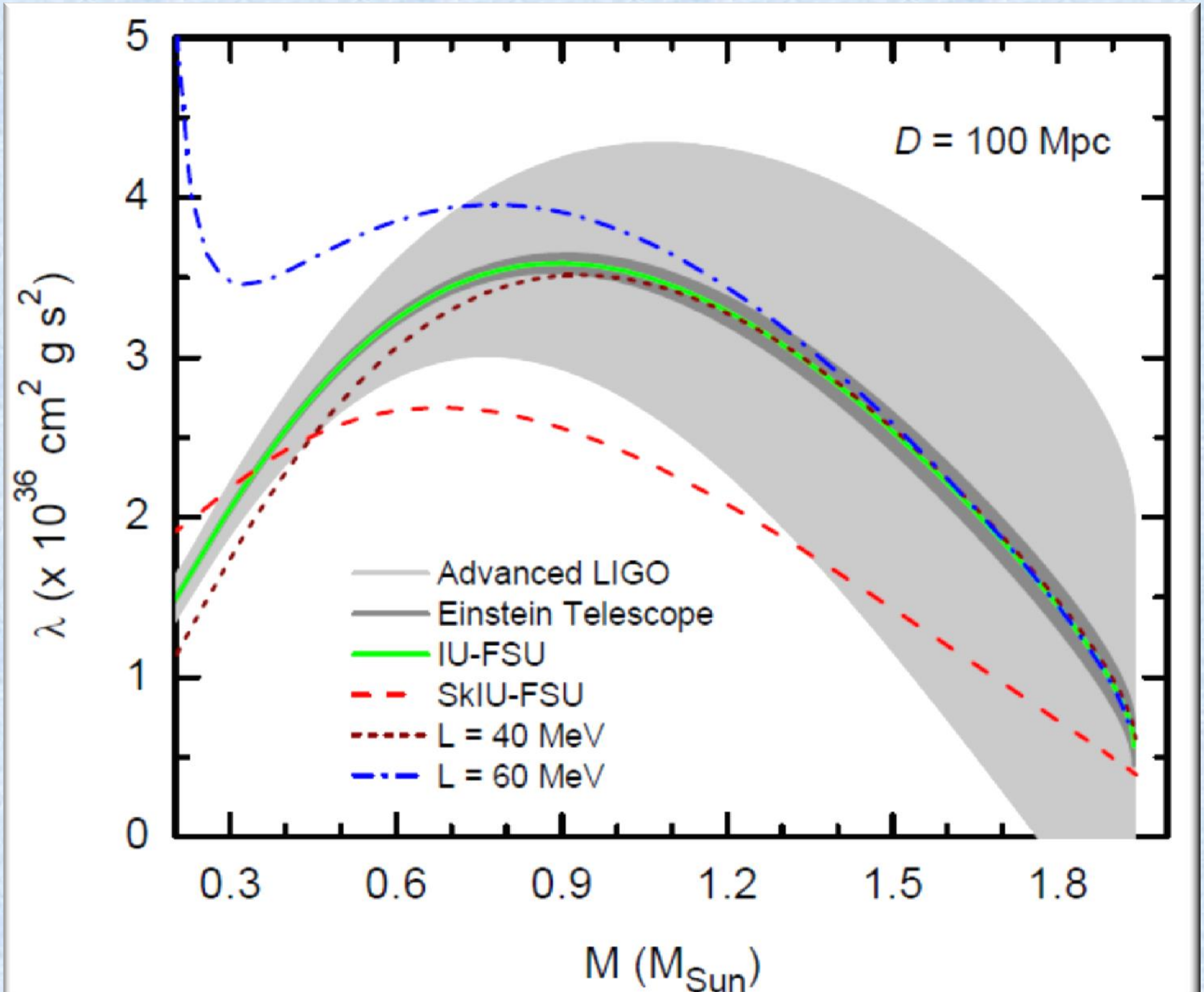

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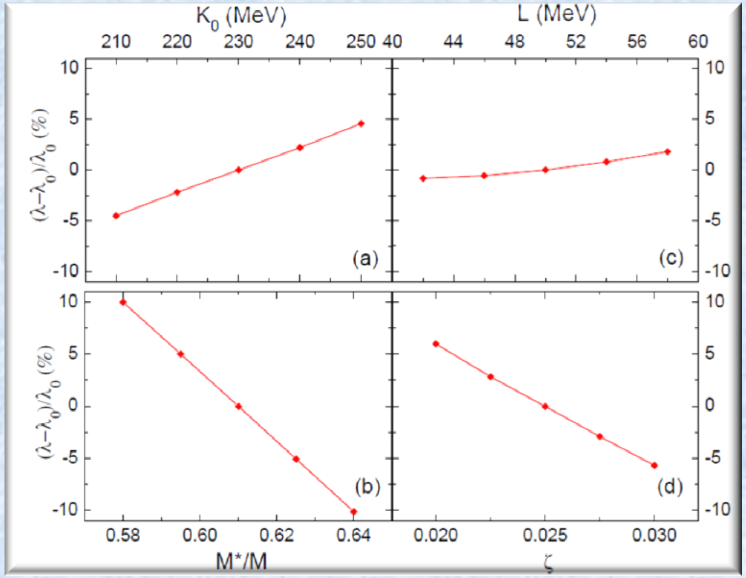

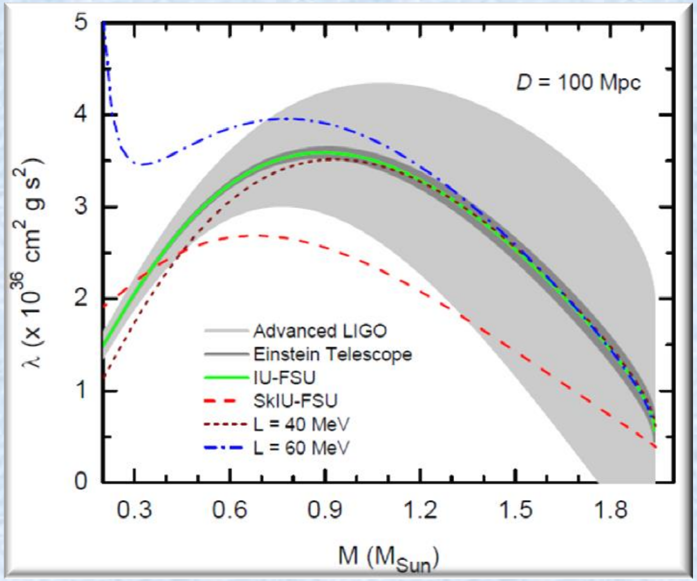

EOS	L	R	k_2	λ	$\Delta\lambda/\lambda$
IU-FSU	47.2	12.49	0.0930	2.828	—
IU-FSU-min	40.0	12.20	0.1054	2.841	+ 0.46 %
IU-FSU-max	60.0	13.07	0.0761	2.906	+ 2.76 %
SkIU-FSU	47.2	11.71	0.0753	1.657	−41.41 %

Fattoyev et al. Phys. Rev. C 87, 015806, 2013 arXiv:1210.3402; Fattoyev et al. 2013, arXiv:1309:5153

Gravitational Wave Signals: Elliptically Deformed Pulsars

Rotating stars produce gravitational waves. The only requirement is the axial asymmetry!

A) Anisotropic stress built up during crystallization period of the solid NS crust may support "mountains". Horowitz and Kadau, PRL 102, 191102 (2009)

B) Due to its violent formation in Supernova, rotation axis and principal axis of moment of inertia may not be aligned. Zimmermann and Szedenitz, PRD 20, 351 (1979)

C) Strong magnetic pressure may distort the star. Bonazolla and Courgoulhon, A&A 312, 675 (1996)

$$h_0 = \frac{16\pi^2 G}{c^4}\frac{\epsilon I_{zz} f^2}{r}$$

- h_0: GW strain amplitude
- I_{zz}: Moment inertia around principal axis
- f: NS's spin frequency
- ϵ: ellipticity

$$\epsilon = \sqrt{\frac{8\pi}{15}}\frac{Q_{22,\max}}{I_{zz}}$$

GW strain amplitude does not depend on the neutron star moment of inertia explicitly but on the quadrupole moment.

$$Q_{22,\max} = 2.4\times 10^{38}\,\mathrm{g\,cm}^2 \left(\frac{\sigma_{\max}}{10^{-2}}\right)\left(\frac{R}{10\,\mathrm{km}}\right)^{6.26}\left(\frac{1.4\,M_\odot}{M}\right)$$

Obtained using a chemically detailed model of crust. Notice this is just an approximation. Ushomirsky et al. MNRAS 319, 902 (2000)

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$$Q_{lm} \equiv \int \delta\rho_{lm}(r) r^{l+2} dr$$

Gravitational Wave Signals:
Elliptically Deformed Pulsars

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$h_0 = \frac{1}{\ldots}$

GW

$Q_{22,\mathrm{n}}$

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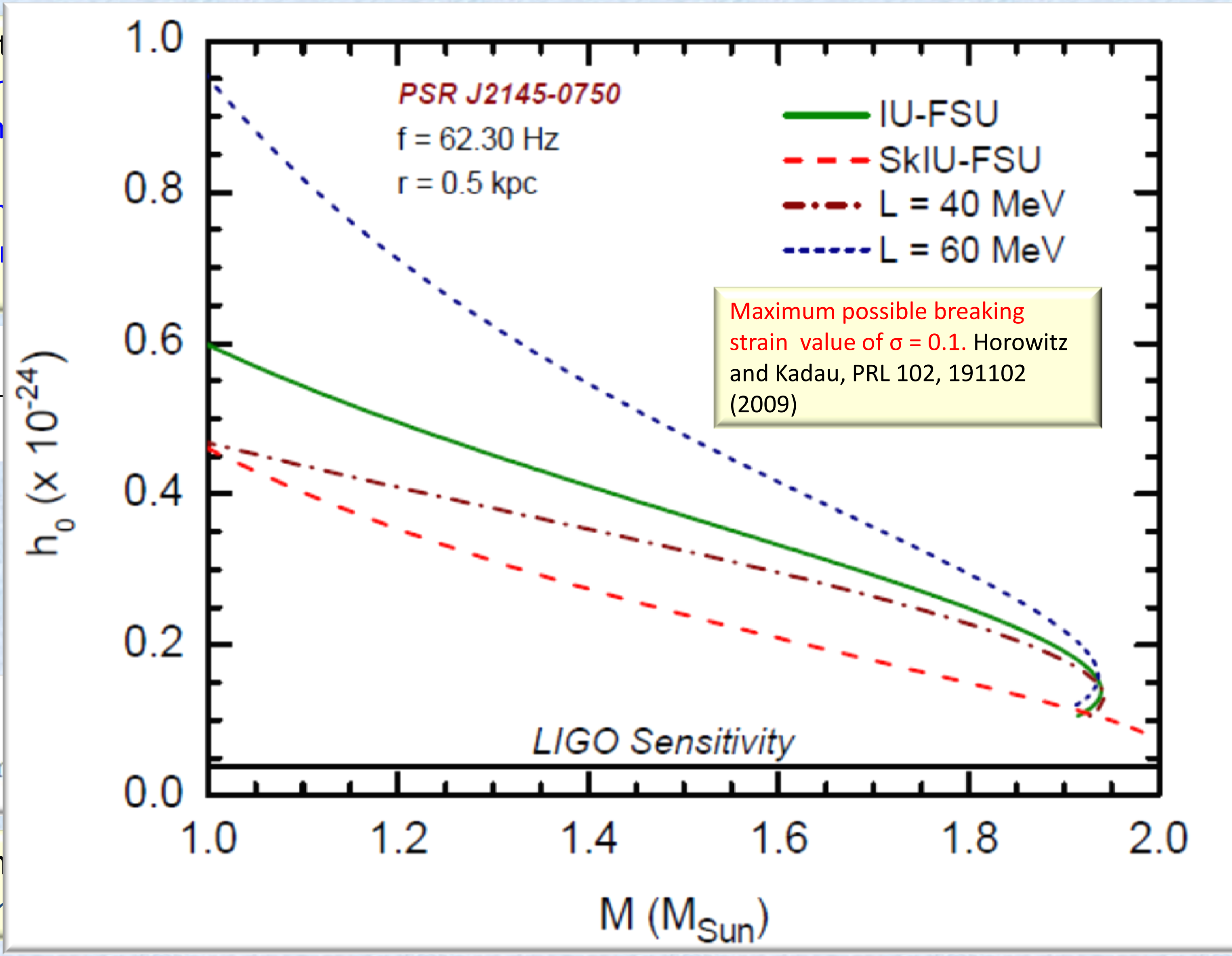


Gravitational Wave Signals: Elliptically Deformed Pulsars

Table 3. Properties of the nearby pulsars considered in this study. The first column identifies the pulsar. The remaining columns are rotational frequency, distance to Earth, the observed [99] and the calculated upper limits on the gravitational wave strain amplitude. Notice that the masses of most of these pulsars are presently unknown and therefore we have adopted a canonical $1.4M_\odot$ mass neutron star in calculating the gravitational wave strain amplitudes, which are all given in units of 1.0×10^{-24}.

Pulsar	f (Hz)	r (kpc)	$h_0^{\rm ob}$	$h_0^{\rm th}$ (IU-FSU)	$h_0^{\rm th}$ (IU-FSU-min)	$h_0^{\rm th}$ (IU-FSU-max)	$h_0^{\rm th}$ (SkIU-FSU)
J0437−4715	173.69	0.1	0.5730	15.9590	13.7350	21.2256	10.6567
J0613−0200	326.60	0.5	0.1110	11.2855	9.7127	15.0097	7.5359
J0751+1807	287.46	0.6	0.1640	7.2855	6.2702	9.6898	4.8649
J1012+5307	190.27	0.5	0.0694	3.8303	3.2965	5.0943	2.5577
J1022+1001	60.78	0.4	0.0444	0.4886	0.4205	0.6498	0.3262
J1024−0719	193.72	0.5	0.0501	3.9704	3.4171	5.2807	2.6513
J1455−3330	125.20	0.7	0.0515	1.1846	1.0195	1.5755	0.7910
J1730−2304	123.11	0.5	0.0593	1.6035	1.3801	2.1327	1.0708
J1744−1134	245.43	0.5	0.1100	6.3730	5.4848	8.4761	4.2556
J1857+0943	186.49	0.9	0.0727	2.0442	1.7593	2.7188	1.3650
J2019+2425	254.16	0.9	0.0923	3.7969	3.2678	5.0499	2.5354
J2124−3358	202.79	0.2	0.0485	10.8773	9.3614	14.4668	7.2634
J2145−0750	62.30	0.5	0.0383	0.4106	0.3534	0.5462	0.2742
J2322+2057	207.97	0.8	0.1120	2.8600	2.4614	3.8038	1.9098

$$Q_{22,\max} = 2.4\times10^{38}\,\mathrm{g\,cm^2}\left(\frac{\sigma_{\max}}{10^{-2}}\right)\left(\frac{R}{10\,\mathrm{km}}\right)^{6.26}\left(\frac{1.4\,M_\odot}{M}\right)$$

$$h_0 = \frac{16\pi^2 G}{c^4}\frac{\epsilon I_{zz} f^2}{r}$$

We have used a maximum breaking strain value of σ = 0.1. Horowitz and Kadau, PRL 102, 191102 (2009)

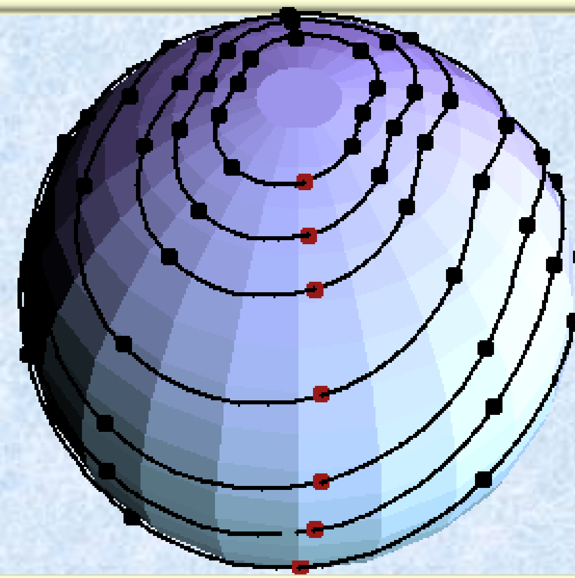
[99] LIGO Collaboration, ApJ713, 671 (2010)

Gravitational Wave Signals:
Neutron-Star Oscillations

Based on the nature of the restoring force the neutron star oscillations can be classified into:

1) *f*undamental f-modes associated with global oscillations of the fluid (frequency range of 1-10 kHz)
2) *g*ravity g-modes associated with the fluid buoyancy (frequency range of 2-100 Hz)
3) *p*ressure p-modes associated with pressure gradient (few kHz frequency range)
4) *r*otational r-modes, associated with the Coriolis force (frequency depends on the stars frequency)
5) purely general-relativistic gravitational *w*ave w-modes, associated with the curvature of spacetime with frequnecy range above 7 kHz.

As seen by a non-rotating observer: r-modes are prograde in this frame
Star rotates faster then r-mode pattern speed



As seen by a co-rotating observer: r-modes are retrograde rotational frame
r-mode pattern seems to move backward

https://pantherfile.uwm.edu/friedman/www/rmode_owen.gif

Gravitational Wave Signals:
R-mode instability (rigid crust approximation)

Mechanism of r-modes:

1. GW radiation tends to drive the r-modes towards unstable growth: the amplitude increases.
2. Viscous dissipation tries to stabilize such an instability of r-modes.

$$\frac{1}{\tau_{\rm GR}} = \frac{32\pi G \Omega^{2l+2}}{c^{2l+5}} \frac{(l-1)^{2l}}{[(2l+1)!!]^2} \left(\frac{l+2}{l+1}\right)^{2l+2} \times \int_0^{R_{\rm t}} \mathcal{E} r^{2l+2} dr \ ,$$

Gravitational radiation timescale

Viscous dissipation timescale at the crust-core boundary layer.

$$\tau_{\rm v} = \frac{2^{l+1}(l+1)!}{l(2l+1)!!\mathcal{I}_l c R_{\rm t}^{2l+2}\sqrt{\mathcal{E}_{\rm t}\Omega\eta_{\rm t}}} \int_0^{R_{\rm t}} \mathcal{E} r^{2l+2} dr$$

Both depend strongly on the EOS. The critical frequency is then defined such that:

$$\tau_{\rm GR} = \tau_{\rm v}$$

We use the electron-electron scattering as the main dissipation mechanism which is true if one considers relatively cool neutron stars with temperature below 1E9 K, above which neutron-neutron scattering might be more relevant.

Gravitational Wave Signals:
R-mode instability (rigid crust approximation)

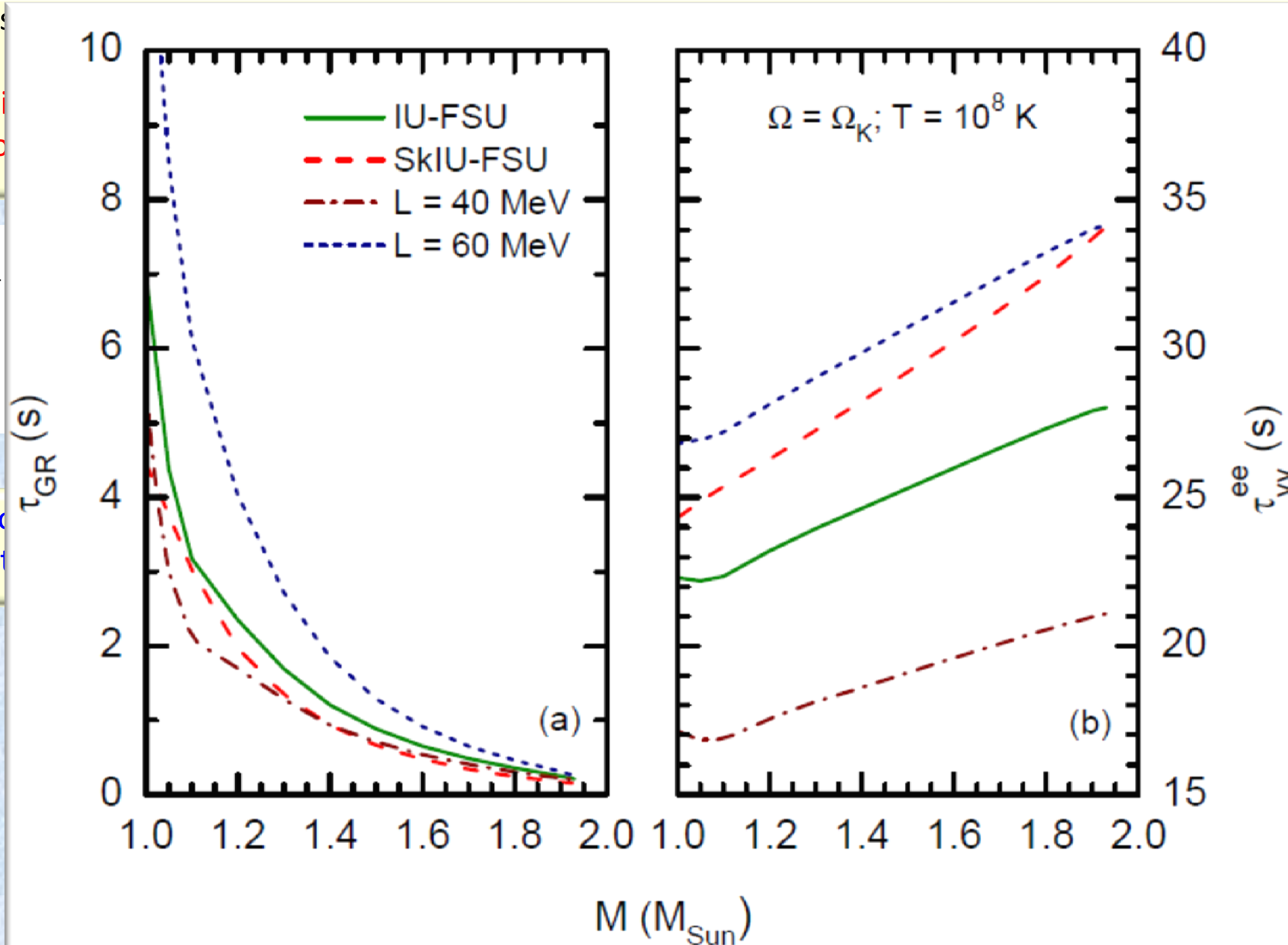
Mechanis

1. Gravi
2. Visco

$\frac{1}{\tau_{\mathrm{GR}}}$

Visco
crust

^{2l+2}dr



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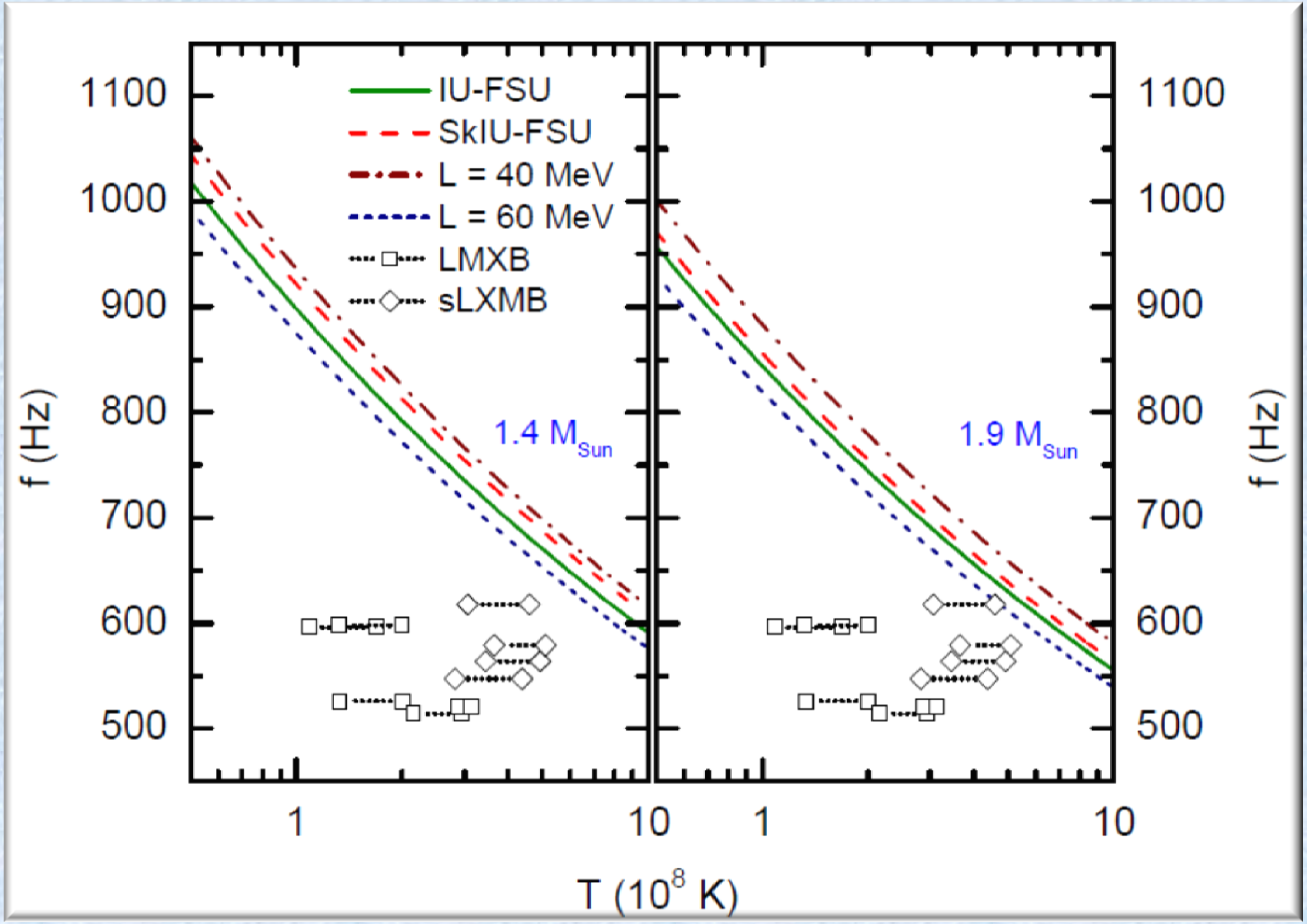

Masses of NSs in LMXB are not accurately measured.
The core temperature are derived from their observed accretion luminosity assuming the cooling is either dominated by modified Urca with normal nucleons (left) or by modified Urca with superfluid neutrons and superconducting protons.

Summary

We have used several EOS of neutron-rich matter satisfying the latest constraints from terrestrial nuclear experiments , state-of-the-art many body calculations for EOS of PNM, and astrophysical observations to probe high-density component of the nuclear symmetry energy with various gravitational wave signals:

1. We have found that tidal polarizability of coalescing neutron stars is the most sensitive probe to the high-density component of the nuclear symmetry energy.
2. *Within a simple approximation* we showed that the gravitational wave strain amplitude from elliptically deformed pulsars is also a sensitive probe to the symmetry energy. Much works need to be done to completely understand the physics of NS crust, in particular its ability to support large ellipticities, microphysics of crust, etc. Observations of GWs from accreting is a promising avenue towards constraining the EOS of neutron-rich nucleonic matter.
3. We have analyzed the sensitivity of the EOS to the r-mode instability line, and showed that it is very sensitive to the nuclear symmetry energy around saturation. Within this simple model we showed that only EOSs with soft symmetry energy with $L < 60$ MeV could be ruled in. A stiffer symmetry energy at saturation could be ruled in only if the subsequent EOS at supra-saturation is softer. However the room for that is much smaller.

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Thank you!