Lifshitz Spacetimes in the Horava-Lifshitz theory in 2+1 dimensions

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Outline

- HL gravity
- Our work and result
- Conclusions and Future Work

Horava-Lifshitz gravity

- Horava-Lifshitz gravity is a new attempt towards quantization of gravity.
- Based on the perspective that Lorentz symmetry appear as an emergent symmetry at long distances, but can be fundamentally absent at high energies.
- As a result, higher order spatial derivative terms allowed in this theory, which changes gravity in UV region.

Anisotropic Scaling

- The key point is that the system exhibits a strong anisotropic scaling between space and time,

$$\mathbf{x} \to \ell \mathbf{x}, \quad t \to \ell^z t, \tag{1}$$

which makes the theory power-counting renormalizable.

- Anisotropic Lifshitz-type scaling has played a central role in the new approach to holographic dual between HL gravity and Lifshitz-type field theory.

Lifshitz spacetime in relativistic gravity

- Lifshitz spacetime solution is a very interesting topic in relativistic gravity.
- Lifshitz spacetimes is invariant under anisotropic scale transformation.
- Obtain Lifshitz spacetime as solutions of the field equations of General Relativity coupled to some matter content (S. Kachru, X. Liu, M. Mulligan, PRD 78,106005(2008))

Question

- In relativistic gravity theory, the coupling to matter is necessary, as the Lifshitz spacetime with $z \neq 1$ does not solve the Einstein equations in the vacuum.(T. Griffin, P. Horava...PRL 110,081602(2013))
- Since HL theory is invariant under the anisotropic scaling, it is natural to ask: can we find Lifshitz solution in the framework of HL theory?

General action of the HL theory

- The general action of the HL theory without the projectability condition in (2+1)-dimensional spacetimes is given by(T. Sotiriou....PRD83,124021(2011))

$$S = \zeta^2 \int dt d^2x N \sqrt{g} \Big(\mathcal{L}_K - \mathcal{L}_V + \zeta^{-2} \mathcal{L}_M \Big), \qquad (2)$$

where $g = \det(g_{ij})$, $\zeta^2 = 1/(16\pi G)$, and

$$\begin{aligned} \mathcal{L}_K &= K_{ij}K^{ij} - \lambda K^2, \\ \mathcal{L}_V &= \gamma_0 \zeta^2 + \gamma_1 R + \frac{1}{\zeta^2}\left[\gamma_2 R^2 + \gamma_3 \Delta R + \gamma_4 R \left(a^i a_i\right)\right] \\ &+ \beta_0 a_i a^i + \frac{1}{\zeta^2}\left[\beta_1 \left(a_i a^i\right)^2 + \beta_2 \left(a^i{}_i\right)^2 \right. \\ &\left. + \beta_3 \left(a_i a^i\right) a^j{}_j + \beta_4 a^{ij} a_{ij}\right], \end{aligned} \qquad (3)$$

Metric

- spherically symmetric static vacuum spacetimes without the projectability condition in $(2+1)$-dimensional spacetimes

$$ds^2 = -r^{2z} f^2(r) dt^2 + \frac{g^2(r)}{r^2} dr^2 + r^2 d^2\theta, \qquad (4)$$

Equation of motion

- $$S_g = 2\pi\zeta^2 \int dtdr\,\mathcal{L}_g(r), \tag{5}$$

  where

  $$\begin{aligned} \mathcal{L}_g(r) &= N\sqrt(g)\,(\mathcal{L}_K - \mathcal{L}_V) \\ &= gf\,\mathcal{L}_V \end{aligned} \tag{6}$$

  To get the Equations of Motion:

  $$\sum_{j=0}^{3} (-1)^j \cdot \frac{d^j}{dr^j} \frac{\partial \mathcal{L}_g}{\partial \left(f^{(j)}(r)\right)} = 0 \tag{7}$$

  $$\sum_{j=0}^{3} (-1)^j \cdot \frac{d^j}{dr^j} \frac{\partial \mathcal{L}_g}{\partial \left(g^{(j)}(r)\right)} = 0 \tag{8}$$

Equation of motion

- $$\frac{r^2}{g^2} = \frac{r^2}{\ell^2} - \mathcal{D}, \qquad (9)$$

  where

$$\begin{aligned} \mathcal{D} &\equiv \frac{\sqrt{C_1}}{\gamma_1^2 (\beta_0 - 2\gamma_1)} \left(\frac{r}{g}\right)^{\frac{\beta_0}{(\beta_0 - \gamma_1)}} \\ &\times \left(\sqrt{\gamma_1^2 + 2\beta_0 g^2 \Lambda} - \gamma_1\right)^{\frac{\gamma_1}{(\beta_0 - \gamma_1)}} \\ &\times \left(\gamma_1^2 + (\beta_0 - \gamma_1)\sqrt{\gamma_1^2 + 2\beta_0 g^2 \Lambda}\right), \\ \ell^2 &\equiv -\frac{\gamma_1^2 (\beta_0 - 2\gamma_1)}{2\Lambda (\beta_0 - \gamma_1)^2}. \end{aligned} \qquad (10)$$

IR Limit

- Coupling constants $\gamma_{2,3}$ and $\beta_{1,\ldots,4}$ are negligible.

  If $\beta_0 = 0$, in this case, we find that

$$\mathcal{D} = \frac{\sqrt{C_1}}{2\gamma_1^2}, \quad \ell^2 = \frac{\gamma_1}{\Lambda}. \tag{11}$$

Then, we obtain that

$$\begin{aligned} g^2 &= \frac{r^2}{-M + \frac{r^2}{\ell^2}}, \\ \hat{f}^2 &= \frac{1}{r^4}\left(-M + \frac{r^2}{\ell^2}\right), \end{aligned} \tag{12}$$

where

$$\hat{f} \equiv r^{z-2} f. \tag{13}$$

The corresponding metric takes the familiar form,

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 d\phi^2, \tag{14}$$

Intermediate Region

- we consider the solutions

$$f(r) = f_0, \quad g(r) = g_0, \tag{15}$$

the spacetime should exhibit Lifshitz scaling

$$\begin{aligned} g_0^2 &= \frac{1}{\zeta^2\left[z\beta_0 - (z-1)\gamma_1\right]}\Big\{2z^2\beta_2 - 2z^3\beta_1 \\ &\quad -(z-1)\Big[4\gamma_2 - z\left[z\beta_3 + \beta_4 + (1+z)\gamma_3\right]\Big] \\ &\quad +z\left[2+(z-1)z\right]\gamma_4\Big\} > 0, \end{aligned} \tag{16}$$

UV Limit

- Coupling constants $\beta_0,\ \gamma_1$ and Λ are negligible.

  Consider the solutions,

$$\hat{f}(r) = \hat{f}_0 r^P , \quad g(r) = g_0 r^Q , \tag{17}$$

  if $\gamma_4 = \beta_{1,2,3,4} = 0$

$$\big(Q,\ P\big) = (1, 0) \tag{18}$$

- it is nothing but a AdS solution.

Conclusions and Future Work

- In IR Limit, we get Schwarzschild Anti-de Sitter solution.
- In Intermediate Region, we get Lifshitz solution.
- In UV Limit, we get several solutions(including AdS solution).
- Connect these three regions!

Thanks!