Post-Newtonian approximations in general covariant theory of Hořava-Lifshitz gravity

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Based on the work:

- K. Lin, S. Mukohyama, AW & T. Zhu, *Post-Newtonian approximations in the Hořava-Lifshitz gravity with extra U(1) symmetry*, arXiv:1310.6666.

- K. Lin & AW, *Static post-Newtonian limits in non-projectable Hořava-Lifshitz gravity with an extra U(1) symmetry*, Phys. Rev. D**87**, 084041 (2013) [arXiv:1212.6794].

- K. Lin, S. Mukohyama & AW, *Solar system tests and interpretation of gauge field and Newtonian prepotential in general covariant Hořava-Lifshitz gravity*, Phys. Rev. D**86**, 104024 (2012) [arXiv:1206.1338].

Outline

① Hořava-Lifshitz Theory of Quantum Gravity

- Minimal Hořava-Lifshitz Gravity
- Challenging Questions
- Different Models

② Universal Coupling with Matter

③ Post-Newtonian Approximations

- PPN Parameters with U(1) symmetry & $N = N(t)$
- PPN Parameters with U(1) symmetry & $N = N(t, \vec{x})$

④ Conclusions

I. Hořava-Lifshitz (HL) Theory of Quantum Gravity

A. Why Quantum Gravity?

- Quantum theory provides a general framework for ALL theories, except for gravity. The universal coupling of gravity to all forms of energy makes it plausible that gravity should be implemented in such a framework, too.

- At the singularities of the Big Bang and black holes, all the physics laws become invalid. Around these singular points, space-time curvatures become so high, Planck physics is highly involved. So, it is expected that quantum effects of gravity become important, and should be taken into account.

I. HL Gravity (Cont.)

Perspectives:

- Construct a theory of quantum gravity in the framework of quantum field theory ☺
- Take the metric as the fundamental variables ☺
- Lorentz symmetry appears only as an emergent symmetry at low energies, but can be fundamentally absent at high energies ☻

I. HL Gravity (Cont.)

- Breaking of the Lorentz symmetry is accomplished by the anisotropic scaling between time and space,

$$t \to b^{-z}t, \quad x^i \to b^{-1}x^i, \ (i = 1, 2, 3),$$

similar to the Lifshitz scalar field (E.M. Lifshitz, Zh. Eksp. Teor. Fiz. 11 (1941), 255; 269).

- Power-counting renormalizabe,

$$z \geq 3.$$

I. HL Gravity (Cont.)

Two Basic Assumptions:

- The space-times possess the foliation-preserving diffeomorphisms,

$$t \to f(t), \quad x^i \to \xi^i(t, x),$$

denoted by Diff$(M, \mathcal{F})$.

- The field equations are *second-order* of time derivatives, and *sixth-order* of spatial derivatives.

I. HL Gravity (Cont.)

- Once the general covariance is broken, the number of independent coupling constants get dramatically increased, $\mathcal{N} > 70$ [Zhu, Shu, Wu & AW, PRD85 (2012) 044053]. To reduce the number, Horava imposed two additional conditions.

- Projectability: The lapse function is a function of t only,

$$N = N(t).$$

This it is preserved by Diff$(M, \mathcal{F})$.

I. HL Gravity (Cont.)

- Detailed Balance:

$$\mathcal{L}_g = \mathcal{L}_K - \mathcal{L}_{(V,D)},$$
$$\mathcal{L}_K = \zeta^2 (K_{ij}K^{ij} - \lambda K^2),$$

where the potential $\mathcal{L}_{(V,D)}$ is given by a superpotential W_g defined only on the leaves of t = Constant,

$$W_g = \frac{1}{w^2}\int_\Sigma \left[\omega_3(\Gamma) + \frac{1}{\mu^2}(R - 2\Lambda)\right],$$

I. HL Gravity (Cont.)

$$\mathcal{L}_{(V,D)} = E_{ij}\mathcal{G}^{ijkl}E_{kl}, \quad E^{ij} = \frac{1}{\sqrt{g}}\frac{\delta W_g}{\delta g_{ij}},$$
$$\mathcal{G}^{ijkl} = \frac{1}{2}(g^{ik}g^{jl} + g^{il}g^{jk}) - \lambda g^{ij}g^{kl}, \tag{1}$$

λ: a coupling constant. Then,

$$\mathcal{N} = 5.$$

— Minimal HL Gravity.

I. HL Gravity (Cont.)

Major Challenges:

- The Minkowski space is not stable [P. Horava, PRD79 (2009) 084008], even without the detailed balance condition (but still with $N = N(t)$) [T. Sotiriou, M. Visser & S. Weinfurtner, JHEP 10 (2009) 033; AW, R. Maartens, PRD81 (2010) 024009].
- Newtonian limit does not exist, because the cosmological constant is strictly negative [H. Lü, J. Mei & C.N. Pope, PRL103 (2009) 091301],

$$G_N \propto \sqrt{-\Lambda}.$$

I. HL Gravity (Cont.)

- It becomes strongly coupled:
  - even with $N = N(t, x)$ (but without a_i terms) [C. Charmousis, G. Niz, A. Padilla, and P.M. Saffin, JHEP 08 (2009) 070; D. Blas, O. Pujolas & S. Sibiryakov, JHEP 10 (2009) 029];
  - even without the detailed balance condition but with $N = N(t)$ [D. Blas, O. Pujolas & S. Sibiryakov, JHEP 10 (2009) 029; K. Koyama & F. Arroja, JHEP 03 (2010) 061; AW & Q. Wu, PRD83 (2011) 044025].

I. HL Gravity (Cont.)

To overcome the above problems, various models have been proposed, including

- The healthy extension (D. Blas, O. Pujolas & S. Sibiryakov, PRL104 (2010) 181302; PLB688 (2010) 350; JHEP04 (2011) 018):

$$
\begin{aligned}
S_g &= \int dt d^3x N\sqrt{g}\mathcal{L}_g, \\
\mathcal{L}_g &= \underbrace{a_1 K_{ij}K^{ij} + a_2 K^2 + a_3 R^3 + a_4 (a_i a^i)^3 + ...}_{(\mathcal{N} \simeq 70)}
\end{aligned}
$$

I. HL Gravity (Cont.)

- All the problems mentioned above are resolved.
- Consistent with solar system tests [D. Blas and H. Sanctuary, PRD84 (2011) 064004; B. Audren, D. Blas, J. Lesgourgues, and S. Sibiryakov, arXiv:1305.0009].
- Consistent with cosmology [T. Kobayashi, Y. Urakawa and M. Yamaguchi, JCAP 1004, 025 (2010); R.-G. Cai, B. Hu, H.-B. Zhang, PRD83 (2011) 084009; E.G.M. Ferreira, R. Brandenberger, PRD86 (2013) 043514].

I. HL Gravity (Cont.)

- The second model [P. Horava & C.M. Melby-Thompson, PRD82 (2010) 064027; AW, Y. Wu, PRD83 (2011) 044031; A.M. da Silva, CQG28 (2011) 055011; Y.-Q. Huang, AW, PRD83 (2011) 104012]:
  - An extra U(1) symmetry

$$U(1) \ltimes \text{Diff}(M,\ \mathcal{F}),$$

    realized by introducing a U(1) gauge field A and a Newtonian prepotential φ.
  - The number of independent coupling constants,

$$\mathcal{N} = 13.$$

I. HL Gravity (Cont.)

- With this extra symmetry, spin-0 gravitons are absent and the theory has the same degree of freedom as GR.

- All the problems mentioned above, including instability, ghosts, and strong coupling, are resolved.

- Consistent with cosmological observations [Y.-Q. Huang, AW, Q. Wu, JCAP10 (2012) 010; Y.-Q. Huang, AW, PRD86 (2012) 103523; Y.-Q. Huang, AW, R. Yousefi, T. Zhu, PRD88 (2013) 023523].

I. HL Gravity (Cont.)

- The third model [T. Zhu, Q. Wu, AW, F.-W. Shu, PRD84 (2011) 101502 (R); T. Zhu, F.-W. Shu, Q. Wu & AW, PRD (2012); K. Lin, S. Mukohyama, AW, T. Zhu, arXiv:1310.6666]:
  - Non-projectable $N = N(t, x)$
  - Enlarged symmetry,

$$U(1) \ltimes \text{Diff}(M,\ \mathcal{F}).$$

  - Detailed balance condition softly broken, so that

$$\mathcal{N} = 15.$$

I. HL Gravity (Cont.)

- All the problems mentioned above, including instability, ghosts, and strong coupling, are resolved.

- Consistent with cosmology [T. Zhu, Y.-Q. Huang, AW, JHEP01 (2013) 138].

- In the following, we shall study the post-Newtonian approximations in the second and third models and work out the PPN parameters in terms of the coupling constants of the models.

II. Universal Coupling with Matter

- The equivalence principle requires that coupling to matter should be universal.

- In the ADM decompositions, (N, N^i, g_{ij}), the shift vector N^i is not U(1) invariant, but

$$\tilde{N}^i \equiv N^i + N g^{ij} \nabla_j \varphi.$$

- We propose a universal coupling of matter with the HL gravity via

$$\left(\tilde{N}, \tilde{N}^i, \tilde{g}_{ij}\right),$$

II. Universal Coupling with Matter (Cont.)

where

$$\tilde{N} \equiv F(\sigma)N, \quad \tilde{g}_{ij} \equiv \Omega^2(\sigma)g_{ij}, \quad \tilde{g}^{ij} \equiv \Omega^{-2}(\sigma)g^{ij}, \tag{2}$$

σ: a scalar, wich is invariant under both U(1) and Diff($M, \mathcal{F}$),

$$\sigma \equiv \frac{A - \mathcal{A}}{N},$$
$$\mathcal{A} \equiv -\dot{\varphi} + N^i\nabla_i\varphi + \frac{1}{2}N\left(\nabla^i\varphi\right)\left(\nabla_i\varphi\right).$$

— Note that in general $F(\sigma)$ and $\Omega(\sigma)$ will depend on matter species, but universality may emerge at low energies.

II. Universal Coupling with Matter (Cont.)

- We assume that such a mechanism exists, and matter propagates in the 4D space-time,

$$
\begin{aligned}
ds^2 &= \gamma_{\mu\nu} dx^\mu dx^\nu \\
&= -\tilde{N}^2 dt^2 + \tilde{g}_{ij}\left(dx^i + \tilde{N}^i dt\right)\left(dx^j + \tilde{N}^j dt\right). \quad (3)
\end{aligned}
$$

- The matter is described by

$$S_M = \int d^4x \sqrt{-\gamma}\mathcal{L}_M(\gamma_{\mu\nu}; \psi_n), \quad (4)$$

ψ_n denotes collectively the matter fields, and

$$T^{\mu\nu} = \frac{1}{\sqrt{-\gamma}} \frac{\delta\left(\sqrt{-\gamma}\mathcal{L}_M\right)}{\delta\gamma_{\mu\nu}}. \quad (5)$$

III. Post-Newtonian Approximations

- We assume the 4D metric takes the form,

$$\gamma_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

$\eta_{\mu\nu} = \text{diag.}\,(-1, 1, 1, 1)$, and

$$\begin{aligned} h_{00} &\sim \mathcal{O}(2) + \mathcal{O}(4), \\ h_{0j} &\sim \mathcal{O}(3), \\ h_{ij} &\sim \mathcal{O}(2) + \mathcal{O}(4), \end{aligned}$$

$\mathcal{O}(n) \equiv \mathcal{O}\left(v^n\right)$, v: 3-velocity of the fluid.

III. Post-Newtonian Approximations (Cont.)

- Using the gauge freedom, it can be shown that $\gamma_{\mu\nu}$ can be cost in the form,

$$
\begin{aligned}
\gamma_{00} &= -1 + 2U - 2\beta U^2 - 2\xi \Phi_W \\
&\quad + (2 + 2\gamma + \alpha_3 + \zeta_1 - 2\xi)\,\Phi_1 \\
&\quad +2\,(1 + 3\gamma - 2\beta + \zeta_2 + \xi)\,\Phi_2 \\
&\quad +2\,(1 + \zeta_3)\,\Phi_3 + 2\,(3\gamma + 3\zeta_4 - 2\xi)\,\Phi_4 \\
&\quad - (\zeta_1 - 2\xi)\,\mathfrak{A} + \zeta_{\rm B}\mathfrak{B}, \\
\gamma_{0i} &= -\frac{1}{2}(3 + 4\gamma + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)\,V_i \\
&\quad -\frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)\,W_i, \\
\gamma_{ij} &= (1 + 2\gamma U)\,\delta_{ij} + \mathbf{h}_{\rm ij}^{(4)},
\end{aligned}
\tag{6}
$$

$\beta, \gamma, \xi, \zeta_i, \alpha_j$ and ζ_B: the PPM parameters to characterize the post-Newtonian limits in the HL theory, and

III. Post-Newtonian Approximations (Cont.)

$$U \equiv \int \frac{\rho(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|} d^3x', \quad \chi \equiv -\int \rho(\mathbf{x}',t)|\mathbf{x}-\mathbf{x}'| d^3x', \quad V_j \equiv \int \frac{\rho(\mathbf{x}',t)v'_j}{|\mathbf{x}-\mathbf{x}'|} d^3x',$$

$$W_j \equiv \int \frac{\rho(\mathbf{x}',t)\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')(x-x')_j}{|\mathbf{x}-\mathbf{x}'|^3} d^3x', \quad U_{ij} \equiv \int \frac{\rho(\mathbf{x}',t)(x-x')_i(x-x')_j}{|\mathbf{x}-\mathbf{x}'|^3} d^3x',$$

$$\Phi_W \equiv \int \rho'\rho'' \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3} \times \left(\frac{\mathbf{x}'-\mathbf{x}''}{|\mathbf{x}-\mathbf{x}''|} - \frac{\mathbf{x}-\mathbf{x}''}{|\mathbf{x}'-\mathbf{x}''|} \right) d^3x' d^3x'',$$

$$\Phi_1 \equiv \int \frac{\rho' v'^2}{|\mathbf{x}-\mathbf{x}'|} d^3x', \quad \Phi_2 \equiv \int \frac{\rho' U'}{|\mathbf{x}-\mathbf{x}'|} d^3x', \quad \Phi_3 \equiv \int \frac{\rho' \Pi'}{|\mathbf{x}-\mathbf{x}'|} d^3x',$$

$$\Phi_4 \equiv \int \frac{p'}{|\mathbf{x}-\mathbf{x}'|} d^3x', \quad \mathfrak{A} \equiv \int \frac{\rho'[\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')]^2}{|\mathbf{x}-\mathbf{x}'|^3} d^3x',$$

$$\mathfrak{B} \equiv \int \frac{\rho'}{|\mathbf{x}-\mathbf{x}'|}(\mathbf{x}-\mathbf{x}') \cdot \frac{d\mathbf{v}'}{dt} d^3x'.$$

III. Post-Newtonian Approximations (Cont.)

— The projectable case $N = N(t)$: The free parameters of the model are

$$(\lambda,\ g_1,\ \varkappa,\ a_1,\ a_2)\,,$$

appearing in the expressions,

$$\begin{aligned}
\mathcal{L}_K &= K_{ij}K^{ij} - \lambda K^2, \quad \mathcal{L}_V^{IR} = \mathbf{g_1} R, \\
F &= 1 - \mathbf{a_1}\sigma, \quad \Omega = 1 - \mathbf{a_2}\sigma, \\
\varkappa &\equiv G/G_N.
\end{aligned}$$

$G \equiv 1/(8\pi M_*^2)$, G_N: the Newtonian constant.

III. Post-Newtonian Approximations (Cont.)

In terms of these coupling constants, the PPN parameters are given by,

$$
\begin{aligned}
\gamma &= \varkappa a_1 - \frac{a_2}{a_1}, \quad \beta = \frac{1+\varkappa a_1}{2}, \\
\alpha_1 &= 4\left[(\varkappa-1) - (a_1-1)\varkappa + \frac{a_2}{a_1}\right], \\
\alpha_2 &= (\varkappa-1) + \left(\frac{1-3\lambda}{1-\lambda}\right)(a_1-1)^2\varkappa, \\
\zeta_1 &= -\zeta_B = \left(\frac{1-3\lambda}{1-\lambda}\right)a_1(a_1-1)\varkappa, \\
\alpha_3 &= \xi = \zeta_2 = \zeta_3 = \zeta_4 = 0.
\end{aligned}
$$

III. Post-Newtonian Approximations (Cont.)

For

$$|a_1 - 1| < 10^{-5}, \quad |a_2| < 10^{-5}, \quad |\varkappa - 1| < 10^{-5},$$
$$(\varkappa - 1), \quad \left|\frac{(a_1 - 1)^2}{1 - \lambda}\right| < 10^{-7},$$

the constraints from all the solar system tests, we find that

$$\begin{aligned}
\gamma &= 1 + (2.1 \pm 2.3) \times 10^{-5}, \\
\beta &= 1 + (-4.1 \pm 7.8) \times 10^{-5}, \\
\alpha_1 &< 10^{-4}, \quad \alpha_2 < 4 \times 10^{-7}, \quad \alpha_3 < 4 \times 10^{-20}, \\
\xi &< 10^{-3}, \quad \Gamma < 1.5 \times 10^{-3}, \quad \zeta_1 < 2 \times 10^{-2}, \\
\zeta_2 &< 4 \times 10^{-5}, \quad \zeta_3 < 10^{-8}, \quad \zeta_4 < 6 \times 10^{-3}, \\
\Gamma &\equiv 4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2,
\end{aligned}$$

are satisfied.

III. Post-Newtonian Approximations (Cont.)

In particular, choosing

$$a_1 = 1, \quad a_2 = 0, \quad g_1 = -1,$$

we find that

$$\gamma = \beta = 1, \quad \alpha_1 = \alpha_2 = \alpha_3 = \xi = 0,$$
$$\zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = \zeta_B = 0,$$

which are precisely the results obtained in GR.

III. Post-Newtonian Approximations (Cont.)

— The nonprojectable case $N = N(t, \vec{x})$: The free parameters of the model are

$$(\lambda,\ \gamma_1,\ \beta_0,\ \varkappa, \sigma_1, \sigma_2,\ a_1,\ a_2)\,,$$

appearing in the expressions,

$$\begin{aligned}
\mathcal{L}_K &= K_{ij}K^{ij} - \lambda K^2, \\
\mathcal{L}_V^{IR} &= \gamma_1 R - \beta_0 a_i a^i - \sigma(\sigma_1 a^i a_i + \sigma_2 \nabla_i a^i), \\
F &= 1 - \mathbf{a}_1\sigma, \quad \Omega = 1 - \mathbf{a}_2\sigma, \\
\varkappa &\equiv G/G_N.
\end{aligned}$$

$G \equiv 1/(8\pi M_*^2)$, G_N: the Newtonian constant.

III. Post-Newtonian Approximations (Cont.)

— The PPN parameters were found explicitly in terms of the coupling constants [arXiv:1310.6666], which will not be given here, but simply note that by properly choosing them, the solar system tests can be easily satisfied.

— In particular, the PPN parameters take exactly their GR values in the cases:

$$(i)\ a_1 = \varkappa = 1, \quad \sigma_2 = 0,$$

$$(ii)\ \sigma_2 = 4(1 - a_1), \quad \beta_0 = -2(\gamma_1 + 1).$$

IV. Conclusions

- The models with U(1) symmetry for both of the projectable and nonprojectbale cases satisfy all the solar system tests, carried out so far.
- Note that only few theories of gravity pass all of these tests, including GR, Brans-Dicke (scale-tensor) theory, and Einstein-aether (vector-tensor) theory.
- But, they are all effective low-energy theories, and are not (perturbatively) renormalizable.

IV. Conclusions (Cont.)

- On the other hand, by construction, the HL theory is power-accounting renormalizable, and recent studies of quantization in (2+1)D spacetimes indicated that it might be indeed renormalizble [C. Anderson, S. Carlip, J.H. Cooperman, P. Horava, R. Kommu, P.R. Zulkowski, Phys. Rev. D85 (2012) 044027].

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