Gravitational Collapse in Hořava-Lifshitz Theory *

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Outline of the Talk

Gravitational Collapse in General Relativity

Hořava-Lifshitz Theory with Projectability

Gravitational Collapse in Hořava-Lifshitz Theory

Summary of Results

Gravitational Collapse in General Relativity

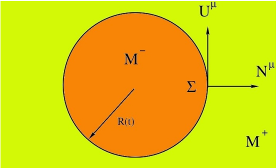
At the end of the life of a star

- White dwarf $\longmapsto$ electron degeneracy pressure due to the Pauli exclusion principle (Chandrasekhar limit)
- Neutron star $\longmapsto$ neutron degeneracy pressure mediated by the strong force (Tolman-Oppenheimer-Volkoff limit)
- Black holes $\longmapsto$ Nothing can stop it from happening! (At least within GR).

In a theory of quantum gravity, it is expected that the formation of singularities in a gravitational collapse is prevented by short-distance quantum effects. It is not like the singularity inside the horizon is replaced with an ultra dense object.

Gravitational Collapse in General Relativity

- Within the framework of general relativity, the dynamical collapse of a homogeneous spherical dust cloud under its own gravity was first considered by Datt (1938) and Oppenheimer and Snyder (1939). It was shown that it always leads to the formation of singularities.



- The spacetime is divided into two regions, the internal M^- and external M^+, where $M^- = \{x^\mu : r < \mathscr{R}(t)\}$, and $M^+ = \{x^\mu : r > \mathscr{R}(t)\}$. The surface $r = \mathscr{R}(t)$ is denoted by Σ.

Gravitational Collapse in General Relativity

In general relativity, there are two common approaches for such studies.

- **Israel's junction conditions (1966)** - relies on the Gauss and Codazzi equations together with the Lanczos equations. An advantage of this method is that it can be applied to the case where the coordinate systems inside and outside a collapsing body are different.
- **Taubs junction conditions (1980)** - relies on distribution theory. In this approach, although the coordinate systems inside and outside the collapsing stars are taken to be the same, the null-hypersurface case can be easily included. Taub's approach was widely used to study colliding gravitational waves and other related issues in general relativity.

Hořava-Lifshitz Theory with Projectability

The HL theory with the projectability condition $N = N(t)$, an arbitrary coupling constant λ and the enlarged symmetry. The fundamental variables are $(N,\ N^i,\ g_{ij},\ A,\ \varphi)$, which transform under $\mathrm{Diff}(M,\ \mathscr{F})$ as

$$\begin{aligned}
\delta N &= \zeta^k \nabla_k N + \dot{N} f + N \dot{f}, \\
\delta N_i &= N_k \nabla_i \zeta^k + \zeta^k \nabla_k N_i + g_{ik} \dot{\zeta}^k + \dot{N}_i f + N_i \dot{f}, \\
\delta g_{ij} &= \nabla_i \zeta_j + \nabla_j \zeta_i + f \dot{g}_{ij}, \\
\delta A &= \zeta^i \partial_i A + \dot{f} A + f \dot{A}, \\
\delta \varphi &= f \dot{\varphi} + \zeta^i \partial_i \varphi, \qquad (1)
\end{aligned}$$

and under the local U(1) symmetry as

$$\begin{aligned}
\delta_\alpha A &= \dot{\alpha} - N^i \nabla_i \alpha, \quad \delta_\alpha \varphi = -\alpha, \\
\delta_\alpha N_i &= N \nabla_i \alpha, \quad \delta_\alpha g_{ij} = 0 = \delta_\alpha N, \qquad (2)
\end{aligned}$$

where α is the generator of the U(1) symmetry.

Hořava-Lifshitz Theory with Projectability

The total action is given by

$$S = \zeta^2 \int dt d^3x N\sqrt{g}\Big(\mathscr{L}_K + \mathscr{L}_\varphi + \mathscr{L}_A + \mathscr{L}_\lambda - \mathscr{L}_V + \zeta^{-2}\mathscr{L}_M\Big), \qquad (3)$$

where $g = \det g_{ij}$, and

$$\begin{aligned}
\mathscr{L}_K &= K_{ij}K^{ij} - \lambda K^2, \\
\mathscr{L}_\varphi &= \varphi \mathscr{G}^{ij}\Big(2K_{ij} + \nabla_i \nabla_j \varphi\Big), \\
\mathscr{L}_A &= \frac{A}{N}\Big(2\Lambda_g - R\Big), \\
\mathscr{L}_\lambda &= \big(1-\lambda\big)\Big[\big(\nabla^2\varphi\big)^2 + 2K\nabla^2\varphi\Big].
\end{aligned} \qquad (4)$$

Here the coupling constant Λ_g, which acts like a 3-dimensional cosmological constant, has the dimension of (length)$^{-2}$. The Ricci and Riemann terms all refer to the three-metric g_{ij}.

Hořava-Lifshitz Theory with Projectability

K_{ij} is the extrinsic curvature, and $\mathcal{G}_{ij}$ is the 3-dimensional "generalized" Einstein tensor defined by

$$\begin{aligned} K_{ij} &= \frac{1}{2N}\left(-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i\right), \\ \mathcal{G}_{ij} &= R_{ij} - \frac{1}{2} g_{ij} R + \Lambda_g g_{ij}. \end{aligned} \tag{5}$$

$\mathcal{L}_V$ denotes the potential part of the action given by

$$\begin{aligned} \mathcal{L}_V &= \zeta^2 g_0 + g_1 R + \frac{1}{\zeta^2}\left(g_2 R^2 + g_3 R_{ij} R^{ij}\right) \\ &+ \frac{1}{\zeta^4}\left(g_4 R^3 + g_5 R\, R_{ij} R^{ij} + g_6 R^i_j R^j_k R^k_i\right) \\ &+ \frac{1}{\zeta^4}\left[g_7 R \nabla^2 R + g_8 \left(\nabla_i R_{jk}\right)\left(\nabla^i R^{jk}\right)\right], \end{aligned} \tag{6}$$

which preserves the parity, where the coupling constants $g_s\,(s = 0, 1, 2, \ldots 8)$ are all dimensionless. $\mathcal{L}_M$ is the matter Lagrangian density.

Gravitational Collapse in Hořava-Lifshitz Theory

We study gravitational collapse of a spherical star with a finite radius in the HL theory with the projectability condition, an arbitrary coupling constant λ, and the extra U(1) symmetry.

For the study of a collapsing star with a finite radius in the HL theory, we follow Taub's approach, because

- We do not know how to generalize the Lanczos equations to the HL theory.
- It turns out to be more convenient when dealing with higher-order derivatives.
- In contrast to the case of general relativity, the foliation structure of the HL theory implies that the coordinate systems inside and outside of the collapsing star are unique.

Gravitational Collapse in Hořava-Lifshitz Theory

The ADM variables for spherically symmetric spacetimes with the projectability condition take the forms

$$\begin{aligned} N &= 1, \\ N^i &= \delta^i_r e^{\mu(r,t)-\nu(r,t)}, \\ g_{ij}dx^i dx^j &= e^{2\nu(r,t)}dr^2 + r^2 d\Omega^2, \end{aligned} \tag{7}$$

in the spherical coordinates $x^i = (r, \theta, \phi)$, where $d\Omega^2 \equiv d\theta^2 + \sin^2\theta\, d\phi^2$. The diagonal case $N^i = 0$ corresponds to $\mu(t,r) = -\infty$. On the other hand, using the $U(1)$ gauge freedom, without loss of generality, we set

$$\varphi = 0, \tag{8}$$

which uniquely fixes the gauge.

Gravitational Collapse in Hořava-Lifshitz Theory

We consider the gravitational collapse of a spherical cloud consisting of a homogeneous and isotropic perfect fluid, described by the FLRW universe,

$$ds^2 = -d\bar{t}^2 + a^2(\bar{t})\left(\frac{d\bar{r}^2}{1-k\bar{r}^2} + \bar{r}^2 d^2\Omega\right),$$

where $k = 0, \pm 1$. Letting $r = a(\bar{t})\bar{r},\ t = \bar{t}$, the corresponding ADM variables take the form (7) with $N^- = 1$, and

$$\begin{aligned} \nu^-(t,r) &= -\frac{1}{2}\ln\left(1 - k\frac{r^2}{a^2(t)}\right), \\ \mu^-(t,r) &= \ln\left(\frac{-\dot{a}(t)r}{\sqrt{a^2(t)-kr^2}}\right), \end{aligned} \tag{9}$$

where $\dot{a} \leq 0$ for a collapsing cloud.

Gravitational Collapse in Hořava-Lifshitz Theory

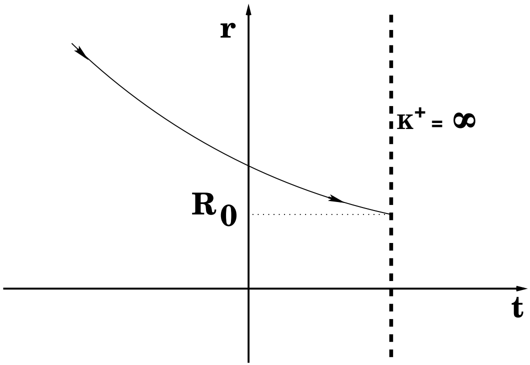


Figure: The evolution of the surface of the collapsing star for $\lambda = 1$ and $\Lambda > 0$, given by $\mathscr{R}(t) = \mathscr{R}_0 \cosh^{\frac{2}{3}}\left(\frac{\sqrt{3\Lambda}}{2}(t_0 - t)\right)$. At the moment $t = t_0$, the star collapses to its minimal radius $\mathscr{R}(t_0) = \mathscr{R}_0$, at which the extrinsic curvature K^+ becomes unbounded, while the four-dimensional Ricci scalar remains finite.

Gravitational Collapse in Hořava-Lifshitz Theory

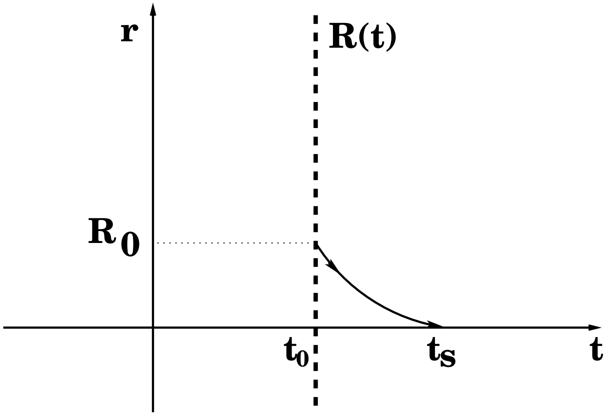


Figure: The evolution of the surface of the collapsing star for $\lambda = 1$ and $\Lambda < 0$, given by $\mathscr{R}(t) = \mathscr{R}_0 \cos^{\frac{2}{3}}\left(\frac{\sqrt{3|\Lambda|}}{2}(t - t_0)\right)$. The star starts to collapse at a time $t = t_i \geq t_0$. At the later time $t = t_s$, at which $\mathscr{R}(t_s) = 0$, the star collapses and a central singularity is formed.

Gravitational Collapse in Hořava-Lifshitz Theory

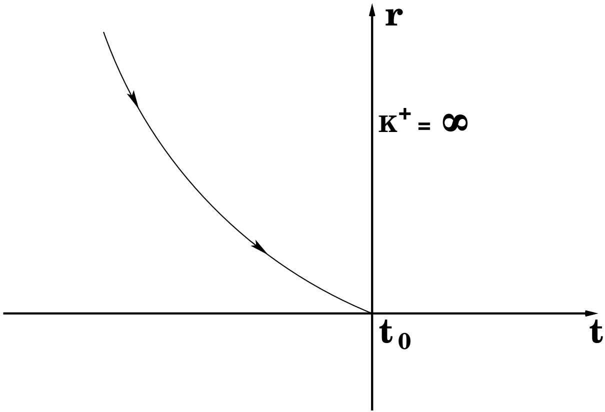


Figure: The evolution of the surface of the collapsing star for $\lambda = 1$ and $\Lambda = 0$, given by $\mathscr{R}(t) = \mathscr{R}_0(t_0 - t)^{\frac{2}{3}}$. At the moment $t = t_i \leq t_0$, the star starts to collapse until the moment $t = t_0$, at which we have $\mathscr{R}(t_0) = 0$, whereby a central singularity is formed.

Summary of Results

- We have studied gravitational collapse of a spherical cloud of fluid with a finite radius in the framework of the nonrelativistic general covariant theory of HL gravity with the projectability condition and an arbitrary coupling constant λ.
- Using distribution theory, we have developed the general junction conditions for such a collapsing spherical body, under the minimal requirement that *the junctions should be mathematically meaningful in the sense of generalized functions.*
- As one of the simplest applications, we have studied a collapsing star that is made of a homogeneous and isotropic perfect fluid, while the external region is described by a stationary spacetime.

Summary of Results

- For the case of a homogeneous and isotropic dust fluid (a perfect fluid with vanishing pressure), we have found explicitly the space-time outside of the collapsing sphere. In particular, in the case $\lambda = 1$, the external spacetimes are described by the Schwarzschild (anti-) de Sitter solutions, written in Painlevé-Gullstrand coordinates.
- It is remarkable that the collapse of a homogeneous and isotropic dust to a Schwarzschild black hole, studied by Oppenheimer and Snyder in general relativity is a particular case.
- In the case $\lambda \neq 1$, the space-time describing the outside of the homogeneous and isotropic dust fluid is not asymptotically flat. Therefore, to obtain an asymptotically flat space-time outside of a collapsing dust fluid, it must not be homogeneous and/or isotropic.

Summary of Results

However, there are fundamental differences.

- First, in general relativity a thin shell does not necessarily appear on the surface of the collapsing sphere, while in the current case we have shown that such a thin shell must exist.
- Second, in general relativity, because of the local conservation of energy of the collapsing body, the energy density of the dust fluid is inversely proportional to the cube of the radius of the fluid, while in the current case it remains a constant, as the conservation law is global and the energy of the collapsing star is not necessarily conserved locally.

Future Directions

Next, we would like to

- Study gravitational collapse of more general fluids, such as perfect fluids with different equations of state, or anisotropic fluids with or without heat flows.
- Generalize the junction conditions of a collapsing star to other versions of Hořava-Lifshitz gravity.
- Study the collapse of more realistic stars i.e., Kerr type exterior axially spherically symmetric rotating vacuum spacetime.

Thanks!

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Thank you for your attention!