# Dynamics of Bianchi Type Scalar-Tensor Cosmology

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**Expanding Universe**

Cosmic expansion is a fascinating primal issue that was confirmed by the scientists in the early 1980's while trying to explore the mysteries related with the galaxies structures.

Current Evidences: Supernova (Ia), Wilkinson Microwave Anisotropy Probe (WMAP), Sloan Digital Sky Survey (SDSS), galactic cluster emission of X-rays, large scale structure, weak lensing etc.

**Existence of Dark Energy**

Baryonic+Dark matter $\nRightarrow$ Total energy density of the universe $\Rightarrow$ "missing energy".

This "missing energy" was termed as dark energy (DE) (exotic component of the universe distribution with large negative pressure). Data obtained from Supernova (Ia)indicates

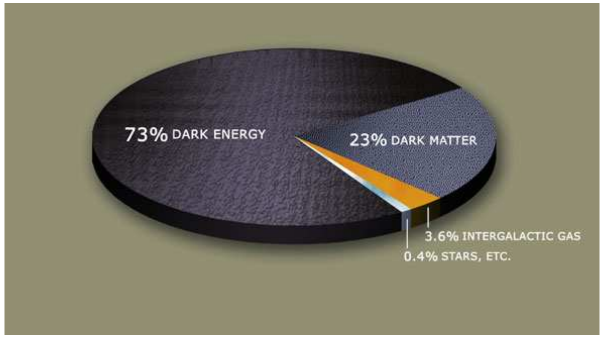


**Figure 1:** Present matter density distribution of cosmos.

**Dark Energy Proposals**

In order to realize the DE mystery, Einstein's gravity is modified by incorporating DE in the following two ways:

Either some exotic matter components are added in the energy-momentum tensor part of the action,
or the whole gravitational action is modified by including some DE source terms. These are:

I. Modified Matter Models;
II. Modified Gravity Models.

Eienstein's General Theory of Relativity (1916)

Action: $S = \int \sqrt{-g} R d^4x + L_m$

Field Equations: $R_{\mu\nu} - 1/2 R g_{\mu\nu} = \kappa T_{\mu\nu}$

## I. Modified Matter Models

- Cosmological constant $(\Lambda)$ is the simplest recognition of DE described by a homogeneous energy density with constant EoS parameter -1.

- Chaplygin gas and its modified versions: Chaplygin gas incorporates both DE as well as DM in a combined way as an exotic fluid background distribution and is described by EoS

  $$P = A/\rho.$$

  Generalized chaplygin gas and modified chaplygin gas have the EoS

  $$p = -\frac{A}{\rho^{\alpha}}, \quad p = A\rho - \frac{B}{\rho^{\alpha}},$$

  where $A,\ B$ both are constants while $\alpha \in [0, 1]$.

- Viscous Forces: Bulk viscosity promotes negative energy field in the fluid and hence can play the role of DE. It can be introduced as a constant or a variable force in the matter sector of the Lagrangian density.

- Scalar field dark energy models like
  - Quintessence: $L = R/16\pi G + X - V(\phi); X = \dot{\phi}^2/2$.
  - Phantom: $\rho_P = -\frac{\dot{\phi}^2}{2} + V(\phi),\ p_P = -\frac{\dot{\phi}^2}{2} - V(\phi)$.
  - k-essence: $L = R/16\pi G + p(\phi, X)$.
  - Tachyon Fields: $\rho_T = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}},\ p_T = -V(\phi)\sqrt{1-\dot{\phi}^2}$.

## II. Modified Gravity Models

- $f(R)$ theory of gravity (Metric f(R) gravity, Palatini $f(R)$ gravity, etc.)
- $f(T)$ theory of gravity, $T$ is the torsion.
- $f(R,T)$ theory of gravity etc.
- Guass-Bonnet theory (additional Guass-Bonnet invariant term)
- Horava-Lifshitz gravity
- Higher Dimensional theories,
- Scalar-tensor theories.

**Scalar-Tensor Gravity**

Scalar-tensor theories of gravity have many significant applications in cosmology (Guth 1981; Bertolami and Martins 2000; Banerjee and Pavon 2001; Sahoo and Singh 2003; Chakraborty and Debnath 2009). In scalar tensor theories, the gravity effects are mediated by both the metric tensor as well as the scalar field. A scalar field is a function that maps each point of the spacetime manifold to a scalar quantity (spin-0 particles).

**Action for Scalar-Tensor Gravity:**

$$S = \int d^4x\sqrt{-g}[U(\phi)R + \omega(\phi)g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi + V(\phi) + L_m]; \quad \alpha, \beta = 0, 1, 2, 3, \quad (1)$$

$U(\phi) > 0$ ensuring that the gravitons carry positive energy,

$R$: Ricci scalar,

$U(\phi)R$: coupling of scalar-field with geometry,

$\omega(\phi)$: coupling function of Brans-Dicke scalar field,

$V(\phi)$: scalar field potential,

$L_m$: matter sector of the Lagrangian density.

## Field Equations

The corresponding field equations are obtained by varying this action using principle of least action with respect to the metric tensor and the extra ingredient scalar field as follows

$$\begin{aligned} U(\phi)G_{\mu\nu} &= T_{\mu\nu} + \omega(\phi)[\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla^\alpha\nabla_\alpha\phi] \\ &+ [\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box]F(\phi) - \frac{1}{2}g_{\mu\nu}V(\phi), \end{aligned} \tag{2}$$

$\Box = \Delta^\mu\Delta_\mu$ is the de'Alembertian operator,

$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}$ is the energy-momentum tensor.

Scalar wave/Klein-Gordan equation providing the evolution of scalar field is

$$2\omega(\phi)\Box\phi+2\nabla_{\gamma}\omega(\phi)\nabla^{\gamma}\phi-\frac{d\omega}{d\phi}\nabla_{\gamma}\phi\nabla^{\gamma}\phi$$
$$+R\frac{dU}{d\phi}-\frac{dV}{d\phi}=0. \quad (3)$$

$T=g^{\mu\nu}T_{\mu\nu}$ represents the trace of energy-momentum tensor.

For different forms of the coupling functions $U$ and $\omega$, we can have different types of scalar-tensor theories.

# Reconstruction Technique

Usually, the modifications of gravity include some unknown functions that cannot be inferred from the fundamental theory. This leads to a complicated issue raising the question that how to choose these functions with a strong cosmological motivation. In this context, the reconstruction technique is considered as one of the possible solutions that can yield a physically significant choice of these functions. The scheme of reconstruction has a long history exploring different cosmological models of DE especially, in the case of modified gravity theories.

Kamenshchik et al. (PLB, 2011) applied the process of reconstruction to flat FRW model in the framework of induced gravity.

The reconstruction technique is an interesting way to determine the nature of scalar field potentials by taking the Hubble parameter as a function of scalar field.

Kamenshchik et al. (PRD, 2013) used this approach to reconstruct the field potential for FRW model in a non-minimally coupled scalar-tensor gravity and explored its nature for different cases describing the cosmic evolution.

Here we apply this technique to LRS Bianchi I universe model and explore the nature of scalar field potentials.

LRS Bianchi type I (BI) universe as an extension of flat FRW model with expansion factors $A$ and $B$ exhibiting the anisotropic and spatially homogeneous characteristics is given by the metric

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)(dy^2 + dz^2). \tag{4}$$

Some parameters yielding the cosmological aspects of the BI universe model are given by

$$\begin{aligned} a(t) &= (AB^2)^{1/3}, \quad V = a^3(t) = AB^2, \\ H(t) &= \frac{1}{3}(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}), \\ H_1 &= \frac{\dot{A}}{A}, \quad H_2 = H_3 = \frac{\dot{B}}{B}, \end{aligned}$$

where $a,\ V,\ H,\ H_1$ and $H_2$ stand for the average scale factor, universe volume, mean Hubble parameter and directional Hubble parameters in directions $x$ and $y,\ z$ , respectively. The expansion and shear scalar for BI universe model turn out to be

$$\Theta = u^a_{;a} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}, \quad \sigma = \frac{1}{\sqrt{3}}(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}).$$

We consider a physical relation between the expansion factors $A = B^m;\ m \neq 0,\ 1.$ This condition is an outcome of the fact that the normal congruence to homogeneous expansion for a spatially homogeneous model corresponds to the proportionality of the shear scalar $\sigma$ and the expansion scalar $\Theta$, i.e., the ratio $\frac{\sigma}{\Theta}$ is constant.

After substituting the Ricci scalar for BI universe model and taking $\omega=\omega_0$, where $\omega_0$ is an arbitrary non-zero constant, the partial integration of the action (1) yields the point-like Lagrangian density given by

$$\begin{aligned}\mathcal{L}(B,\phi,\dot{B},\dot{\phi}) &= 2(m+2)B^{(m+1)}\frac{dU}{d\phi}\dot{B}\dot{\phi} \\ &+ 2B^{m}\dot{B}^{2}(1+2m)U(\phi)-\frac{\omega_0}{2}B^{m+2}\dot{\phi}^{2} \\ &+ V(\phi)B^{m+2}, \qquad (5)\end{aligned}$$

The field equations obtained by the variation of this Lagrangian density are

$$2(1+2m)U(\phi)H_2^2+2(m+2)H_2\dot{U}=\frac{\omega_0}{2}\dot{\phi}^2+V(\phi), \quad (6)$$

$$-2(m+2)\ddot{U}-4(1+2m)\dot{U}H_2-4(1+2m)U(\phi)\frac{\ddot{B}}{B}$$
$$-2(1+2m)mU(\phi)H_2^2=(m+2)[\frac{\omega_0}{2}\dot{\phi}^2-V(\phi)], \quad (7)$$

$$\omega_0\ddot{\phi}+\omega_0(m+2)\dot{\phi}H_2+\frac{dV}{d\phi}=[2(m+2)\frac{\ddot{B}}{B}$$
$$+2(m^2+m+1)H_2^2]\frac{dU}{d\phi}, \quad (8)$$

where $\frac{dU}{d\phi}=\frac{\dot{U}}{\dot{\phi}}$, $\frac{d^2U}{d\phi^2}=\frac{\ddot{U}}{\dot{\phi}^2}-\frac{\dot{U}\ddot{\phi}}{\dot{\phi}^3}$. $m=1$ reduces to FRW universe.

Using Eqs.(6) and (7), we have

$$2(m+2)\ddot{U}+4(1+2m)U\dot{H}_2+\omega_0(m+2)\dot{\phi}^2$$
$$-2(m^2+2)H_2\dot{U}=0. \quad (9)$$

Assume that the directional Hubble parameters are functions of scalar field, i.e., $H_1=mH_2=mY(\phi)$, where $Y(\phi)$ is an unknown to be determined. Using $\frac{dU}{dt}=\frac{dU}{d\phi}\dot{\phi}$, $\frac{d^2U}{dt^2}=\frac{d^2U}{d\phi^2}\dot{\phi}^2+\frac{dU}{d\phi}\ddot{\phi}$ , the above equation becomes

$$2(m+2)U_{,\phi\phi}\dot{\phi}^2+2(m+2)U_{,\phi}\ddot{\phi}$$
$$+4(1+2m)UY_{,\phi}\dot{\phi}+\omega_0(m+2)\dot{\phi}^2$$
$$-2(m^2+2)Y(\phi)U_{\phi}\dot{\phi}=0. \quad (10)$$

Let $\dot{\phi} = F(\phi)$ and consequently, we have $\phi(t) = \int F(\phi)dt$. For $F(\phi) \equiv 0$ , $\phi$ is constant and consequently, the GR equations can be recovered. Using these values, Eq.(10) can be written as

$$\begin{aligned} &2(m+2)U_{,\phi\phi}F + 2(m+2)U_{,\phi}F_{,\phi} \\ &+4(1+2m)UY_{,\phi} + \omega_0(m+2)F \\ &-2(m^2+2)Y(\phi)U_{\phi} = 0. \end{aligned} \tag{11}$$

This equation contains three unknowns namely $(U,\ F,\ Y)$. Thus if any pair of these functions are given then the third unknown can be determined.

For example, if both $U$ and $F$ are known, then $Y$ can be found from the equation

$$\begin{aligned} Y(\phi) &= -U^{(m^2+2)/2(1+2m)} \\ &\times \int^{\phi} U^{-(m^2+2)/(2(1+2m))}(m+2)[2U_{,\tilde{\phi}\tilde{\phi}}F \\ &+ 2U_{,\tilde{\phi}}F_{,\tilde{\phi}} + \omega_0 F]d\tilde{\phi} + c_1 U^{(m^2+2)/(2(1+2m))}. \end{aligned} \tag{12}$$

If both $F$ and $Y$ are known, then $U$ can be determined by the equation

$$U_{,\phi\phi} + [\frac{2(m+2)F_{,\phi} - 2(m^2+2)Y}{2(m+2)F}]U_{,\phi} + \frac{2(1+2m)}{m+2}\frac{Y_{,\phi}}{F}U(\phi) + \frac{\omega_0}{2} = 0. \quad (13)$$

Finally, if $U$ and $Y$ are given, then $F$ is given by

$$\int[\frac{d}{d\phi}(\exp(\int^{\phi}\frac{2(m+2)U_{,\tilde{\phi}\tilde{\phi}}+\omega_0(m+2)}{2(m+2)U_{,\tilde{\phi}}}d\tilde{\phi})F)]d\phi-c_2$$

$$=\int^{\phi}(\exp(\int^{\phi^\star}\frac{2(m+2)U_{,\tilde{\phi}\tilde{\phi}}+\omega_0(m+2)}{2(m+2)U_{,\tilde{\phi}}}d\tilde{\phi})F)$$

$$\times[\frac{2(m^2+2)YU_{,\phi^\star}}{2(m+2)U_{,\phi^\star}}-\frac{4(1+2m)UY_{,\phi^\star}}{2(m+2)U_{,\phi^\star}}]d\phi^\star. \quad (14)$$

The field equation (6) leads to the scalar field potential

$$V(\phi) = 2(1+2m)U(\phi)Y^2 + 2(m+2)YU_{,\phi}F - \frac{\omega_0}{2}F^2. \quad (15)$$

Usually, the Hubble parameter or the directional Hubble parameters are expressed in terms of cosmic time or the average scale factor. However, it is more interesting to express the directional Hubble parameters in terms of logarithmic average scale factor, a new dimensionless variable given by $N = \ln B^{(m+2)/3}$ or equivalently, $\exp(N) = B^{(m+2)/3}$.

Thus, the time derivative can be expressed as

$$\begin{aligned}\frac{d}{dt} &= \frac{(m+2)}{3}H_2\frac{d}{dN},\\ \frac{d^2}{dt^2} &= \frac{(m+2)^2}{9}[\frac{1}{2}(H_2^2)'\frac{d}{dN}+H_2^2\frac{d^2}{dN^2}].\end{aligned} \tag{16}$$

In the onward discussion, we denote the time derivative by dot while the prime indicates the derivative with respect to $N$.

Using the above relations, Eq.(11) can be written as

$$\frac{(m+2)^2}{9}U'(H_2^2)' + \frac{2(m+2)^2}{9}H_2^2U'' + \frac{2(1+2m)}{3}U(H_2^2)' + \frac{\omega_0(m+2)^2}{9}H_2^2(\phi')^2 - \frac{2(m^2+2)}{3}U'H_2^2 = 0. \quad (17)$$

For the sake of simplicity, let us assume that $W(\phi) = H_2^2$ and $\hat{F} = \frac{d\phi}{dN}$. Thus, the derivatives of the directional Hubble parameter and interaction function $U$ can be defined as

$$(H_2^2)' = \frac{dH_2^2}{dN} = W_{,\phi}\hat{F}, \quad U' = U_{,\phi}\hat{F}, \quad U'' = \hat{F}[U_{,\phi\phi}\hat{F}+U_{,\phi}\hat{F}_{,\phi}].$$

Equation (17) can be written as

$$\frac{(m+2)^2}{9}U_{,\phi}W_{,\phi}\hat{F} + \frac{2(m+2)^2}{9}W(U_{,\phi\phi}\hat{F} + U_{,\phi}\hat{F}_{,\phi}) + \frac{2(1+2m)}{3}UW_{,\phi} + \frac{\omega_0(m+2)^2}{9}W\hat{F} - \frac{2(m^2+2)}{3}U_{,\phi}W = 0. \quad (18)$$

This equation also involves three unknown functions $W,\ \hat{F},\ U$ that are to be found. If the functions $\hat{F}$ and $U$ are specified then the function $W$ can be determined by the equation

$$\begin{aligned}\frac{W_{,\phi}}{W} &= -[\frac{2(m+2)^2}{9}(U_{,\phi\phi}\hat{F}+U_{,\phi}\hat{F}_{,\phi})\\ &+ \frac{\omega_0(m+2)^2}{9}\hat{F}-\frac{2(m^2+2)}{3}U_{,\phi}]\\ &\times [\frac{(m+2)^2}{9}U_{,\phi}\hat{F}+\frac{2(1+2m)U}{3}]^{-1}. \qquad (19)\end{aligned}$$

Using Eq.(6), the respective scalar field potential becomes

$$V(\phi) = 2(1+2m)U(\phi)W + 2(m+2)WU_{,\phi}\hat{F} - \frac{\omega_{bd}}{2}W\hat{F}^2. \quad (20)$$

Substituting the relation $F = \frac{m+2}{3}Y\hat{F}$ in Eq.(11), we have

$$2\frac{Y_{,\phi}}{Y} = -[\frac{2(m+2)^2}{9}(U_{,\phi\phi}\hat{F} + U_{,\phi}\hat{F}_{,\phi}) + \frac{\omega_0(m+2)^2}{9}\hat{F} - \frac{2(m^2+2)}{3}U_{,\phi}] \times [\frac{(m+2)^2}{9}U_{,\phi}\hat{F} + \frac{2(1+2m)U}{3}]^{-1}. \quad (21)$$

Thus Eq.(18) plays the same role as does Eq.(11). The only difference is that Eq.(11) yields unknowns in terms of cosmic time while Eq.(18) provides unknowns in terms of new variable $N$. Both these equations serve as the key relations in this reconstruction scheme.

**Example**

Now we discuss the nature of scalar field potentials by using the above reconstruction scheme for the following example.

**Quadratic Non-minimally Interacted Models**

Let us consider a specific choice of the function $U$, representing the non-minimal interaction of scalar field with geometry

$$U(\phi) = \gamma\phi^2 + c_0\phi + J_0, \tag{22}$$

where $\gamma,\ c_0$ and $J_0$ are arbitrary constants. By taking different values of these constants, we have different cases of scalar-tensor theories. Substituting this value in Eqs.(12) and (14), we have two determining equations for the functions $F$ and $Y$

$$\begin{aligned} F(\phi) &= (2\gamma\phi+c_0)^{-(1+\frac{\omega_0}{4\gamma})}\int^{\phi}[(2\gamma\tilde{\phi}+c_0)^{(1+\frac{\omega_0}{4\gamma})}(2(m^2+2) \\ &\times Y(\tilde{\phi})(2\gamma\tilde{\phi}+c_0)-4(1+2m)(\gamma\tilde{\phi}^2+c_0\tilde{\phi}+J_0)Y_{,\tilde{\phi}})] \\ &\times [2(m+2)(2\gamma\tilde{\phi}+c_0)]^{-1}d\tilde{\phi}+c_2(2\gamma\phi+c_0)^{-(1+\frac{\omega_0}{4\gamma})},\quad (23) \end{aligned}$$

$$\begin{aligned} Y(\phi) &= (\gamma\phi^2+c_0\phi+J_0)^{\frac{m^2+2}{2(1+2m)}}[-\int^{\phi}(\gamma\tilde{\phi}^2+c_0\tilde{\phi} \\ &+ J_0)^{-(m^2+2)/2(1+2m)}(m+2)[4\gamma F+2(2\gamma\tilde{\phi}+c_0)F_{,\tilde{\phi}} \\ &+ \omega_0 F]d\tilde{\phi}]+c_1(\gamma\phi^2+c_0\phi+J_0)^{\frac{m^2+2}{2(1+2m)}}. \quad (24) \end{aligned}$$

Integration of these equations is difficult however, the solution can be discussed by taking specific values of the anisotropy parameter $m$. If we take $F(\phi)=f_1\phi+f_0$ with $f_0$ and $f_1$ as constants, then Eq.(24) yields

$$\begin{aligned}
Y(\phi) &= (\gamma\phi^2+c_0\phi+J_0)^{\frac{m^2+2}{2(1+2m)}}[-\int^{\phi}(\gamma\tilde{\phi}^2+c_0\tilde{\phi}\\
&+ J_0)^{-(m^2+2)/2(1+2m)}(m+2)[4\gamma f_1\tilde{\phi}+4\gamma f_0+4\gamma f_1\tilde{\phi}\\
&+ \omega_0(f_1\phi+f_0)]d\tilde{\phi}]+c_1(\gamma\phi^2+c_1\phi+J_0)^{\frac{m^2+2}{2(1+2m)}}.
\end{aligned}$$

In order to be particular, we choose $c_0 = 0$ instead of specifying $m$ and using the binomial expansion, the above can be integrated as follows

$$\begin{aligned} Y(\phi) &= -(\gamma\phi^2 + J_0)^{\frac{m^2+2}{2(1+2m)}}(m+2)J_0^{-\frac{m^2+2}{2(1+2m)}}[2\gamma f_1\phi^2 \\ &+ 4\gamma f_0\phi + 2\gamma f_1\phi^2 + 2f_1c_0\phi + \omega_0 f_1\frac{\phi^2}{2} + \omega_0 f_0\phi \\ &- \frac{(m^2+2)}{2(1+2m)}\frac{\gamma_1}{J_0}(\gamma f_1\phi^4 + 4\gamma f_0\frac{\phi^3}{3} + \gamma f_1\phi^4 + 2f_1c_0\frac{\phi^3}{3} \\ &+ \frac{\omega_0 f_1\phi^4}{4} + \frac{\omega_0 f_0\phi^3}{3})] + c_1(\gamma\phi^2 + J_0)^{\frac{m^2+2}{2(1+2m)}}. \end{aligned} \tag{25}$$

The respective field potential (15) can be written as

$$
\begin{aligned}
V(\phi) &= 2(1+2m)(\gamma\phi^2+J_0)[-(\gamma\phi^2+J_0)^{\frac{m^2+2}{2(1+2m)}}(m+2)J_0^{-\frac{m^2+2}{2(1+2m)}} \\
&\times [2\gamma f_1\phi^2+4\gamma f_0\phi+2\gamma f_1\phi^2+2f_1c_0\phi+\omega_0 f_1\frac{\phi^2}{2}+\omega_0 f_0\phi \\
&- \frac{(m^2+2)}{2(1+2m)}\frac{\gamma_1}{J_0}(\gamma f_1\phi^4+4\gamma f_0\frac{\phi^3}{3}+\gamma f_1\phi^4+2f_1c_0\frac{\phi^3}{3} \\
&+ \frac{\omega_0 f_1\phi^4}{4}+\frac{\omega_0 f_0\phi^3}{3})]+c_1(\gamma\phi^2+J_0)^{\frac{m^2+2}{2(1+2m)}}]^2+2(m+2) \\
&\times [-(\gamma\phi^2+J_0)^{\frac{m^2+2}{2(1+2m)}}(m+2)J_0^{-\frac{m^2+2}{2(1+2m)}}[2\gamma f_1\phi^2+4\gamma f_0\phi
\end{aligned}
$$

$$
\begin{aligned}
+ &\ 2\gamma f_1\phi^2 + 2f_1c_0\phi + \omega_0 f_1\frac{\phi^2}{2} + \omega_0 f_0\phi - \frac{(m^2+2)}{2(1+2m)} \\
\times &\ \frac{\gamma_1}{J_0}(\gamma f_1\phi^4 + 4\gamma f_0\frac{\phi^3}{3} + \gamma f_1\phi^4 + 2f_1c_0\frac{\phi^3}{3} + \frac{\omega_0 f_1\phi^4}{4} \\
+ &\ \frac{\omega_0 f_0\phi^3}{3})] + c_1(\gamma\phi^2 + J_0)^{\frac{m^2+2}{2(1+2m)}}](2\gamma\phi)(f_0 + f_1\phi) \\
- &\ \frac{\omega_0}{2}(f_0 + f_1\phi)^2. \qquad (26)
\end{aligned}
$$

This shows that the polynomial form of the directional Hubble parameter and exponential form of the scalar field ($\phi(t) = c\exp(f_1 t) - \frac{f_0}{f_1}$) gives rise to polynomial form of the field potential that can involve negative powers depending upon the values of anisotropy parameter $m$. Thus the obtained potential is of power law nature.

## Concluding Remarks

- This technique does not require any specification of the Hubble parameter or scalar field as a function of cosmic time or scale factor.
- We have discussed the general form of the field potential in terms of the functions $(U,\ F,\ Y)$. For any pair of these functions, the third unknown can easily be determined.
- The field potential is of power law nature either positive or reverse power law in terms of scalar field depending upon $m$.


Thank You