f(T) Gravity and Cosmology

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Goal

- We investigate cosmological scenarios in a universe governed by torsional modified gravity

- Note:

  A consistent or interesting cosmology is not a proof for the consistency of the underlying gravitational theory

Talk Plan

- 1) Introduction: Gravity as a gauge theory, modified Gravity
- 2) Teleparallel Equivalent of General Relativity and f(T) modification
- 3) Perturbations and growth evolution
- 4) Bounce in f(T) cosmology
- 5) Non-minimal scalar-torsion theory
- 6) Black-hole solutions
- 7) Solar system and growth-index constraints
- 8) Conclusions-Prospects

Introduction

- Einstein 1916: **General Relativity**:
  energy-momentum source of spacetime Curvature
  Levi-Civita connection: Zero Torsion

- Einstein 1928: **Teleparallel Equivalent of GR:**
  Weitzenbock connection: Zero Curvature

- **Einstein-Cartan** theory: energy-momentum source of Curvature, spin source of Torsion
  [Hehl, Von Der Heyde, Kerlick, Nester Rev.Mod.Phys.48]

Introduction

- **Gauge Principle**: global symmetries replaced by local ones:

  The group generators give rise to the compensating fields

  It works perfect for the standard model of strong, weak and E/M interactions

  $$SU(3) \times SU(2) \times U(1)$$

- Can we apply this to gravity?

Introduction

- Formulating the gauge theory of gravity (mainly after 1960):
- Start from Special Relativity

⇒ Apply (Weyl-Yang-Mills) gauge principle to its Poincaré-group symmetries

⇒ Get Poinaré gauge theory:

Both curvature and torsion appear as field strengths

- Torsion is the field strength of the translational group

(Teleparallel and Einstein-Cartan theories are subcases of Poincaré theory)

[Blagojevic, Hehl, Imperial College Press, 2013]

Introduction

- One could extend the gravity gauge group (SUSY, conformal, scale, metric affine transformations) obtaining SUGRA, conformal, Weyl, metric affine gauge theories of gravity

- In all of them torsion is always related to the gauge structure.
- Thus, a possible way towards gravity quantization would need to bring torsion into gravity description.

Introduction

- 1998: Universe acceleration

$\Rightarrow$Thousands of work in Modified Gravity

(f(R), Gauss-Bonnet, Lovelock, nonminimal scalar coupling, nonminimal derivative coupling, Galileons, Hordenski, massive etc)

[Copeland, Sami, Tsujikawa Int.J.Mod.Phys.D15], [Nojiri, Odintsov Int.J.Geom.Meth.Mod.Phys. 4]

- Almost all in the curvature-based formulation of gravity

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- Almost all in the curvature-based formulation of gravity

- So question: Can we modify gravity starting from its torsion-based formulation?

torsion $\Rightarrow$ gauge ? $\Rightarrow$ quantization

modification $\Rightarrow$ full theory ? $\Rightarrow$ quantization

Teleparallel Equivalent of General Relativity (TEGR)

- Let's start from the simplest tosion-based gravity formulation, namely TEGR:
- Vierbeins e_A^{μ} : four linearly independent fields in the tangent space

$$g_{\mu\nu}(x) = \eta_{AB}\, e_{\mu}^{A}(x)\, e_{\nu}^{B}(x)$$

- Use curvature-less Weitzenböck connection instead of torsion-less Levi-Civita one: $\Gamma_{\nu\mu}^{\lambda\{W\}} = e_A^{\lambda}\partial_{\mu} e_{\nu}^{A}$
- Torsion tensor:

$$T_{\mu\nu}^{\lambda} = \Gamma_{\nu\mu}^{\lambda\{W\}} - \Gamma_{\mu\nu}^{\lambda\{W\}} = e_A^{\lambda}\left(\partial_{\mu} e_{\nu}^{A} - \partial_{\nu} e_{\mu}^{A}\right)$$

[Einstein 1928], [Pereira: Introduction to TG]

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- Lagrangian (imposing coordinate, Lorentz, parity invariance, and up to 2nd order in torsion tensor)

$$L \equiv T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}^{\rho} T_{\nu}^{\nu\mu}$$

- **Completely equivalent** with GR at the level of equations

[Einstein 1928], [Hayaski,Shirafuji PRD 19], [Pereira: Introduction to TG]

f(T) Gravity and f(T) Cosmology

- f(T) Gravity: Simplest torsion-based modified gravity
- Generalize T to f(T) (inspired by f(R))

$$S = \frac{1}{16\pi G}\int d^4x \, e \, [T + f(T)] + S_m$$ [Bengochea, Ferraro PRD 79], [Linder PRD 82]

- Equations of motion:

$$e^{-1}\partial_\mu\left(e e_A^\rho S_\rho^{\mu\nu}\right)(1+f_T) - e_A^\lambda T^\rho_{\mu\lambda} S_\rho^{\nu\mu} + e_A^\rho S_\rho^{\mu\nu}\partial_\mu(T) f_{TT} - \frac{1}{4}e_A^\nu [T + f(T)] = 4\pi G e_A^\rho \mathrm{T}_\rho^{\nu\{\mathrm{EM}\}}$$

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- f(T) Cosmology: Apply in FRW geometry:

$$e_\mu^A = diag(1,a,a,a) \ \Rightarrow\ ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$$ (not unique choice)

- Friedmann equations:

$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} - 2 f_T H^2$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 + f_T - 12H^2 f_{TT}}$$

- Find easily

$$T = -6H^2$$

f(T) Cosmology: Background

- Effective Dark Energy sector:

$$\rho_{DE} = \frac{3}{8\pi G}\left[-\frac{f}{6} + \frac{T}{3} f_T\right]$$

$$w_{DE} = -\frac{f - Tf_T + 2T^2 f_{TT}}{[1 + f_T + 2Tf_{TT}][f - 2Tf_T]}$$

[Linder PRD 82]

- Interesting cosmological behavior: Acceleration, Inflation etc
- At the background level indistinguishable from other dynamical DE models

f(T) Cosmology: Perturbations

- Can I find imprints of f(T) gravity? Yes, but need to go to perturbation level

$$e_\mu^0 = \delta_\mu^0(1+\psi)\ ,\ e_\mu^\alpha = \delta_\mu^\alpha \alpha(1-\phi) \quad \Rightarrow \quad ds^2 = (1+2\psi)dt^2 - a^2(1-2\phi)\delta_{ij}dx^i dx^j$$

- Obtain Perturbation Equations:

$$L.H.S = R.H.S$$
$$\ldots$$
$$L.H.S = R.H.S$$

[Chen, Dent, Dutta, Saridakis PRD 83],
[Dent, Dutta, Saridakis JCAP 1101]

- Focus on growth of matter overdensity $\delta \equiv \frac{\delta\rho_m}{\rho_m}$ go to Fourier modes:

$$3H\left(1+f_T-12H^2 f_{TT}\right)\dot{\phi}_k + \left[(3H^2+k^2/a^2)(1+f_T)-36H^4 f_{TT}\right]\phi_k + 4\pi G\rho_m\delta_k = 0$$

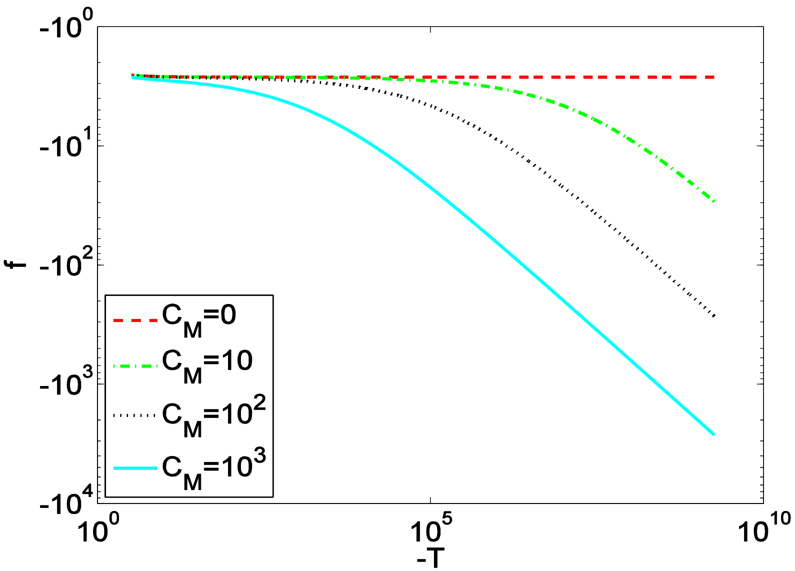
[Chen, Dent, Dutta, Saridakis PRD 83]

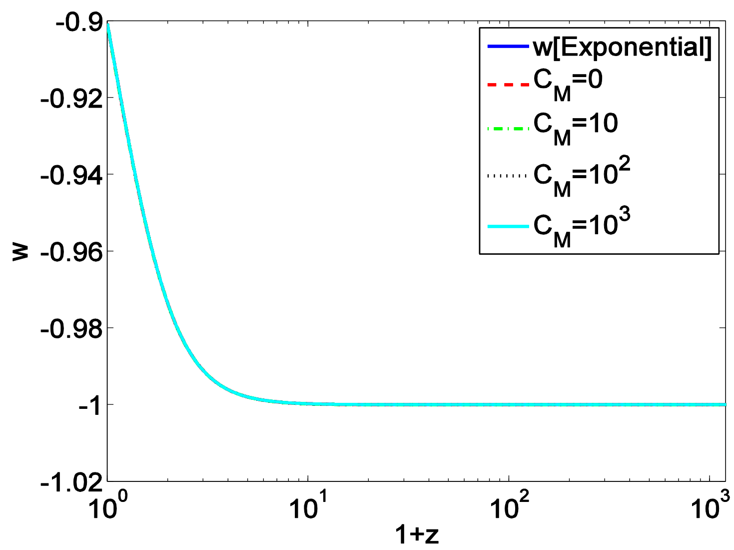
f(T) Cosmology: Perturbations

- Application: Distinguish f(T) from quintessence
- 1) Reconstruct f(T) to coincide with a given **quintessence** scenario:

$$f(H) = 16\pi G H \int \frac{\rho_Q}{H^2} dH \; + CH \quad \text{with} \quad \rho_Q = \dot{\phi}^2/2 + V(\phi) \quad \text{and} \quad H = \sqrt{-T/6}$$

[Dent, Dutta, Saridakis JCAP 1101]

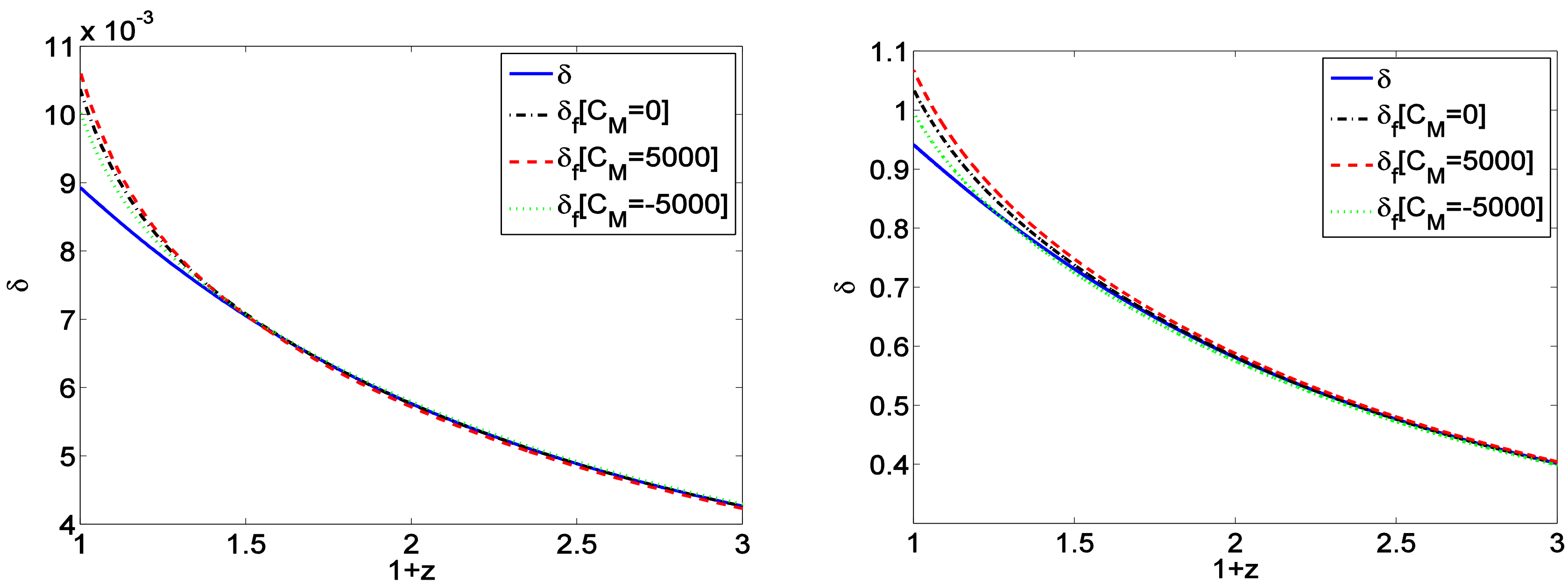




f(T) Cosmology: Perturbations

- Application: Distinguish f(T) from quintessence
- 2) Examine evolution of matter overdensity $\delta \equiv \frac{\delta\rho_m}{\rho_m}$

[Dent, Dutta, Saridakis JCAP 1101]



Bounce and Cyclic behavior

- Contracting ($H<0$), bounce ($H=0$), expanding ($H>0$)
  near and at the bounce $\dot{H}>0$

- Expanding ($H>0$), turnaround ($H=0$), contracting $H<0$
  near and at the turnaround $\dot{H}<0$

Bounce and Cyclic behavior in f(T) cosmology

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$$H^2=\frac{8\pi G}{3}\rho_m-\frac{f(T)}{6}-2f_T H^2$$

$$\dot{H}=-\frac{4\pi G(\rho_m+p_m)}{1+f_T-12H^2 f_{TT}}$$

- Bounce and cyclicity can be easily obtained

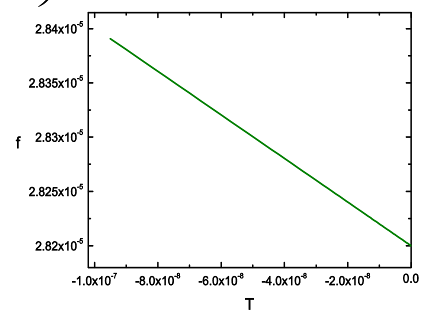
[Cai, Chen, Dent, Dutta, Saridakis CQG 28]

Bounce in f(T) cosmology

- Start with a bounching scale factor: $a(t)=a_B\left(1+\frac{3}{2}\sigma\, t^2\right)^{1/3}$

$$\Rightarrow t(T)=\pm\left(-\frac{4}{3T}-\frac{2}{3\sigma}+\frac{4\sqrt{T\sigma^3+\sigma^4}}{3T\sigma^2}\right)$$

$$\Rightarrow f(t)=\frac{4t}{(2+3\sigma t^2)M_p^2}\left[\frac{\rho_{mB}}{t}+\frac{6tM_p^2\sigma^2}{2+3\sigma t^2}+\sqrt{6\sigma}\,\rho_{mB}\ ArcTan\left(\sqrt{\frac{3s}{2}}t\right)\right]$$



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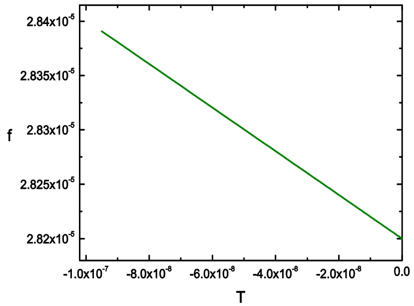

- Examine the full perturbations:

$$\ddot{\phi}_k + \alpha\dot{\phi}_k + \mu^2\phi_k + c_s^2\frac{k^2}{a^2}\phi_k = 0$$

with α, μ^2, c_s^2 known in terms of $H, \dot{H}, f, f_T, f_{TT}$ and matter

$$\left(\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2}{a^2}h_{ij}\right) - \frac{12H\dot{H}f_{TT}}{1+f_T}\dot{h} = 0$$

- $\Longrightarrow$ **Primordial** power spectrum: $P_\zeta = \frac{\sigma}{288\pi^2 M_p^2}$
- $\Longrightarrow$ **Tensor-to-scalar** ratio: $r \approx 2.8\times10^{-3}$

[Cai, Chen, Dent, Dutta, Saridakis CQG 28]

Non-minimally coupled scalar-torsion theory

- In curvature-based gravity, apart from $R+f(R)$ one can use $R+\xi R\varphi$^2
- Let's do the same in torsion-based gravity:

$$S=\int d^4x\, e\left[\frac{T}{2\kappa^2}+\frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi+\xi T\varphi^2\right)-V(\varphi)+L_m\right]$$

[Geng, Lee, Saridakis, Wu PLB704]

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[Geng, Lee, Saridakis, Wu PLB704]

- Friedmann equations in FRW universe:

$$H^2=\frac{\kappa^2}{3}(\rho_m+\rho_{DE})$$

$$\dot{H}=-\frac{\kappa^2}{2}(\rho_m+p_m+\rho_{DE}+p_{DE})$$

with effective Dark Energy sector:

$$\rho_{DE}=\frac{\dot{\varphi}^2}{2}+V(\varphi)-3\xi H^2\varphi^2$$

$$p_{DE}=\frac{\dot{\varphi}^2}{2}-V(\varphi)+4\xi H\varphi\dot{\varphi}+\xi\left(3H^2+2\dot{H}\right)\varphi^2$$

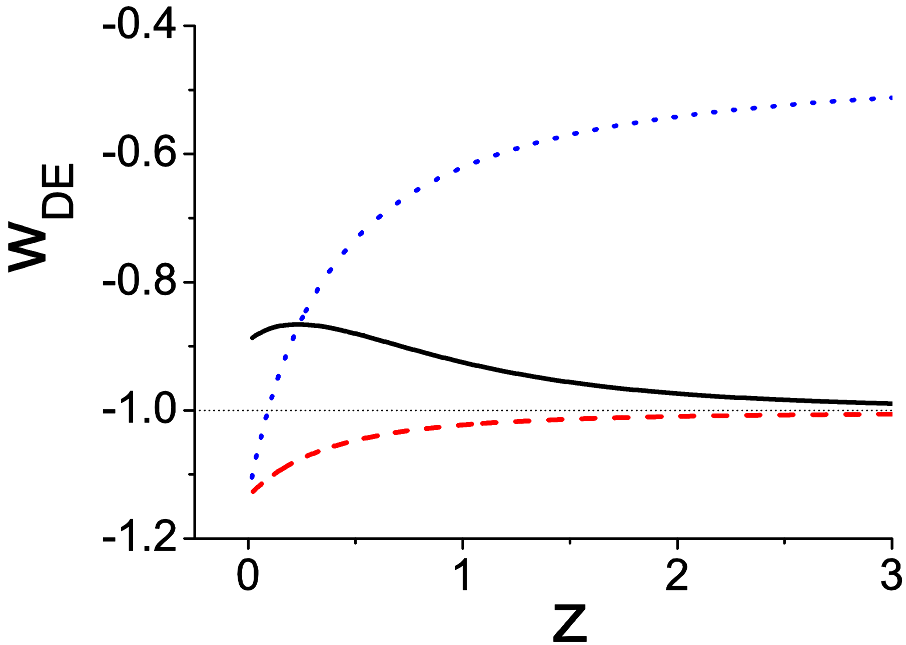
- **Different** than non-minimal quintessence!

[Geng, Lee, Saridakis,Wu PLB 704]

(no conformal transformation in the present case)

Non-minimally coupled scalar-torsion theory

- Main advantage: Dark Energy may lie in the phantom regime or/and experience the phantom-divide crossing
- Teleparallel Dark Energy:

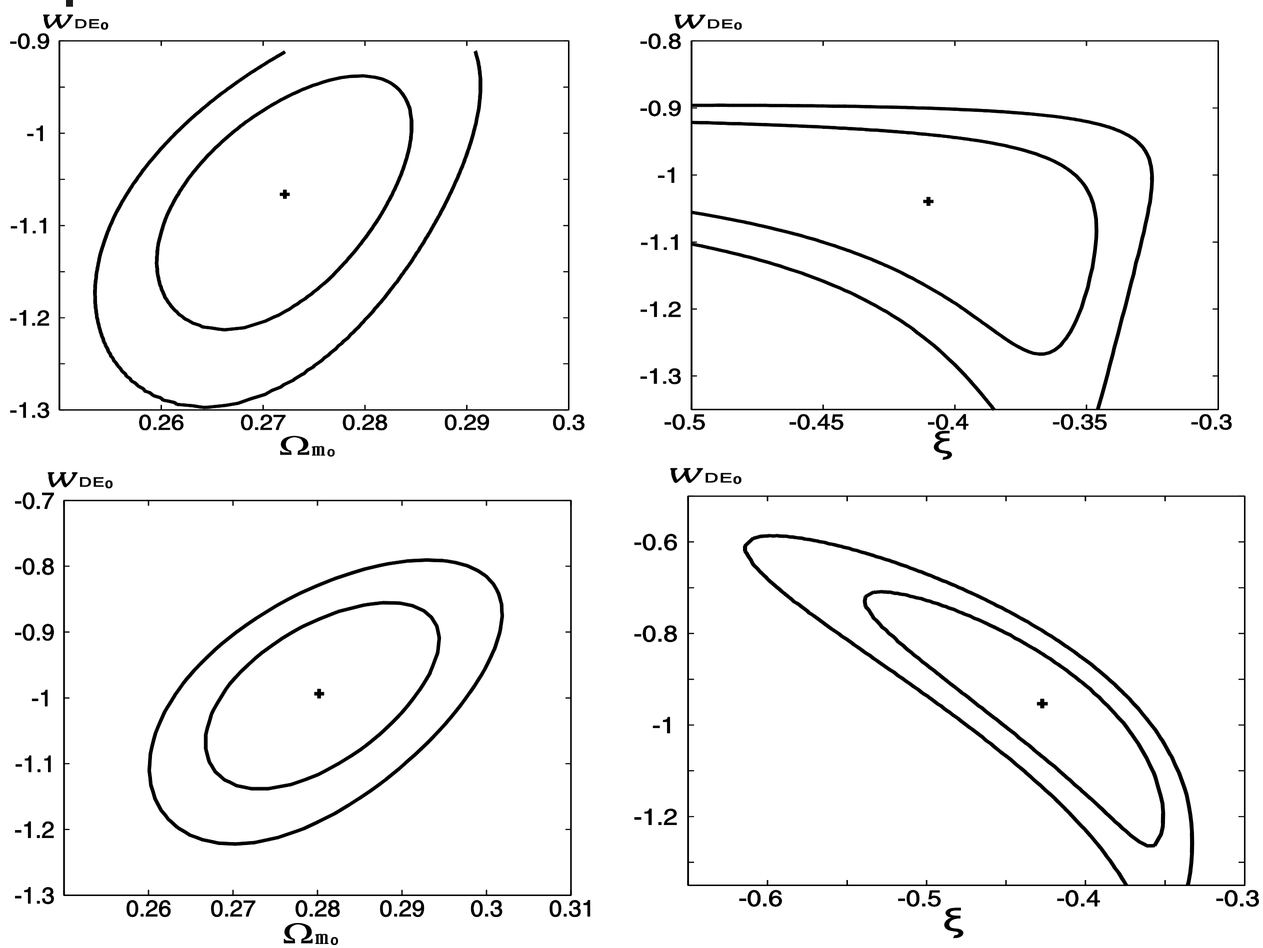


[Geng, Lee, Saridakis, Wu PLB 704]

Observational constraints on Teleparallel Dark Energy

- Use observational data (SNIa, BAO, CMB) to constrain the parameters of the theory
- Include matter and standard radiation: $\rho_M = \rho_{M0}/a^3, \rho_r = \rho_{r0}/a^4, 1+z = 1/a$
- We fit $\Omega_{M0}, \Omega_{DE0}, w_{DE0}, \xi$ for various $V(\varphi)$

Observational constraints on Teleparallel Dark Energy



Exponential potential

Quartic potential

[Geng, Lee, Saridkis   JCAP 1201]

Phase-space analysis of Teleparallel Dark Energy

- Transform cosmological system to its autonomous form:

$$x = \frac{\kappa\dot{\varphi}}{\sqrt{6}H},\quad y = \frac{\kappa\sqrt{V(\varphi)}}{\sqrt{3}H},\quad z = \sqrt{|\xi|}\kappa\varphi$$

$$\Rightarrow \Omega_m \equiv \frac{\rho_m}{3H^2} = 1 - x^2 - y^2 + z^2\,\mathrm{sgn}(\xi),\qquad \Omega_{DE} \equiv \frac{\rho_{DE}}{3H^2} = x^2 + y^2 - z^2\,\mathrm{sgn}(\xi)$$

$$w_{DE} = w_{DE}(x, y, z, \xi)$$

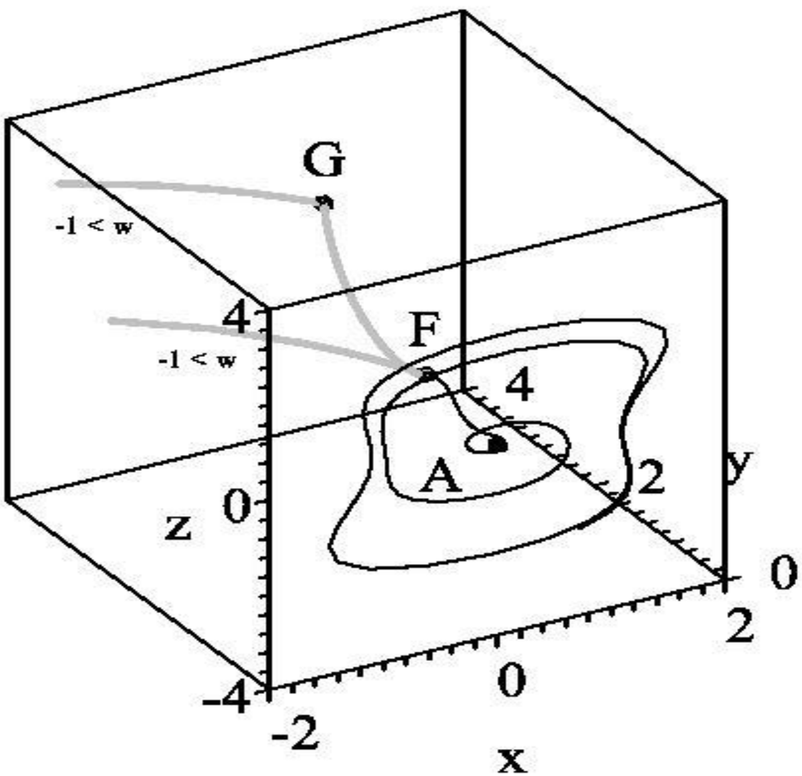
[Xu, Saridakis, Leon, JCAP 1207]

$$\Rightarrow X' = f(X), \qquad X'|_{X=X_C} = 0$$

- Linear Perturbations: $X = X_C + U \;\Longrightarrow U' = QU$
- Eigenvalues of Q determine type and stability of C.P

Phase-space analysis of Teleparallel Dark Energy

- Apart from usual quintessence points, there exists an extra stable one for $\lambda^2 < \xi$ corresponding to $\Omega_{DE} = 1,\ w_{DE} = -1,\ q = -1$



- At the critical points $w_{DE} \geq -1$ however during the evolution it can lie in quintessence or phantom regimes, or experience the phantom-divide crossing!

[Xu, Saridakis, Leon, JCAP 1207]

Exact charged black hole solutions

- Extend f(T) gravity in D-dimensions (focus on D=3, D=4):

$$S = \frac{1}{2\kappa}\int d^D x \; e\left[T + f(T) - 2\Lambda\right]$$

- Add E/M sector: $L_F = -\frac{1}{2} F \wedge {}^*F$ with $F = dA, \; A \equiv A_\mu dx^\mu$
- Extract **field equations**: $\boxed{L.H.S = R.H.S}$

[Gonzalez, Saridakis, Vasquez, JHEP 1207]

[Capozzielo, Gonzalez, Saridakis, Vasquez, JHEP 1302]

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- Look for spherically symmetric solutions:

$$e^0 = F(r)dt,\ e^1 = \frac{1}{G(r)}dr,\ e^2 = r dx_1,\ ,e^3 = r dx_2,\ \cdots$$

$$\Rightarrow ds^2 = F(r)^2 dt^2 - \frac{1}{G(r)^2}dr^2 - r^2 \sum_1^{D-2} dx_i^2$$

- Radial Electric field: $E_r = \frac{Q}{r^{D-2}}$ $\Rightarrow F(r)^2, G(r)^2$ known

[Gonzalez, Saridakis, Vasquez, JHEP 1207], [Capozzielo, Gonzalez, Saridakis, Vasquez, JHEP 1302]

Exact charged black hole solutions

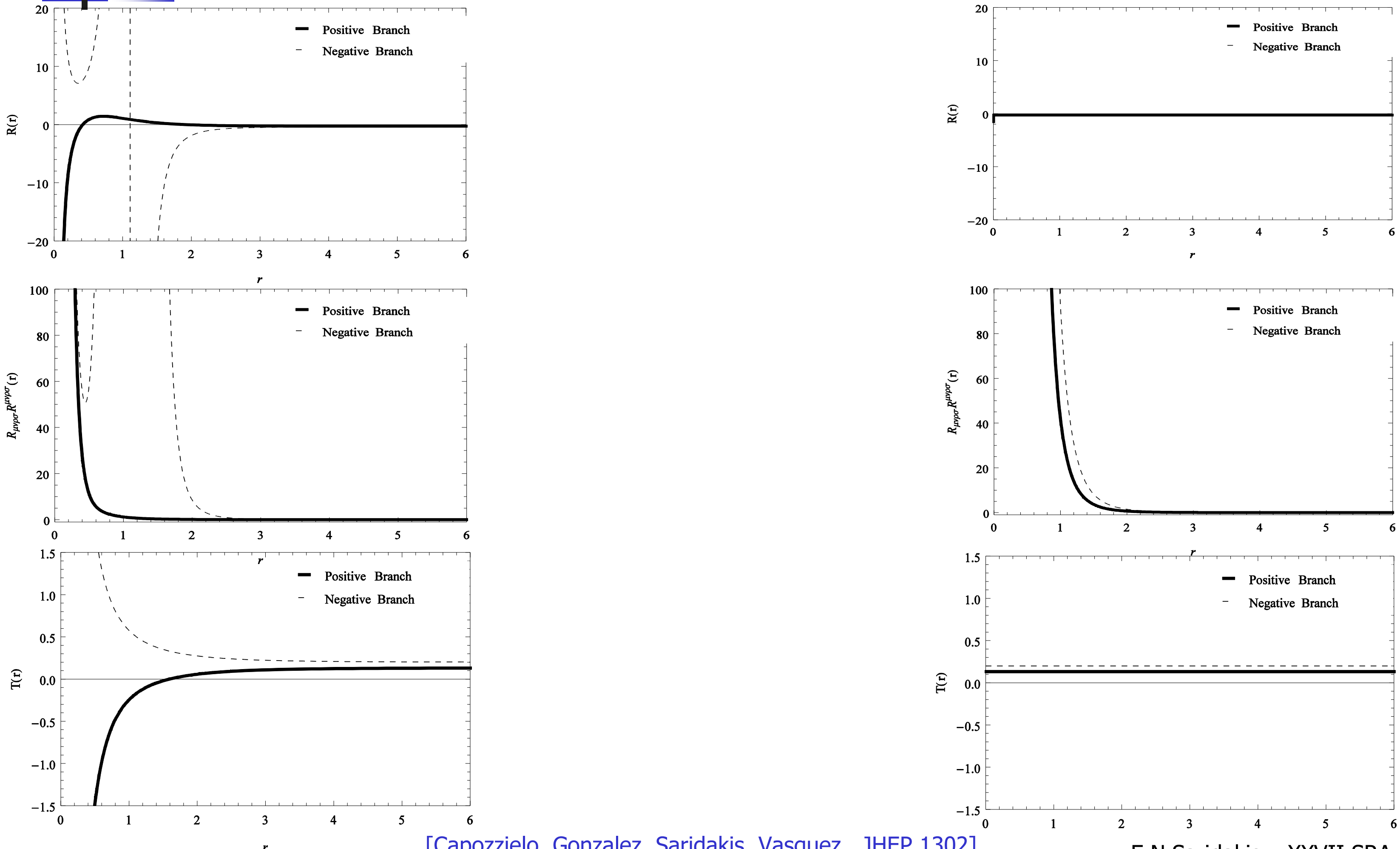
- Horizon and singularity analysis:

- 1) Vierbeins, Weitzenböck connection, Torsion invariants:
  T(r) known $\Longrightarrow$ obtain horizons and singularities

- 2) Metric, Levi-Civita connection, Curvature invariants:
  R(r) and Kretschmann $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}(r)$ known
  $\Longrightarrow$ obtain horizons and singularities

[Gonzalez, Saridakis, Vasquez, JHEP1207], [Capozzielo, Gonzalez, Saridakis, Vasquez, JHEP 1302]

Exact charged black hole solutions



[Capozzielo, Gonzalez, Saridakis, Vasquez, JHEP 1302]

Exact charged black hole solutions

- More singularities in the curvature analysis than in torsion analysis!
  (some are naked)
- The differences disappear in the $f(T)=0$ case, or in the uncharged case.

- Should we go to quartic torsion invariants?

- $f(T)$ brings novel features.

- E/M in torsion formulation was known to be nontrivial (E/M in Einstein-Cartan and Poinaré theories)

Solar System constraints on f(T) gravity

- Apply the black hole solutions in Solar System:
- Assume corrections to TEGR of the form $f(T)=\alpha T^2+O(T^3)$

$$\Rightarrow F(r)^2=1-\frac{2GM}{c^2r}-\frac{\Lambda}{3}r^2+\alpha\left[-6\Lambda-\frac{6}{r^2}-\frac{4GM\Lambda}{c^2r}\right]$$

$$\Rightarrow G(r)^2=1-\frac{2GM}{c^2r}-\frac{\Lambda}{3}r^2+\alpha\left[\frac{8\Lambda}{3}-\frac{24}{r^2}-2\Lambda^2r^2-\frac{2GM}{c^2r}\left(8\Lambda-\frac{8}{r^2}\right)\right]$$

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- Use data from Solar System orbital motions:

$$\Delta U_{f(T)} \leq 6.2\times 10^{-10}$$

[Iorio, Saridakis, Mon.Not.Roy.Astron.Soc 427)

T<<1 so consistent

- f(T) divergence from TEGR is very small
- This was already known from cosmological observation constraints up to

$O(10^{-1} - 10^{-2})$ [Wu, Yu, PLB 693], [Bengochea PLB 695]

- With Solar System constraints, much more stringent bound.

Growth-index constraints on f(T) gravity

- Perturbations: $\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{eff}\ \rho_m \delta_m$, clustering growth rate: $\frac{d\ \ln\delta_m}{d\ \ln a} = \Omega_m^{\gamma}(a)$
- γ(z): Growth index. $G_{eff} = \frac{1}{1+f'(T)}$

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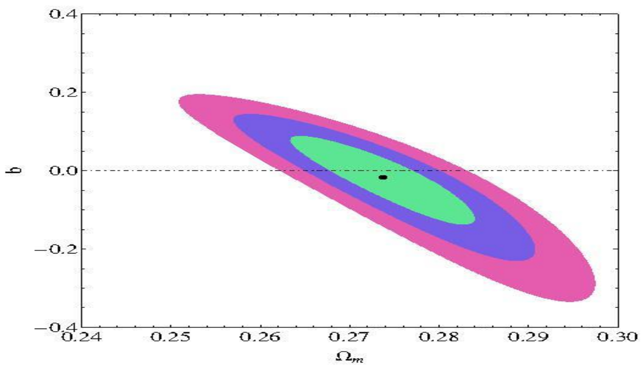

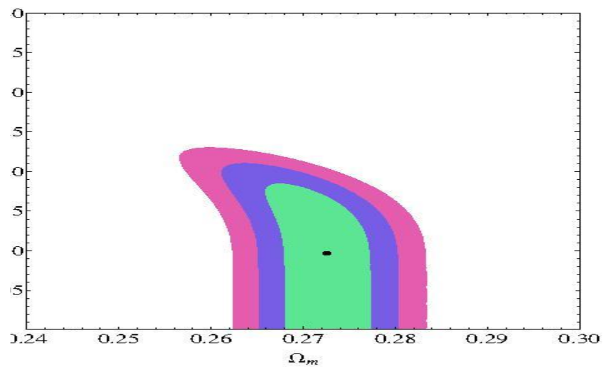

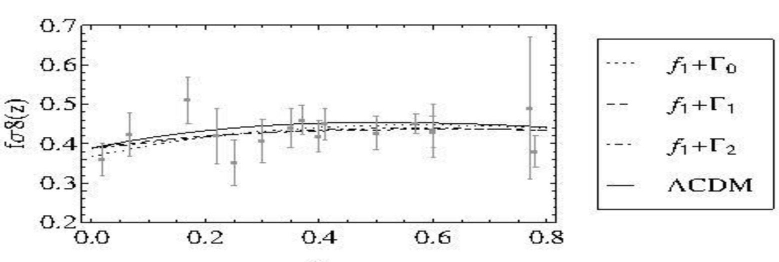

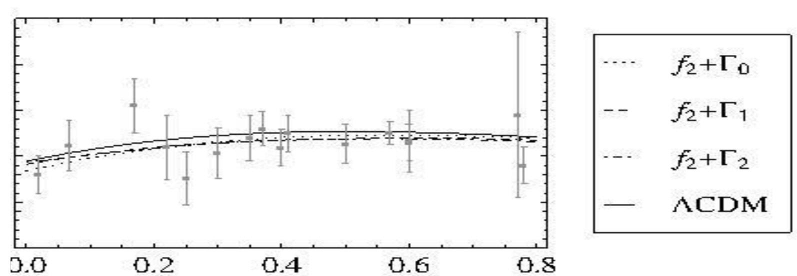

- Viable f(T) models are practically indistinguishable from ΛCDM.

[Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]

Open issues of f(T) gravity

- f(T) cosmology is very interesting. But f(T) gravity and nonminially coupled teleparallel gravity has many open issues [Li, Sotiriou, Barrow PRD 83a], [Geng,Lee,Saridakis,Wu PLB 704]

- For nonlinear f(T), Lorentz invariance is not satisfied
- Equivalently, the vierbein choices corresponding to the same metric are not equivalent (extra degrees of freedom) [Li,Sotiriou,Barrow PRD 83c], [Li,Miao,Miao JHEP 1107]

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- Black holes are found to have different behavior through curvature and torsion analysis [Capozzielo, Gonzalez, Saridakis, Vasquez JHEP 1302]
- Thermodynamics also raises issues [Bamba,Geng JCAP 1111], [Miao,Li,Miao JCAP 1111]
- Cosmological, Solar System and Growth Index observations constraint f(T) very close to linear-in-T form

Gravity modification in terms of torsion?

- So can we modify gravity starting from its torsion formulation?
- The simplest, a bit naïve approach, through f(T) gravity is interesting, but has open issues
- Additionally, f(T) gravity is not in correspondence with f(R)
- Even if we find a way to modify gravity in terms of torsion, will it be still in 1-1 correspondence with curvature-based modification?
- What about higher-order corrections, but using torsion invariants (similar to Gauss Bonnet, Lovelock, Hordenski modifications)?
- Can we modify gauge theories of gravity themselves? E.g. can we modify Poincaré gauge theory?

Conclusions

- i) Torsion appears in all approaches to gauge gravity, i.e to the first step of quantization.
- ii) Can we modify gravity based in its torsion formulation?
- iii) Simplest choice: f(T) gravity, i.e extension of TEGR
- iv) f(T) cosmology: Interesting phenomenology. Signatures in growth structure.
- v) We can obtain bouncing solutions
- vi) Non-minimal coupled scalar-torsion theory $T+\xi T\varphi^2$: Quintessence, phantom or crossing behavior.
- vii) Exact black hole solutions. Curvature vs torsion analysis.
- viii) Solar system constraints: f(T) divergence from T less than 10^{-10}
- ix) Growth Index constraints: Viable f(T) models are practically indistinguishable from ΛCDM.
- x) Many open issues. Need to search for other torsion-based modifications too.

Outlook

- Many subjects are open. Amongst them:
- i) Examine thermodynamics thoroughly.
- ii) Extend f(T) gravity in the braneworld.
- iii) Understand the extra degrees of freedom and the extension to non-diagonal vierbeins.
- iv) Try to modify TEGR using higher-order torsion invariants.
- v) Try to modify Poincaré gauge theory (extremely hard!)
- vi) What to quantize? Metric, vierbeins, or connection?
- vii) Convince people to work on the subject!

THANK YOU!