f(T) Gravity and Cosmology

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Goal

- We investigate cosmological scenarios in a universe governed by torsional modified gravity

Note:

A consistent or interesting cosmology is not a proof for the consistency of the underlying gravitational theory
Talk Plan

1) **Introduction**: Gravity as a gauge theory, modified Gravity

2) Teleparallel Equivalent of General Relativity and $f(T)$ modification

3) Perturbations and growth evolution

4) Bounce in $f(T)$ cosmology

5) Non-minimal scalar-torsion theory

6) Black-hole solutions

7) Solar system and growth-index constraints

8) Conclusions-Prospects
Introduction

- Einstein 1916: **General Relativity:**
  energy-momentum source of spacetime Curvature
  Levi-Civita connection: Zero Torsion

- Einstein 1928: **Teleparallel Equivalent of GR:**
  Weitzenbock connection: Zero Curvature

- **Einstein-Cartan** theory: energy-momentum source of Curvature, spin source of Torsion
  [Hehl, Von Der Heyde, Kerlick, Nester Rev.Mod.Phys.48]
Introduction

- **Gauge Principle**: global symmetries replaced by local ones:

The group generators give rise to the compensating fields

It works perfect for the standard model of strong, weak and E/M interactions

\[ SU(3) \times SU(2) \times U(1) \]

- Can we apply this to gravity?
Introduction

- Formulating the **gauge theory** of gravity (mainly after 1960):
  - Start from **Special Relativity**
  - Apply (Weyl-Yang-Mills) **gauge principle** to its **Poincaré-group** symmetries
  - Get **Poincaré gauge theory**: Both curvature and torsion appear as field strengths

- **Torsion** is the **field strength** of the **translational group**
  (Teleparallel and Einstein-Cartan theories are subcases of **Poincaré theory**)

[Blagojevic, Hehl, Imperial College Press, 2013]
One could extend the gravity gauge group (SUSY, conformal, scale, metric affine transformations) obtaining SUGRA, conformal, Weyl, metric affine gauge theories of gravity.

In all of them torsion is always related to the gauge structure.

Thus, a possible way towards gravity quantization would need to bring torsion into gravity description.
Introduction

1998: Universe acceleration

⇒ Thousands of work in Modified Gravity

(f(R), Gauss-Bonnet, Lovelock, nonminimal scalar coupling,
nonminimal derivative coupling, Galileons, Hordenski, massive etc)


Almost all in the curvature-based formulation of gravity
Introduction

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    (f(R), Gauss-Bonnet, Lovelock, nonminimal scalar coupling,
    nonminimal derivative coupling, Galileons, Hordenski, massive etc)

- Almost all in the curvature-based formulation of gravity

- So question: Can we modify gravity starting from its torsion-based formulation?
  torsion \[ \Rightarrow \] gauge \[ ? \] \[ \Rightarrow \] quantization
  modification \[ \Rightarrow \] full theory \[ ? \] \[ \Rightarrow \] quantization
Teleparallel Equivalent of General Relativity (TEGR)

- Let's start from the simplest torsion-based gravity formulation, namely TEG:
  - **Vierbeins** $e^\mu_A$ : four linearly independent fields in the tangent space
    \[ g_{\mu\nu}(x) = \eta_{AB} \ e^A_\mu(x) \ e^B_\nu(x) \]
  - Use curvature-less Weitzenböck connection instead of torsion-less Levi-Civita one: \[ \Gamma^A_{\nu\mu} = e^A_B \partial_\mu e^B_\nu \]
  - **Torsion tensor**:
    \[ T^\lambda_{\mu\nu} = \Gamma^A_{\nu\mu} \Gamma^A_{\lambda\mu} - \Gamma^A_{\mu\nu} \Gamma^A_{\lambda\mu} = e^A_\lambda \left( \partial_\mu e^A_\nu - \partial_\nu e^A_\mu \right) \]
    [Einstein 1928], [Pereira: Introduction to TG]
Teleparallel Equivalent of General Relativity (TEGR)

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  - **Vierbeins** \( e^\mu_A \): four linearly independent fields in the tangent space
  \[
g_{\mu\nu}(x) = \eta_{AB} \quad e^A_\mu(x) \quad e^B_\nu(x)
\]
  - Use **curvature-less Weitzenböck connection** instead of **torsion-less Levi-Civita** one:
    \[
    \Gamma^A_{\mu\nu} = e^A_\mu \partial_\nu e^A_\lambda - e^A_\lambda \partial_\nu e^A_\mu
    \]
  - **Torsion tensor**:
    \[
    T^\lambda_{\mu\nu} = \Gamma^A_{\lambda\mu\nu} - \Gamma^A_{\lambda\mu\nu} = e^A_\mu \partial_\nu e^A_\lambda - e^A_\lambda \partial_\nu e^A_\mu
    \]
  - **Lagrangian** (imposing coordinate, Lorentz, parity invariance, and up to 2\textsuperscript{nd} order in torsion tensor)
    \[
    L \equiv T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T^\rho_{\rho\mu} T_{\nu}^\mu
    \]

- **Completely equivalent** with GR at the level of equations

[Einstein 1928], [Hayashi, Shirafuji PRD 19], [Pereira: Introduction to TG]
f(T) Gravity and f(T) Cosmology

- **f(T) Gravity**: Simplest torsion-based modified gravity
- Generalize T to \( f(T) \) (inspired by \( f(R) \))

\[
S = \frac{1}{16\pi G} \int d^4 x \left[ T + f(T) \right] + S_m \quad \text{[Bengochea, Ferraro PRD 79], [Linder PRD 82]}
\]

- Equations of motion:

\[
e^{-1} \partial_\mu \left( e e_A^\rho S^\mu_\rho \right) (1 + f_T) - e_A^\lambda T^\rho_\mu S^\mu_\rho + e_A^\rho S^\mu_\sigma \partial_\mu (T) f_{TT} - \frac{1}{4} e_A^\nu [T + f(T)] = 4\pi G e^\rho_\lambda T^{\nu(EM)}
\]
f(T) Gravity and f(T) Cosmology

- **f(T) Gravity**: Simplest torsion-based modified gravity

- Generalize T to $f(T)$ (inspired by $f(R)$)

$$ S = \frac{1}{16\pi G} \int d^4x \, e \left[ T + f(T) \right] + S_m \quad \text{[Bengochea, Ferraro PRD 79], [Linder PRD 82]} $$

- Equations of motion:

$$ e^{-\phi} \left( e e_A^\rho S_A^\mu \nu (1 + f_T) - e_A^\rho T^\mu \nu \rho + e_A^\rho S^\mu \nu \phi(T) f_{TT} - \frac{1}{4} e_A^\nu [T + f(T)] \right) = 4\pi G e_A^\rho T^\nu (\text{EM}) $$

- **f(T) Cosmology**: Apply in FRW geometry:

$$ e^A = \text{diag} (1, a, a, a) \Rightarrow ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j \quad \text{(not unique choice)} $$

- Friedmann equations:

$$ H^2 = \frac{8\pi G}{3} \rho_m - \frac{f(T)}{6} - 2 f_T H^2 $$

$$ \dot{H} = - \frac{4\pi G (\rho_m + p_m)}{1 + f_T - 12 H^2 f_{TT}} $$

- Find easily

$$ T = -6H^2 $$
**f(T) Cosmology: Background**

- **Effective Dark Energy sector:**
  \[ \rho_{DE} = \frac{3}{8\pi G} \left[ -\frac{f}{6} + \frac{T}{3} f_T \right] \]
  \[ w_{DE} = -\frac{f - Tf_T + 2T^2 f_{TT}}{[1 + f_T + 2Tf_{TT}][f - 2Tf_T]} \]
  [Linder PRD 82]

- Interesting cosmological behavior: **Acceleration, Inflation etc**
- At the **background level indistinguishable** from other **dynamical DE models**
Can I find imprints of f(T) gravity? Yes, but need to go to perturbation level

$$e_\mu^0 = \delta_\mu^0 (1 + \psi) \quad , \quad e_\mu^\alpha = \delta_\mu^\alpha \alpha (1 - \phi) \quad \Rightarrow \quad ds^2 = (1 + 2\psi)dt^2 - a^2 (1 - 2\phi)\delta_{ij}dx^i dx^j$$

Obtain Perturbation Equations:

$$L.H.S = R.H.S$$

... [Chen, Dent, Dutta, Saridakis PRD 83],
[Chen, Dent, Dutta, Saridakis JCAP 1101]

Focus on growth of matter overdensity \( \delta = \frac{\delta \rho_m}{\rho_m} \) go to Fourier modes:

$$3H(1 + f_T - 12H^2 f_{TT})\dot{\phi}_k + \left[(3H^2 + k^2/a^2)(1 + f_T) - 36H^4 f_{TT}\right]\phi_k + 4\pi G\rho_m \delta_k = 0$$

[Chen, Dent, Dutta, Saridakis PRD 83]
f(T) Cosmology: Perturbations

**Application:** Distinguish f(T) from quintessence

1) **Reconstruct** f(T) to **coincide** with a given **quintessence** scenario:

\[
f(H) = 16\pi GH \int \frac{\rho_0}{H^2} dH + CH \quad \text{with} \quad \rho_0 = \dot{\phi}^2/2 + V(\phi) \quad \text{and} \quad H = \sqrt{-T/6}
\]

[Dent, Dutta, Saridakis JCAP 1101]
**f(T) Cosmology: Perturbations**

- **Application:** Distinguish $f(T)$ from quintessence
- **2)** Examine evolution of matter overdensity \( \delta \equiv \frac{\delta \rho_m}{\rho_m} \)

[Dent, Dutta, Saridakis JCAP 1101]
Bounce and Cyclic behavior

- Contracting \((H < 0)\), bounce \((H = 0)\), expanding \((H > 0)\) near and at the bounce \(\dot{H} > 0\)

- Expanding \((H > 0)\), turnaround \((H = 0)\), contracting \(H < 0\) near and at the turnaround \(\dot{H} < 0\)
Bounce and Cyclic behavior in $f(T)$ cosmology

- Contracting ($H < 0$), bounce ($H = 0$), expanding ($H > 0$) near and at the bounce $\dot{H} > 0$

- Expanding ($H > 0$), turnaround ($H = 0$), contracting $H < 0$ near and at the turnaround $\dot{H} < 0$

$$H^2 = \frac{8\pi G}{3} \rho_m - \frac{f(T)}{6} - 2f_T H^2$$

$$\dot{H} = -\frac{4\pi G (\rho_m + p_m)}{1 + f_T - 12H^2 f_{TT}}$$

- Bounce and cyclicity can be easily obtained

[Cai, Chen, Dent, Dutta, Saridakis CQG 28]
Bounce in f(T) cosmology

- Start with a **bouncing scale factor**: \( a(t) = a_B \left( 1 + \frac{3}{2} \sigma t^2 \right)^{1/3} \)

\[
\Rightarrow t(T) = \pm \left( -\frac{4}{3T} - \frac{2}{3\sigma} + \frac{4\sqrt{T\sigma^3 + \sigma^4}}{3T\sigma^2} \right)
\]

\[
\Rightarrow f(t) = \frac{4t}{(2 + 3\sigma^2)M_p^2} \left[ \rho_{mb} \frac{6t M_p^2 \sigma^2}{t} + \frac{6\sigma \rho_{mb}}{2 + 3\sigma^2} \text{ArcTan} \left( \sqrt{\frac{3t}{2}} \right) \right]
\]
Bounce in f(T) cosmology

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\[
\Rightarrow f(t) = \frac{4t}{(2 + 3\sigma^2)M_p^2} \left[ \rho_{m_B} \frac{6tM_p^2\sigma^2}{t} + \frac{\rho_{m_B} \sqrt{6\sigma} \rho_{m_B} \text{ArcTan} \left( \sqrt{\frac{3s}{2t}} \right)}{2 + 3\sigma^2} \right]
\]

- Examine the full perturbations:

\[
\ddot{\phi}_k + \alpha \dot{\phi}_k + \mu^2 \phi_k + c_s^2 \frac{k^2}{a^2} \phi_k = 0
\]

with \( \alpha, \mu^2, c_s^2 \) known in terms of \( H, \dot{H}, f, f_T, f_{TT} \) and matter

\[
\left( \ddot{h}_{ij} + 3H \dot{h}_{ij} - \frac{\nabla^2}{a^2} h_{ij} \right) - \frac{12H \dot{H} f_{TT}}{1 + f_T} \dot{h} = 0
\]

\[\Rightarrow \text{Primordial power spectrum: } P_\psi = \frac{\sigma}{288\pi^2 M_p^2} \]

\[\Rightarrow \text{Tensor-to-scalar ratio: } r \approx 2.8 \times 10^{-3} \]

[Cai, Chen, Dent, Dutta, Saridakis  CQG 28]
Non-minimally coupled scalar-torsion theory

- In curvature-based gravity, apart from \( R + f(R) \) one can use \( R + \xi R \phi^2 \).
- Let’s do the same in torsion-based gravity:

\[
S = \int d^4 x \ e \left[ \frac{T}{2\kappa^2} + \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi + \xi T \phi^2 \right) - V(\phi) + L_m \right]
\]

[Geng, Lee, Saridakis, Wu PLB704]
Non-minimally coupled scalar-torsion theory

- In curvature-based gravity, apart from $R + f(R)$ one can use $R + \xi R \varphi^2$
- Let’s do the same in torsion-based gravity:

$$ S = \int d^4 x \ e \left[ \frac{T}{2\kappa^2} + \frac{1}{2} \left( \partial_{\mu} \varphi \partial^{\mu} \varphi + \xi T \varphi^2 \right) - V(\varphi) + L_m \right] $$

- Friedmann equations in FRW universe:

$$ H^2 = \frac{\kappa^2}{3} (\rho_m + \rho_{DE}) $$

$$ \dot{H} = -\frac{\kappa^2}{2} (\rho_m + p_m + \rho_{DE} + p_{DE}) $$

with effective Dark Energy sector: \( \rho_{DE} = \frac{\dot{\varphi}^2}{2} + V(\varphi) - 3\xi H^2 \varphi^2 \)

\( p_{DE} = \frac{\dot{\varphi}^2}{2} - V(\varphi) + 4\xi H \varphi \dot{\varphi} + \xi \left( 3H^2 + 2\dot{H} \right) \varphi^2 \)

- Different than non-minimal quintessence!

(no conformal transformation in the present case)

[Geng, Lee, Saridakis, Wu  PLB704]
Non-minimally coupled scalar-torsion theory

- Main advantage: Dark Energy may lie in the **phantom regime** or/and experience the **phantom-divide crossing**
- **Teleparallel Dark Energy:**

\[ \text{[Geng, Lee, Saridakis, Wu  PLB 704]} \]
Observational constraints on Teleparallel Dark Energy

- Use **observational** data (SNIa, BAO, CMB) to **constrain** the parameters of the theory
- Include **matter** and standard radiation: \( \rho_M = \rho_{M0}/a^3, \rho_r = \rho_{r0}/a^4, 1 + z = 1/a \)
- We fit \( \Omega_{M0}, \Omega_{DE0}, w_{DE0}, \xi \) for various \( V(\phi) \)
Observational constraints on Teleparallel Dark Energy

Exponential potential

Quartic potential

[Geng, Lee, Saridakis  JCAP 1201]
Phase-space analysis of Teleparallel Dark Energy

- Transform cosmological system to its autonomous form:

\[ x = \frac{\kappa \phi}{\sqrt{6}H}, \quad y = \frac{\kappa \sqrt{V(\phi)}}{\sqrt{3}H}, \quad z = \sqrt{\xi} \kappa \phi \]

\[ \Rightarrow \Omega_m \equiv \frac{\rho_m}{3H^2} = 1 - x^2 - y^2 + z^2 \operatorname{sgn}(\xi), \quad \Omega_{DE} \equiv \frac{\rho_{DE}}{3H^2} = x^2 + y^2 - z^2 \operatorname{sgn}(\xi) \]

\[ w_{DE} = w_{DE}(x, y, z, \xi) \]

\[ \Rightarrow X' = f(X), \quad X'\big|_{X = X_C} = 0 \]

- Linear Perturbations: \( X = X_C + U \quad \Rightarrow \quad U' = Q U \)

- Eigenvalues of \( Q \) determine type and stability of C.P.

[Xu, Saridakis, Leon, JCAP 1207]
Phase-space analysis of Teleparallel Dark Energy

- Apart from usual quintessence points, there exists an extra stable one for $\lambda^2 < \xi$ corresponding to $\Omega_{DE} = 1, w_{DE} = -1, q = -1$

- At the critical points $w_{DE} \geq -1$ however during the evolution it can lie in quintessence or phantom regimes, or experience the phantom-divide crossing!

[Xu, Saridakis, Leon, JCAP 1207]
Exact charged black hole solutions

- **Extend** f(T) gravity in **D-dimensions** (focus on D=3, D=4):
  \[ S = \frac{1}{2\kappa} \int d^D x \ e[T + f(T) - 2\Lambda] \]

- **Add** E/M sector: \( L_F = -\frac{1}{2} F \wedge^* F \) with \( F = dA, \ A \equiv A_\mu dx^\mu \)

- **Extract** field equations: \( \text{L.H.S} = \text{R.H.S} \)  
  
  [Gonzalez, Saridakis, Vasquez, JHEP 1207]  
  [Capozzielo, Gonzalez, Saridakis, Vasquez, JHEP 1302]
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\[ S = \frac{1}{2\kappa} \int d^D x \ e [T + f(T) - 2\Lambda] \]

- Add E/M sector: \( L_r = -\frac{1}{2} F \wedge F \) with \( F = dA, \ A = A_\mu dx^\mu \)

- Extract field equations: \( L.H.S = R.H.S \)

- Look for spherically symmetric solutions:

\[ e^0 = F(r) dt, \ e^1 = \frac{1}{G(r)} dr, \ e^2 = r dx_1, \ e^3 = r dx_2, \cdots \]

\[ \Rightarrow ds^2 = F(r)^2 dt^2 - \frac{1}{G(r)^2} dr^2 - r^2 \sum_{i=1}^{D-2} dx_i^2 \]

- Radial Electric field: \( E_r = \frac{Q}{r^{D-2}} \Rightarrow F(r)^2, G(r)^2 \) known

[Gonzalez, Saridakis, Vasquez, JHEP 1207], [Capozzielo, Gonzalez, Saridakis, Vasquez, JHEP 1302]
 Exact charged black hole solutions

- Horizon and singularity analysis:
  - 1) Vierbeins, Weitzenböck connection, Torsion invariants: 
    T(r) known $\Rightarrow$ obtain horizons and singularities
  - 2) Metric, Levi-Civita connection, Curvature invariants: 
    R(r) and Kretschmann $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}(r)$ known
    $\Rightarrow$ obtain horizons and singularities

[Gonzalez, Saridakis, Vasquez, JHEP1207], [Capozzielo, Gonzalez, Saridakis, Vasquez, JHEP 1302]
Exact charged black hole solutions

[Capozzielo, Gonzalez, Saridakis, Vasquez, JHEP 1302]
Exact charged black hole solutions

- More singularities in the curvature analysis than in torsion analysis! (some are naked)
- The differences disappear in the $f(T)=0$ case, or in the uncharged case.

- Should we go to quartic torsion invariants?

- $f(T)$ brings novel features.

- $E/M$ in torsion formulation was known to be nontrivial ($E/M$ in Einstein-Cartan and Poincaré theories)
Solar System constraints on f(T) gravity

- Apply the black hole solutions in Solar System:
- Assume corrections to TEGR of the form $f(T) = \alpha T^2 + O(T^3)$

\[ F(r)^2 = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda}{3} r^2 + \alpha \left[ -6\Lambda - \frac{6}{r^2} - \frac{4GM\Lambda}{c^2 r} \right] \]

\[ G(r)^2 = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda}{3} r^2 + \alpha \left[ \frac{8\Lambda}{3} - \frac{24}{r^2} - 2\Lambda^2 r^2 - \frac{2GM}{c^2 r} \left\{ 8\Lambda - \frac{8}{r^2} \right\} \right] \]
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\[ f(T) = \alpha T^2 + O(T^3) \]

\[ \Rightarrow F(r)^2 = 1 - \frac{2GM}{c^2r} - \frac{\Lambda}{3} r^2 + \alpha \left[ -6\Lambda - \frac{6}{r^2} - \frac{4GM\Lambda}{c^2r} \right] \]

\[ \Rightarrow G(r)^2 = 1 - \frac{2GM}{c^2r} - \frac{\Lambda}{3} r^2 + \alpha \left[ \frac{8\Lambda}{3} - \frac{24}{r^2} - 2\Lambda^2 r^2 - \frac{2GM}{c^2r} \left( 8\Lambda - \frac{8}{r^2} \right) \right] \]

- Assume corrections to TEGR of the form

- Use data from Solar System orbital motions:

\[ \Delta U_{f(T)} \leq 6.2 \times 10^{-10} \]

T<<1 so consistent

- f(T) divergence from TEGR is very small

- This was already known from cosmological observation constraints up to \( O(10^{-1} – 10^{-2}) \) [Wu, Yu, PLB 693], [Bengochea PLB 695]

- With Solar System constraints, much more stringent bound.
Growth-index constraints on $f(T)$ gravity

- Perturbations: $\ddot{\delta}_m + 2H\delta_m = 4\pi G_{\text{eff}} \rho_m \delta_m$, clustering growth rate:
  \[
  \frac{d}{d \ln a} \ln \delta_m = \Omega_m^\gamma(a)
  \]
- $\gamma(z)$: Growth index. $G_{\text{eff}} = \frac{1}{1 + f'(T)}$
Growth-index constraints on f(T) gravity

- Perturbations: \( \ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{\text{eff}} \rho_m \delta_m \), clustering growth rate:

\[
\frac{d}{d \ln a} \frac{\ln \delta_m}{\ln a} = \Omega_m^\gamma(a)
\]

- \( \gamma(z) \): Growth index. \( G_{\text{eff}} = \frac{1}{1 + f'(T)} \)

- Viable f(T) models are practically indistinguishable from \( \Lambda \text{CDM} \).

[Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]
Open issues of f(T) gravity

- f(T) cosmology is very interesting. But f(T) gravity and nonminimally coupled teleparallel gravity has many open issues [Li, Sotiriou, Barrow PRD 83a], [Geng, Lee, Saridakis, Wu PLB 704]

- For nonlinear f(T), Lorentz invariance is not satisfied
- Equivalently, the vierbein choices corresponding to the same metric are not equivalent (extra degrees of freedom) [Li, Sotiriou, Barrow PRD 83c], [Li, Miao, Miao JHEP 1107]
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- Black holes are found to have different behavior through curvature and torsion analysis [Capozziello, Gonzalez, Saridakis, Vasquez JHEP 1302]

- Thermodynamics also raises issues [Bamba, Geng JCAP 1111], [Miao, Li, Miao JCAP 1111]

- Cosmological, Solar System and Growth Index observations constraint f(T) very close to linear-in-T form
Gravity modification in terms of torsion?

- So can we modify gravity starting from its torsion formulation?

- The simplest, a bit naïve approach, through f(T) gravity is interesting, but has open issues.

- Additionally, f(T) gravity is not in correspondence with f(R).

- Even if we find a way to modify gravity in terms of torsion, will it be still in 1-1 correspondence with curvature-based modification?

- What about higher-order corrections, but using torsion invariants (similar to Gauss Bonnet, Lovelock, Hordenski modifications)?

- Can we modify gauge theories of gravity themselves? E.g. can we modify Poincaré gauge theory?
Conclusions

i) **Torsion** appears in all approaches to **gauge gravity**, i.e to the first step of quantization.

ii) Can we **modify** gravity based in its **torsion formulation**?

iii) Simplest choice: **f(T) gravity**, i.e extension of **TEGR**

iv) **f(T) cosmology**: Interesting phenomenology. Signatures in growth structure.

v) We can obtain **bouncing solutions**

vi) **Non-minimal coupled scalar-torsion theory** $\mathcal{T} + \xi \mathcal{T} \phi^2$: Quintessence, phantom or crossing behavior.

vii) Exact **black hole** solutions. **Curvature vs torsion** analysis.

viii) **Solar system constraints**: $f(T)$ divergence from $\mathcal{T}$ less than $10^{-10}$

ix) **Growth Index constraints**: Viable $f(T)$ models are practically indistinguishable from $\Lambda$CDM.

x) Many **open issues**. Need to search for other torsion-based modifications too.
Outlook

- Many subjects are open. Amongst them:
  - i) Examine thermodynamics thoroughly.
  - ii) Extend $f(T)$ gravity in the braneworld.
  - iii) Understand the extra degrees of freedom and the extension to non-diagonal vierbeins.
  - iv) Try to modify TEGR using higher-order torsion invariants.
  - v) Try to modify Poincaré gauge theory (extremely hard!)
  - vi) What to quantize? Metric, vierbeins, or connection?
  - vii) Convince people to work on the subject!
THANK YOU!