Development of a multidimensional relativistic radiative transfer code

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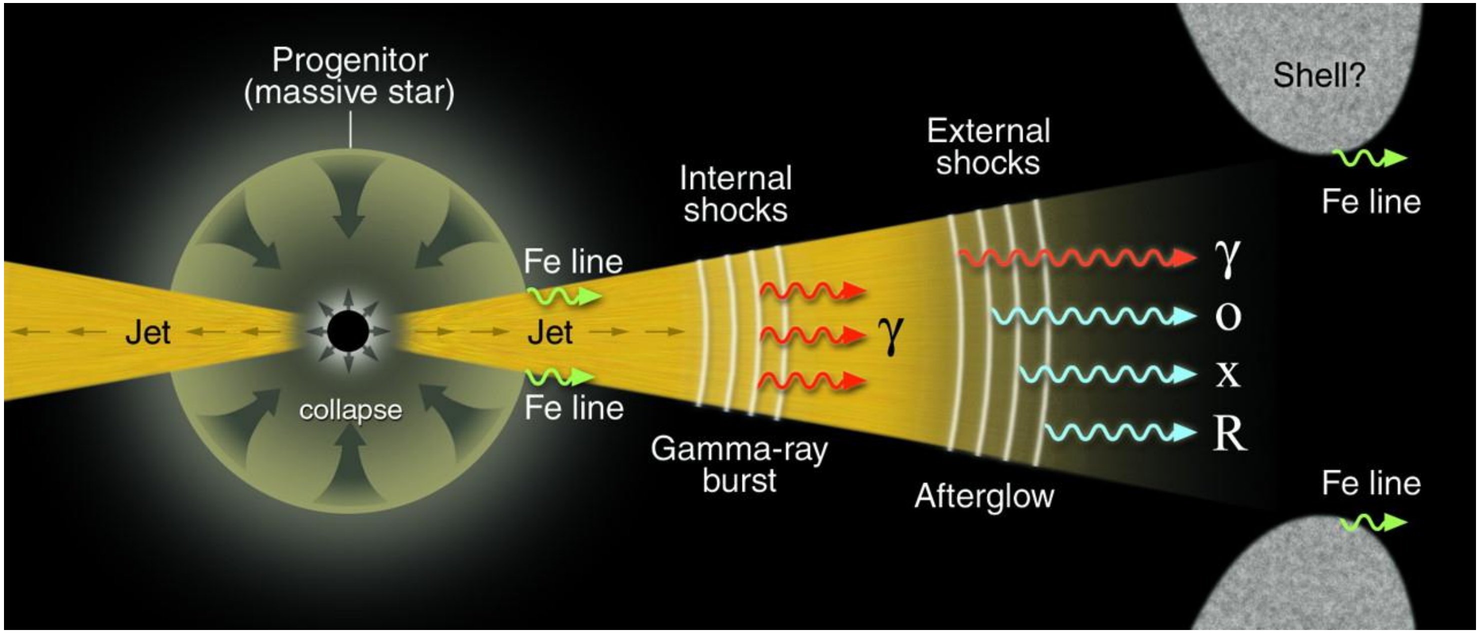
Outline

* Motivation
  * Radiation in gamma-ray bursts
* Multidimensional relativistic RT cal.
  * Spherical Harmonic Discrete Ordinate Method (SHDOM)
  * Implementation of physical processes for relativistic radiative transfer cal.
* Test problems

Gamma-ray bursts (GRBs)

- Gamma-ray emission from relativistic jets



- The emission mechanism is under debate.
- We need to study it with quantitative comparisons between obs. and theory.

Previous numerical studies

- Relativistic Monte Carlo simulation (on steady flow)

E.g., Giannois06; Pe'er08; Beloborodov10; Ito+13; Ruffini+13; Shibata, NT in prep. (Talk on GRB II)

- Relativistic hydrodynamics + a superposition of black body radiation from a surface (e.g., $\tau_{sc}=1$)

E.g., Lazzati+11; Mizuta+11; Nagakura+11

- Spherical relativistic radiation hydrodynamics

Tolstov+13 (rel. rad. transfer: Beloborodov11)

However, GRBs involve **relativistic jets with $E_r \sim E_m$**.
Therefore, a multid. rela. rad. hyd. cal. is necessary.
Since even a **multid. rela. rad. trans. cal.** is not available, we develop it as the 1st step toward GRB rad. cal..

Multidimensional multigroup radiative transfer equation

* 6-dimensional Boltzmann equation

$$\frac{1}{c}\frac{\partial I\left(t,s,\nu,\Omega\right)}{\partial t}+\boldsymbol{n}\cdot\nabla I\left(t,s,\nu,\Omega\right)$$
$$=\eta\left(t,s,\nu,\Omega\right)-\sigma_{\mathrm{a}}\left(t,s,\nu,\Omega\right)I\left(t,s,\nu,\Omega\right)$$
$$+\int_0^\infty d\nu'\int d\Omega'\left[\frac{\nu}{\nu'}\sigma_{\mathrm{s}}\left(t,s,\nu'\to\nu,\Omega'\to\Omega\right)I\left(t,s,\nu',\Omega'\right)\right.$$
$$\left.-\sigma_{\mathrm{s}}\left(t,s,\nu\to\nu',\Omega\to\Omega'\right)I\left(t,s,\nu,\Omega\right)\right]$$

Pomraning73

* We concentrate on spatially 2-dimensional equation (3-dimension for photon direction).

Spherical Harmonic Discrete Ordinate Method (SHDOM)

* Solve static monochromatic radiative transfer eq.

$$\boldsymbol{n} \cdot \nabla I\left(s, \Omega\right) = \eta_{\mathrm{all}}\left(s, \Omega\right) - \alpha\left(s, \Omega\right) I\left(s, \Omega\right)$$

Evans+98, Pincus&Evans09

  * Ray tracing in a discrete ordinate space

$$I\left(s\right) = \exp\left[-\int_0^s \alpha\left(s'\right) ds'\right] I(0) + \int_0^s \exp\left[-\int_{s'}^s \alpha\left(s''\right) ds''\right] S\left(s'\right) \alpha\left(s'\right) ds'$$

  * Deriving source function with spherical harmonics expansion

$$S\left(\Omega\right) = \sum_{lm} Y_{lm}\left(\Omega\right) S_{lm} \quad P\left(\cos\Theta\right) = \sum_{l=0}^{N_{\mathrm{L}}} \chi_l \mathscr{P}_l\left(\cos\Theta\right)$$

$$S_{lm} = \frac{\omega \chi_l}{2l+1} I_{lm} + T_{lm}$$

In order to apply to GRBs,

several revisions are required.

1. Time dependence
   * Light velocity is finite for the relativistic medium.
2. Lorentz transformation
   * Relativistic beaming
   * Energy variation
3. Compton scattering
   * Photon energy is as high as electron rest mass.

1. Time dependence

- Time-dependent radiative transfer eq.

$$\frac{1}{c}\frac{\partial I\left(s,\nu,\Omega\right)}{\partial t}+\boldsymbol{n}\cdot\nabla I\left(s,\nu,\Omega\right)=\eta_{\mathrm{all}}\left(s,\nu,\Omega\right)-\alpha\left(s,\nu,\Omega\right)I\left(s,\nu,\Omega\right)$$

- First, we discretize the time-derivative term,

$$\frac{\partial I(s,\nu,\Omega)}{\partial t}=\frac{I^{n+1}(s,\nu,\Omega)-I^{n}(s,\nu,\Omega)}{\Delta t}$$

$$\tilde{\alpha}=\alpha+\frac{1}{c\Delta t}\qquad\tilde{\eta}_{\mathrm{all}}=\eta_{\mathrm{all}}+\frac{I^{n}}{c\Delta t}$$

$$\boldsymbol{n}\cdot\nabla I^{n+1}=\tilde{\eta}_{\mathrm{all}}-\tilde{\alpha}I^{n+1}$$

Baron+09, Hiller&Dessart12, Jack+12

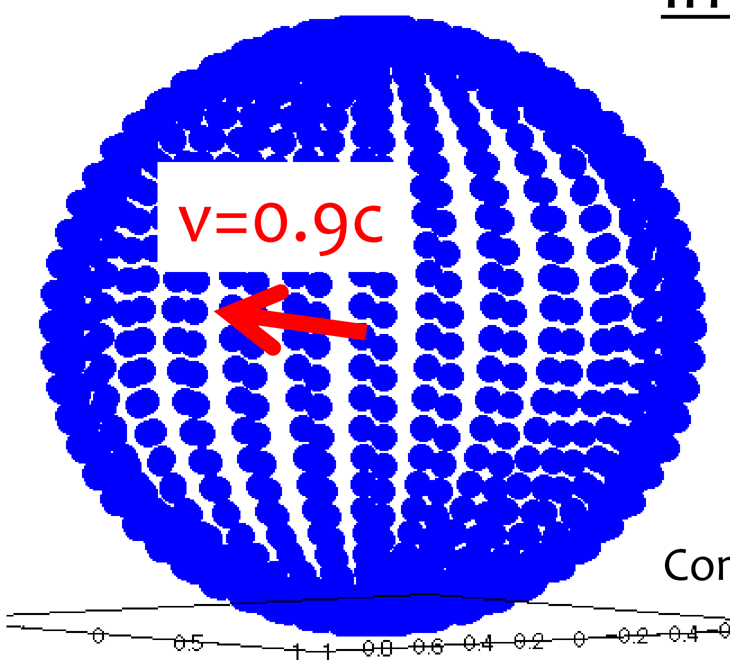
2. Lorentz transformation

- Photon energy
- Direction

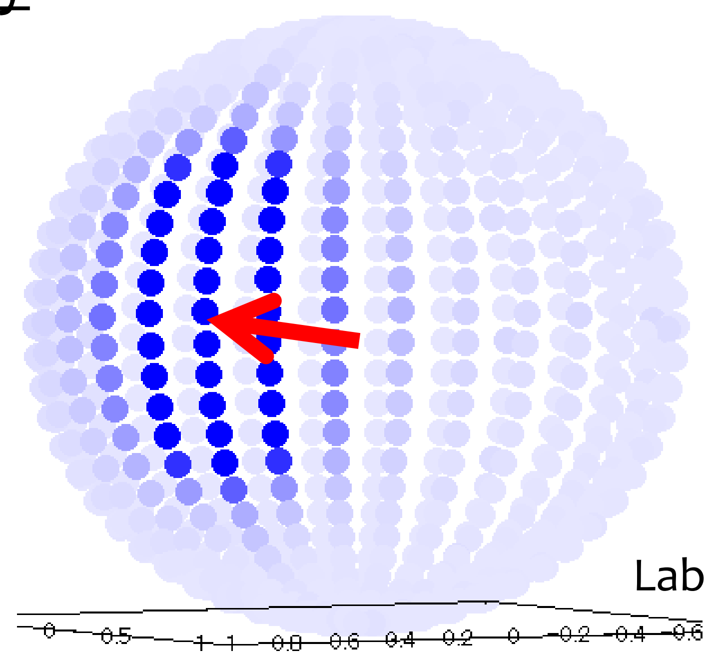
$$\nu_0 = \Gamma\nu\left(1 - \frac{\boldsymbol{n}\cdot\boldsymbol{v}}{c}\right) \quad \boldsymbol{n}_0 = \frac{\nu}{\nu_0}\left\{\boldsymbol{n} - \Gamma\frac{\boldsymbol{v}}{c}\left[1 - \frac{\Gamma}{\Gamma+1}\frac{\boldsymbol{n}\cdot\boldsymbol{v}}{c}\right]\right\}$$

E.g., Mihalas & Weibel Miharas 84

<u>Intensity</u>



Comoving frame



Laboratory frame

3. Compton scattering

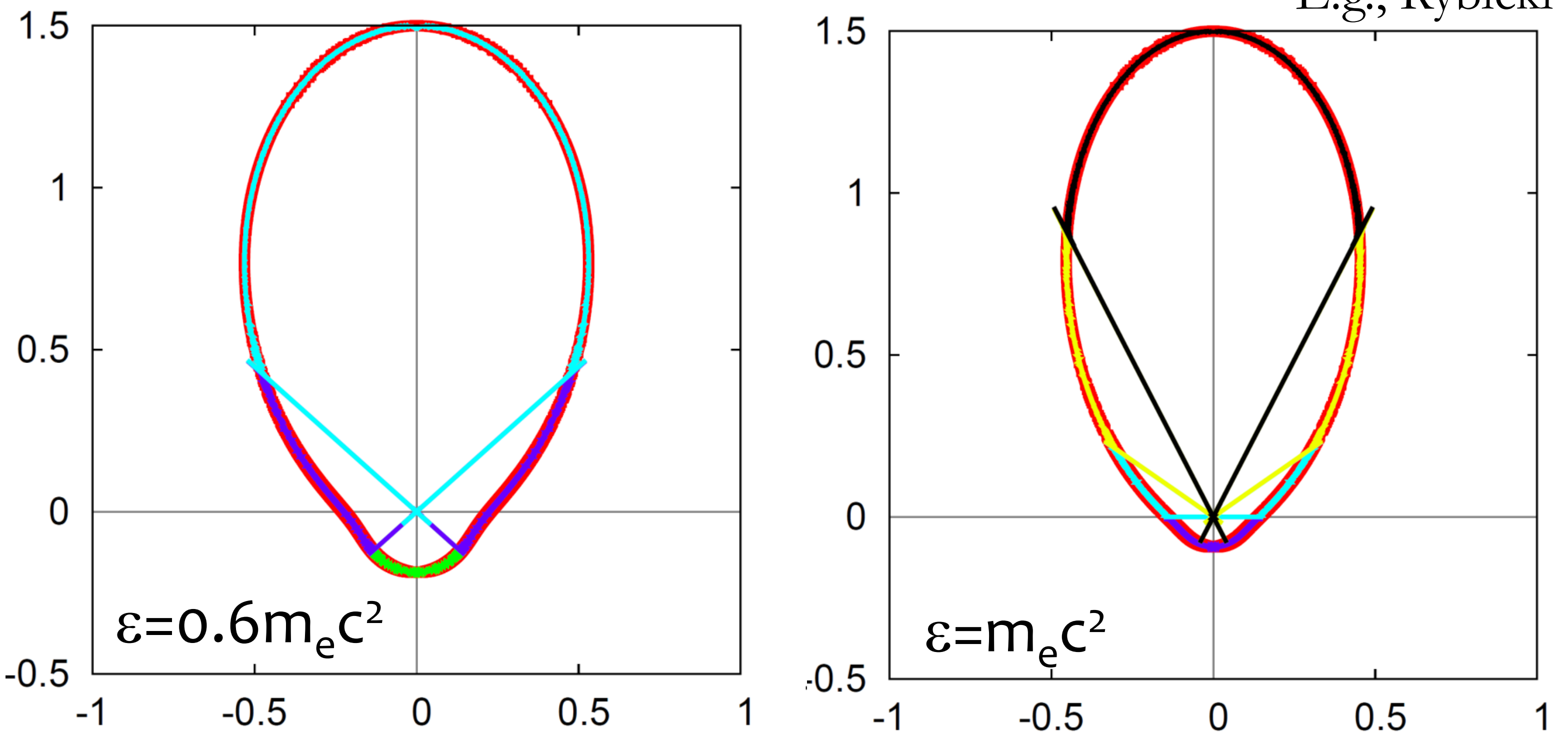
- Photon energy

$$\varepsilon_1 = \frac{\varepsilon}{1 + \frac{\varepsilon}{mc^2}(1 - \cos\theta)}$$

- Differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\frac{\varepsilon_1}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon_1} + \frac{\varepsilon_1}{\varepsilon} - \sin^2\theta\right)$$

E.g., Rybicki & Lightman 85



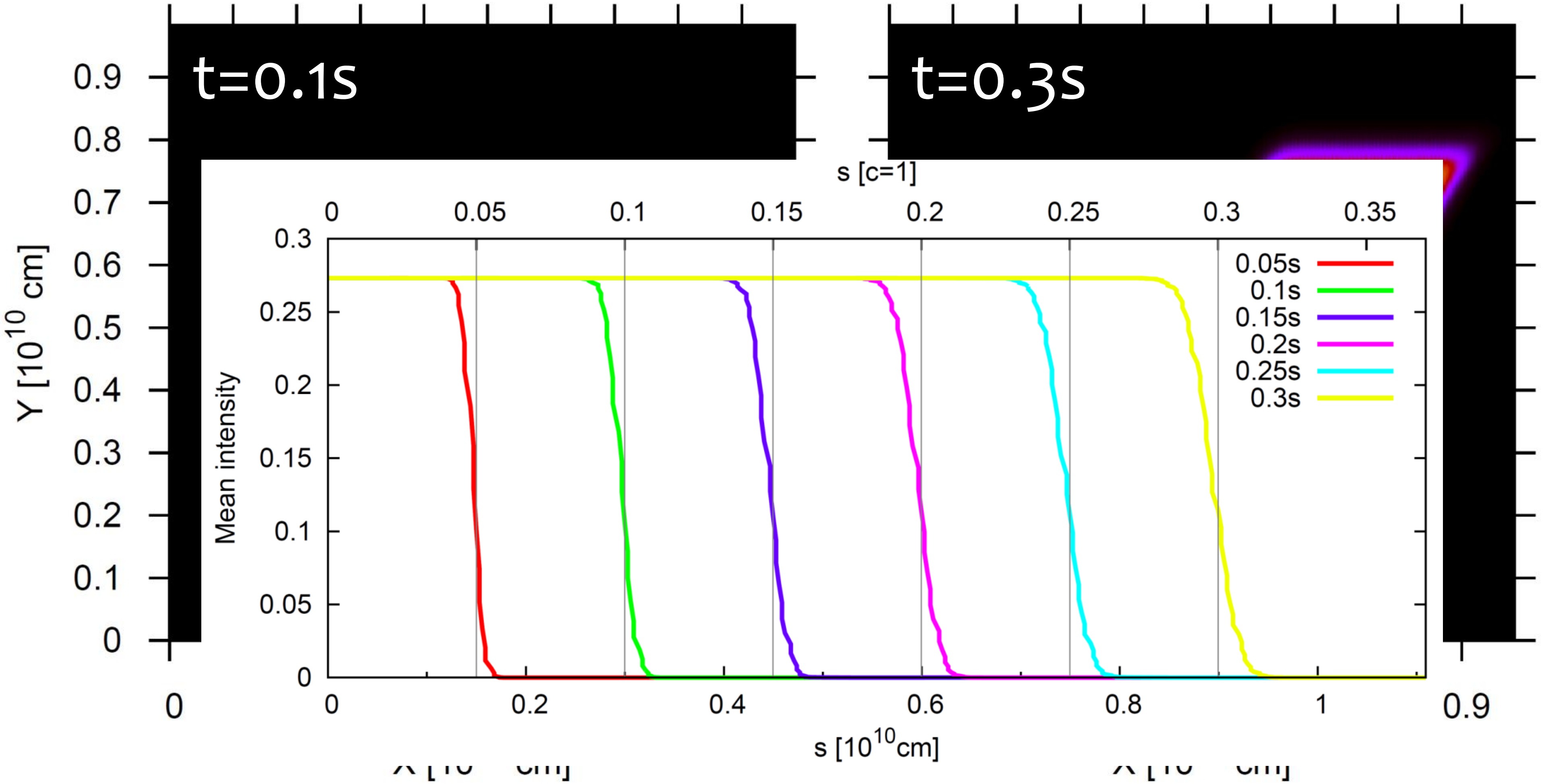
Intensity

ε_1=0.2m_ec^2
0.4m_ec^2
0.6m_ec^2
0.8m_ec^2
1.0m_ec^2

Test problems

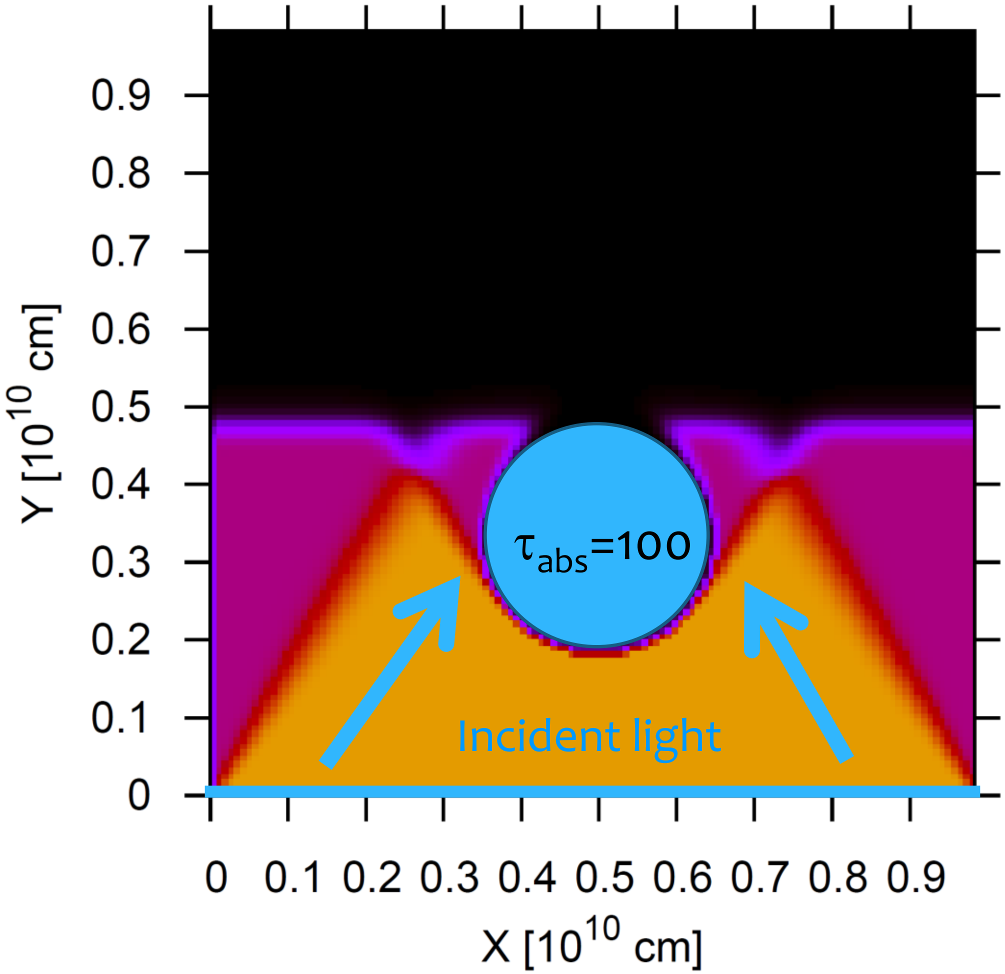
Beam test

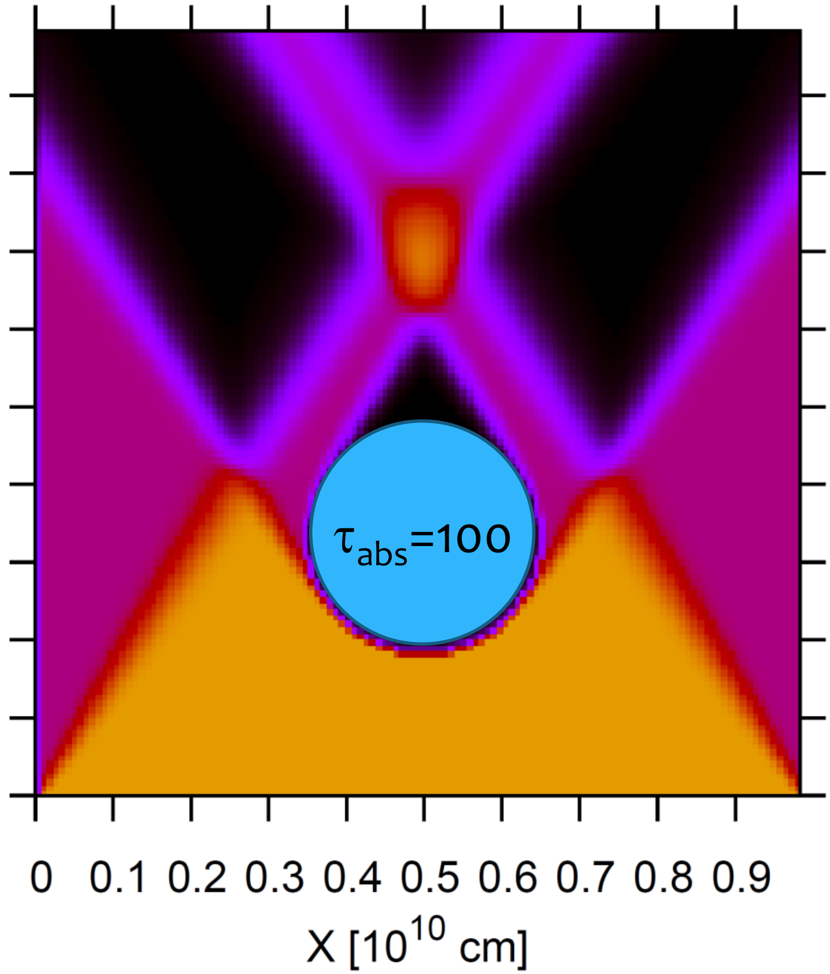
- Optically thin medium: 512(x)x512(y)x4(θ)x8(ϕ)



Two beam with shadow

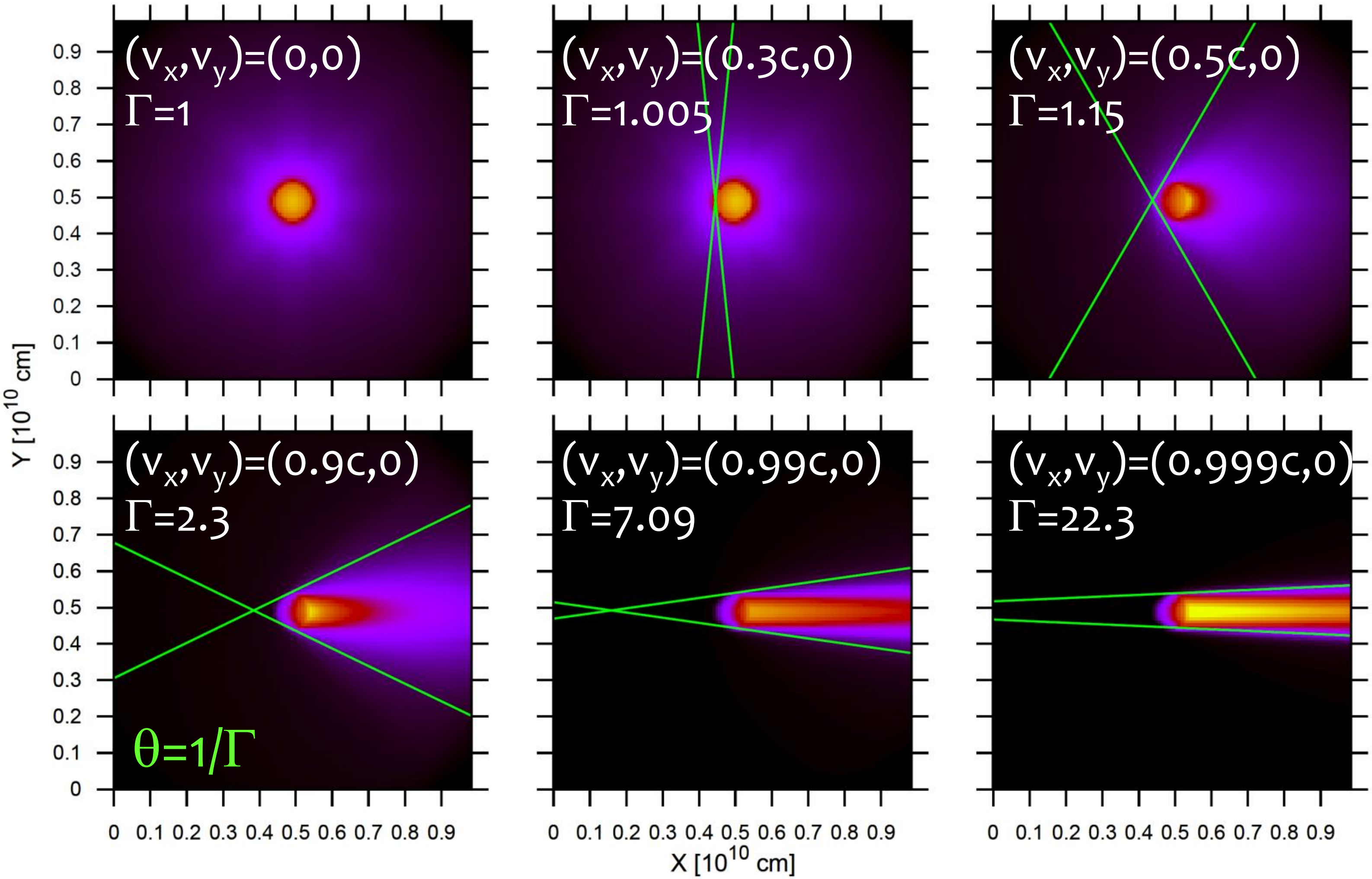
- Optically thick cylinder:128(x)x128(y)x32(θ)x64(ϕ)





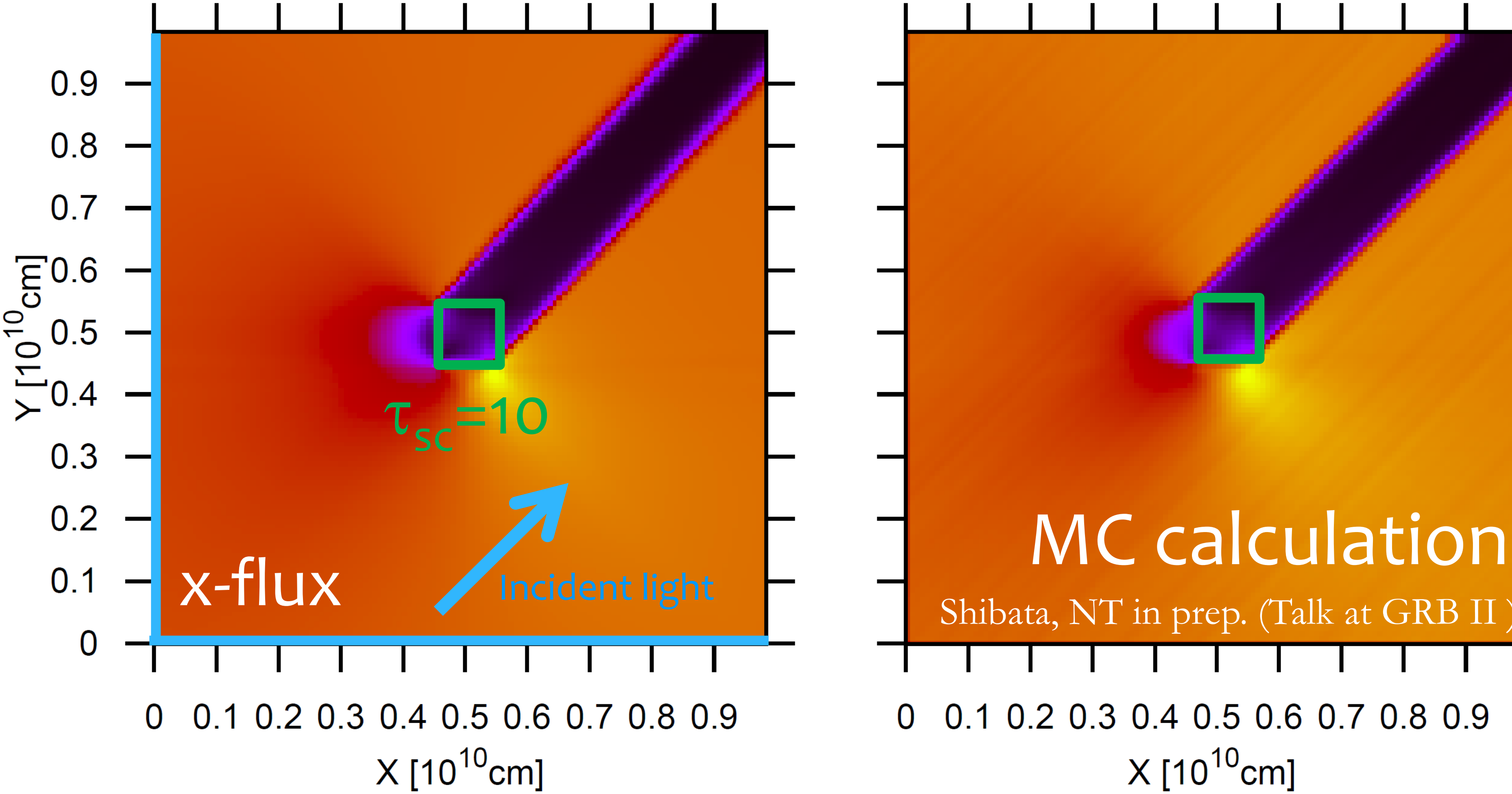
Relativistic beaming

* Thermal cylinder: 128(x)x128(y)x32(θ)x64(φ)



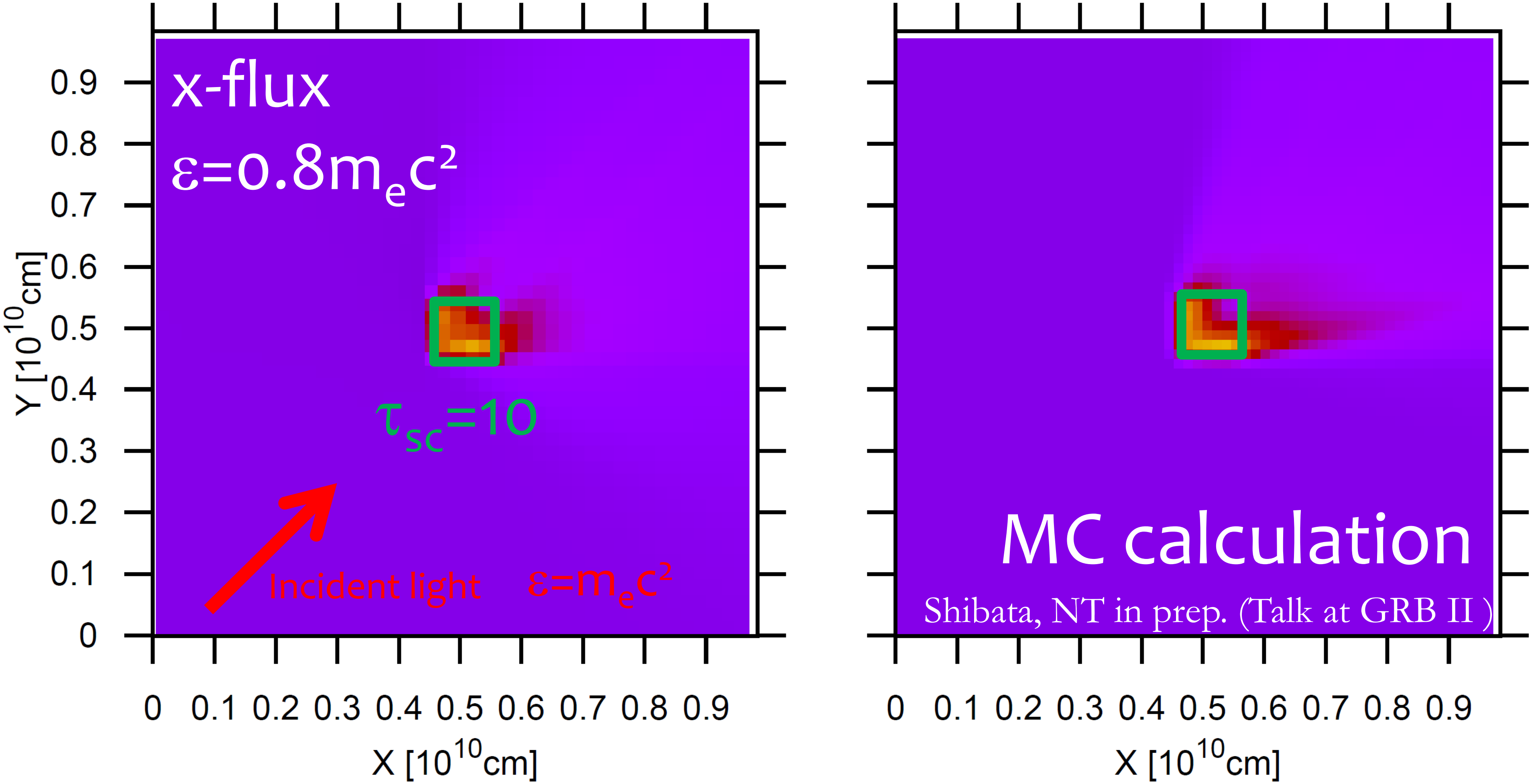
Comparison with MC cal. - Thomson scattering-

* Optically thick cuboid: 128(x)x128(y)x16(θ)x32(ϕ)



Comparison with MC cal. -Compton scattering-

* Optically thick cuboid: 64(x)x64(y)x64(θ)x128(φ)x10(ν)



Summary

* A multidimensional time-dependent relativistic radiative transfer code is essential to numerically study the radiation from GRBs.
* We develop the code with implementing time dependence, Lorentz transformation, and compton scattering to SHDOM code.
* This is the 1st step to realize a multidimensional relativistic radiation hydrodynamics calculation.
* Next step: we will incorporate the code with a relativistic hydrodynamics code (NT09).