Self-forced evolutions for comparable and intermediate mass-ratio coalescences

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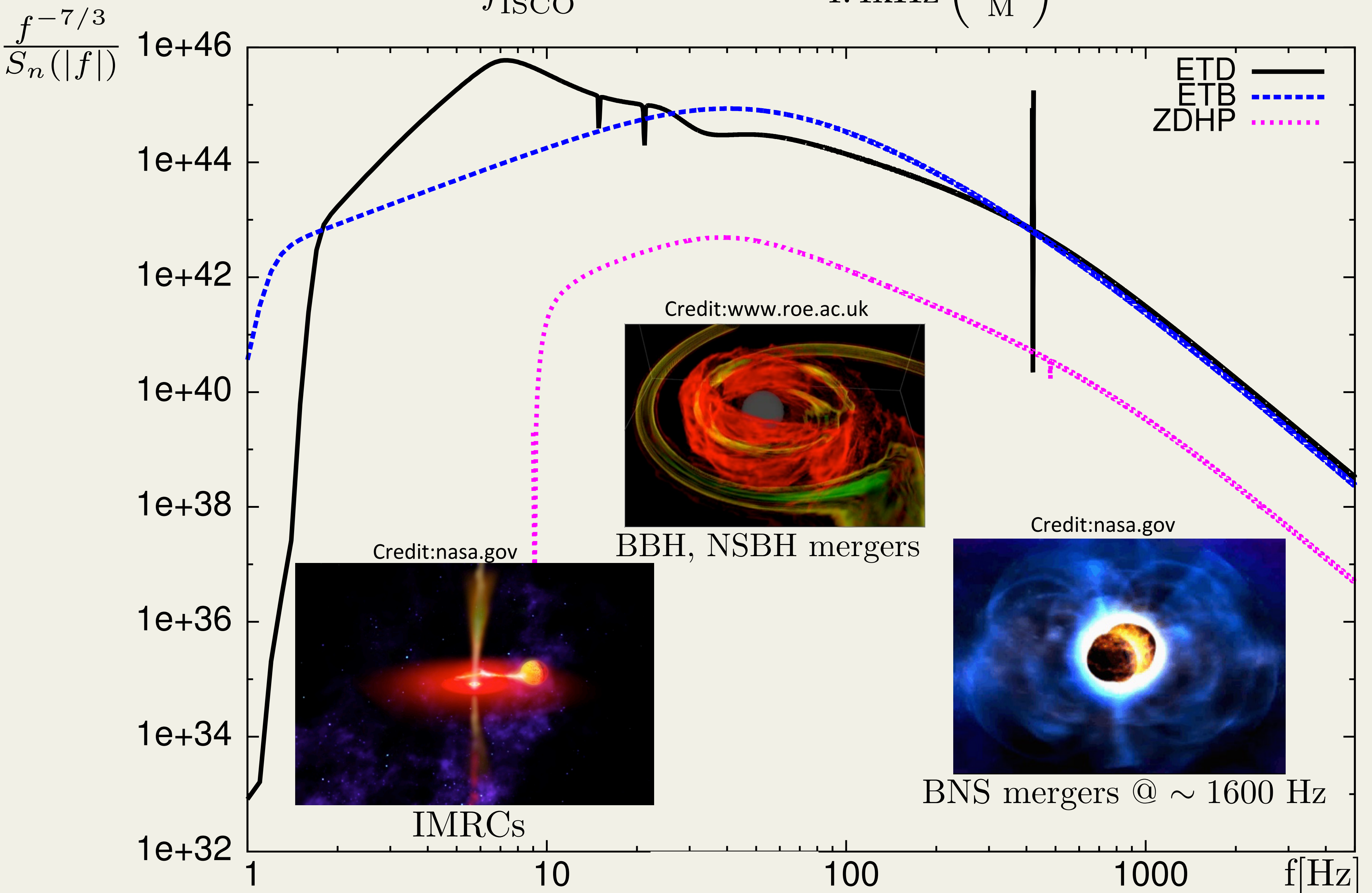
Motivation

Second generation gravitational wave (GW) detectors may directly detect gravitational radiation emitted by the inspiral of Binary Neutron Stars (BNSs), inspiral-merger-ringdown (IMR) of Binary Black Holes (BBHs), NS-BH binaries and Intermediate Mass Ratio Coalescences (IMRCs) (CQG 27:173001,2010; ApJ 681: 1431,2008)

Event	Realistic event rate per year	Optimal Horizon Distance [Mpc]
Inspiral of BNS	40	445
IMR of NSBH	10	927
IMR of BBH	20	2187
IMR of IMRC	1 - 10	2822

Sensitivity of ground-based GW detectors

$$f_{\rm ISCO}^{\rm dominant\ harmonic} = 4.4{\rm kHz}\left(\frac{M_\odot}{M}\right)$$



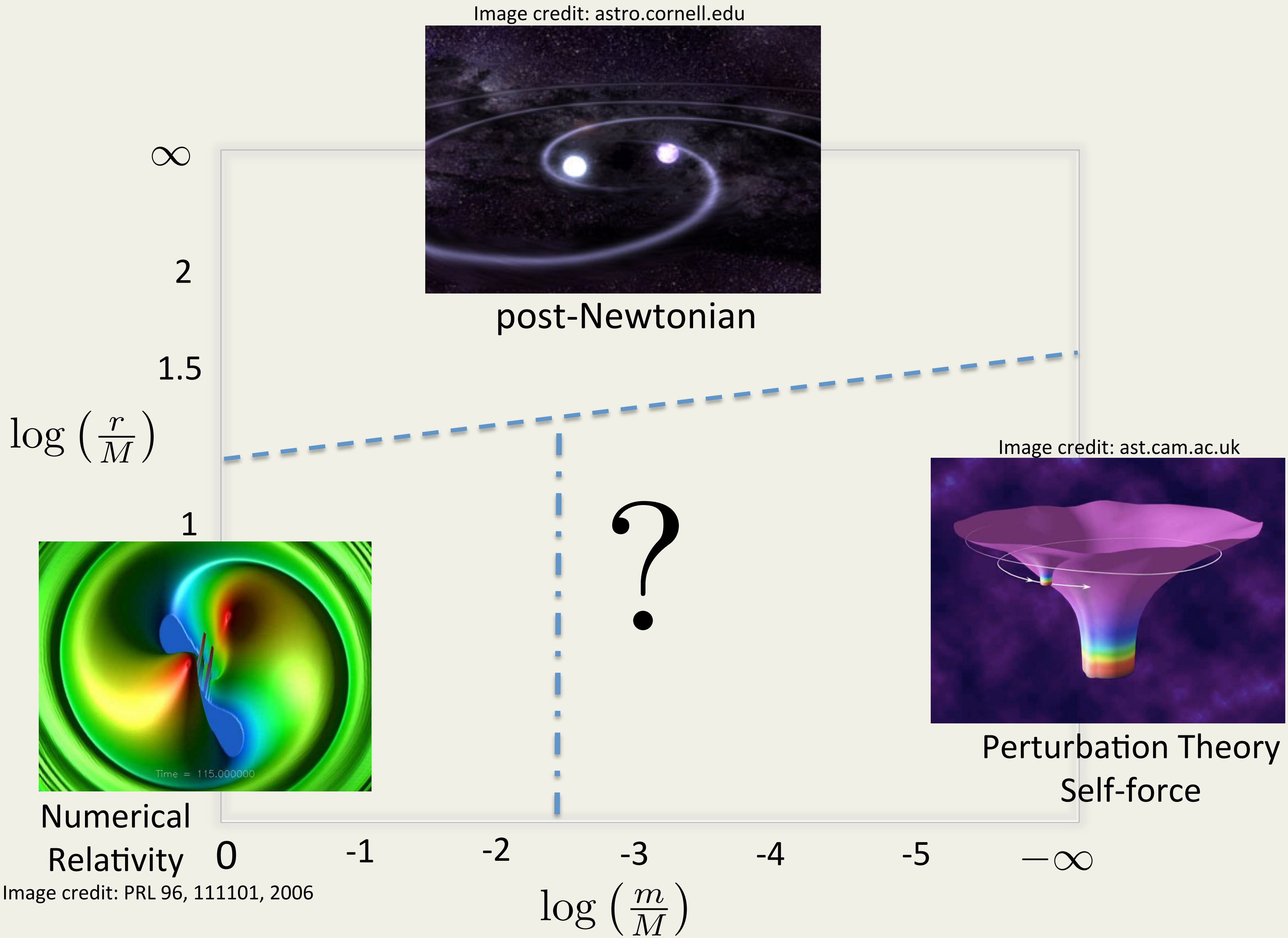
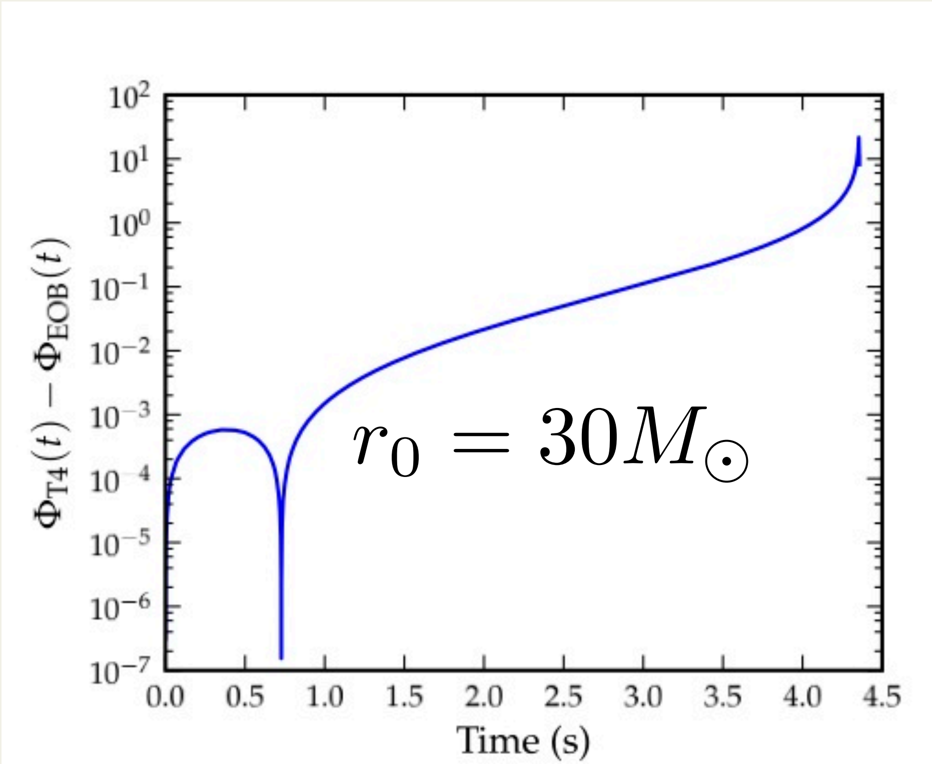


Image credit: astro.cornell.edu
post-Newtonian
∞
2
1.5
$\log\left(\frac{r}{M}\right)$
1
?
Image credit: ast.cam.ac.uk
Perturbation Theory
Self-force
Time = 115.000000
Numerical
Relativity
0
-1
-2
-3
-4
-5
$-\infty$
$\log\left(\frac{m}{M}\right)$
Image credit: PRL 96, 111101, 2006

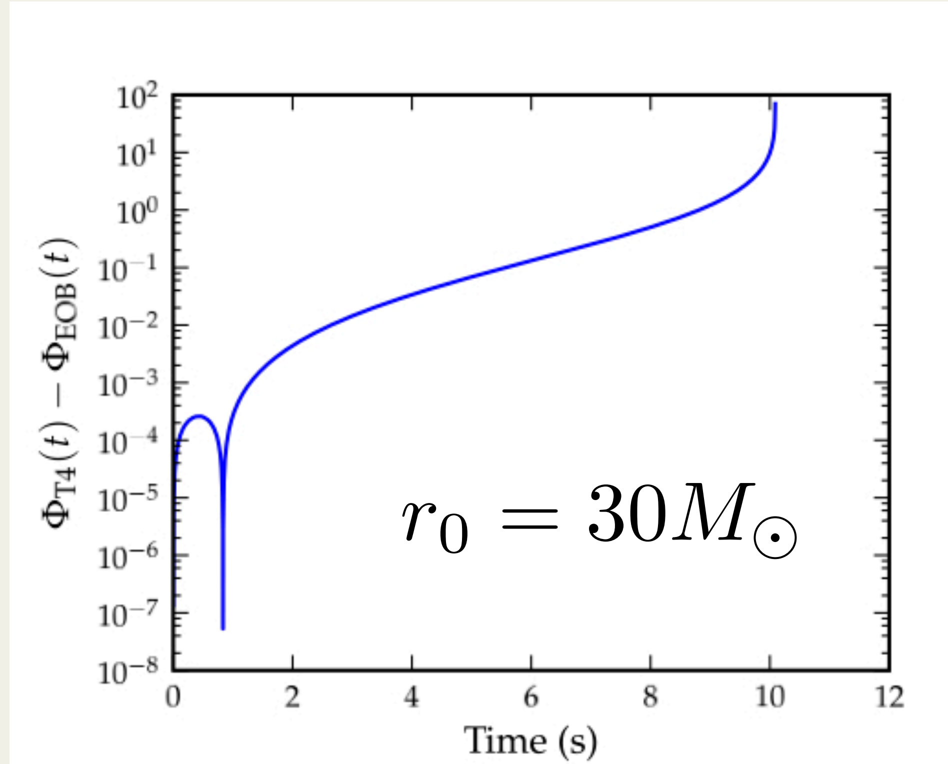
Motivation

PN expansions are not reliable for moderate and highly asymmetric mass-ratio systems

$(1M_\odot, 6M_\odot)$



$(1M_\odot, 10M_\odot)$



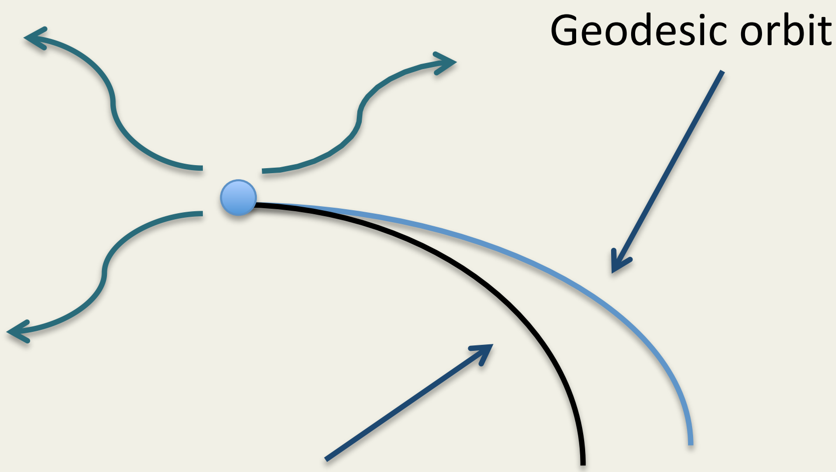
Motivation

- Effective One Body formalism developed by Damour et al provides a possible analytic fitting framework for binaries with mass-ratios 1:1 to 1:8.

- We propose an approach that is entirely based on the self-force (SF) formalism to construct a complete waveform model from inspiral to ringdown, and which provides a unified framework to model binaries with mass-ratios that range from the comparable to the extreme mass-ratio limit.

What is the self-force

Electromagnetic self-force



- An accelerated charge generates a field that behaves as outgoing radiation in the wave zone and removes energy from the particle
- In the near zone this field interacts with the particle and gives rise to a SF that prevents it from following a geodesic of the background spacetime
- Electromagnetic SF: accelerated charge interacts with its own field

Gravitational self-force

- In curved spacetime we use the dictionary

Charge of the particle → mass of the inspiraling object
Vector potential → metric perturbation

- Up-to-date formalism is valid at first order in mass-ratio and takes into account the internal structure of compact objects. SF conservative corrections are now known all the way to the light-ring: Phys. Rev. D 86, 104041

- SF has two components: radiative (loss of energy and angular momentum) and conservative (shift in orbital parameters)

- Interpretation: accelerated motion on a background spacetime or geodesic motion in a perturbed spacetime

Construction of waveform model

The orbital elements, energy E and angular momentum L are given by:

$$E(x,\eta) = \frac{1-2x}{\sqrt{1-3x}} - 1 + \eta E_{\mathrm{SF}}(x,\eta) \qquad L(x,\eta) = \frac{1}{\sqrt{x(1-3x)}} + \eta L_{\mathrm{SF}}(x,\eta)$$

$$E_{\mathrm{SF}} = E_{\mathrm{SF}}(a(x),\eta), \quad L_{\mathrm{SF}} = L_{\mathrm{SF}}(a(x),\eta) \quad \text{with} \quad \mathrm{a(x)} \propto \mathrm{h}^{\mathrm{G,R}}_{\alpha,\beta}\mathrm{u}^{\alpha}\mathrm{u}^{\beta}$$

The metric perturbation is defined by $g_{\alpha\beta} \to \eta_{\alpha\beta} + h_{\alpha\beta}$

The gauge invariant frequency parameter x is given by

$$x = (M\Omega)^{2/3}$$

Construction of the waveform

These expressions for the orbital elements reproduce with great accuracy:

- the energy and angular momentum evolution extracted by numerical simulations (Phys.Rev.Lett.108:131103,2012)

- the periastron advance of spinless binaries with mass-ratios 1:1 to 1:8 (Phys.Rev.Lett.107:141101,2011)

Construction of waveform model

The orbital evolution is given by:

$$\frac{d\phi}{d\mathrm{t}} = \frac{u^2(1-2u)L(x,\eta)}{E(x,\eta)}$$

with

$$u(x) = x\left\{1 + \eta\left(\tfrac{1}{6}a'(x) + \tfrac{2}{3}\left(\tfrac{1-2x}{\sqrt{1-3x}} - 1\right)\right)\right\}$$

The inspiral trajectory is generated using the simple relation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}E}{\mathrm{d}t}\,\frac{\mathrm{d}x}{\mathrm{d}E}$$

Waveform construction

Expanding the flux F(x) and orbital elements:

First order radiative SF:
Teukolsky fluxes

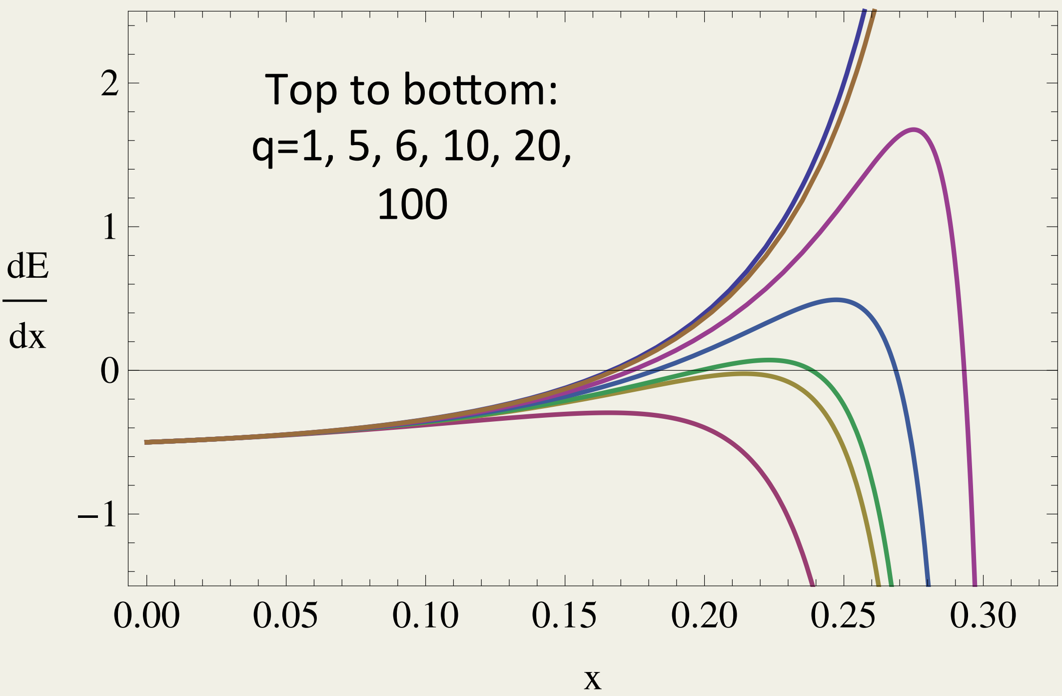
$$F(x) \to F_{(1)}(\eta^2) + F_{(2)}(\eta^3)$$

$$E(x,\eta) \to E_{(1)}(\eta) + E_{(2)}(\eta^2)$$

First order
conservative SF
enters here!

We only know first order corrections to the energy flux. We have derived second order dissipative corrections by ensuring that the phase evolution of the self-force model reproduces results from numerical simulations

Transition to merger



Crossing point at the ISCO:

$$x = \frac{1}{6}\left(1 + 0.83401\eta + 4.59483\eta^2\right)$$

In the vicinity of the ISCO the inspiral scheme breaks down

Smoothly connect late inspiral evolution with the plunge phase by extending the transition scheme of Ori and Thorne (PRD 62, 124022, 2000)

Transition scheme

Radial evolution is governed by

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = -\frac{\partial V}{\partial r} + \eta F_{\mathrm{SF}}$$

where the effective potential is given by

$$V(r, E, L) = \left(1 - \frac{2}{r}\right)\left(1 + \frac{L^2}{r^2}\right)$$

and

$$u(x) = x\left\{1 + \eta\left(\tfrac{1}{6}a'(x) + \tfrac{2}{3}\left(\tfrac{1-2x}{\sqrt{1-3x}} - 1\right)\right)\right\}$$

Linearise the orbital elements and the effective potential using the relations:

$$E(x) \to E(x_{\mathrm{ISCO}}) + \Omega_{\mathrm{ISCO}}\,\xi, \qquad L(x) \to L(x_{\mathrm{ISCO}}) + \xi$$

Model the orbital phase evolution from the transition scheme to the light ring using the scheme developed by Baker et al (PRD 78, 044046, 2008)

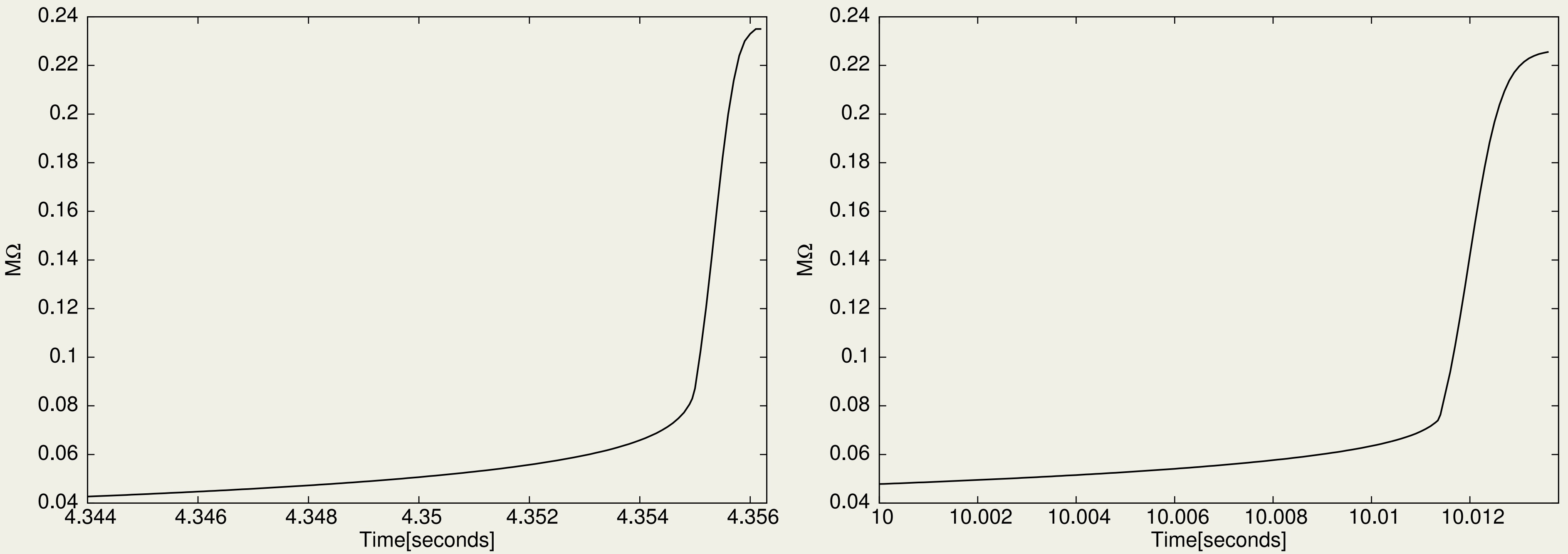
$$\frac{d\phi}{d\mathrm{t}} \to \Omega_{\mathrm{f}} - (\Omega_{\mathrm{f}} - \Omega_{\mathrm{i}})\, e^{-2(t-t_0)/b}$$

Plunge

Radial evolution is determined by modified geodesic equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{u^2(1-2u)}{E(x,\eta)}\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^{-1}\left[E(x,\eta)^2 - V\left(u(x), E(x,\eta), L(x,\eta)\right)\right]^{1/2}$$

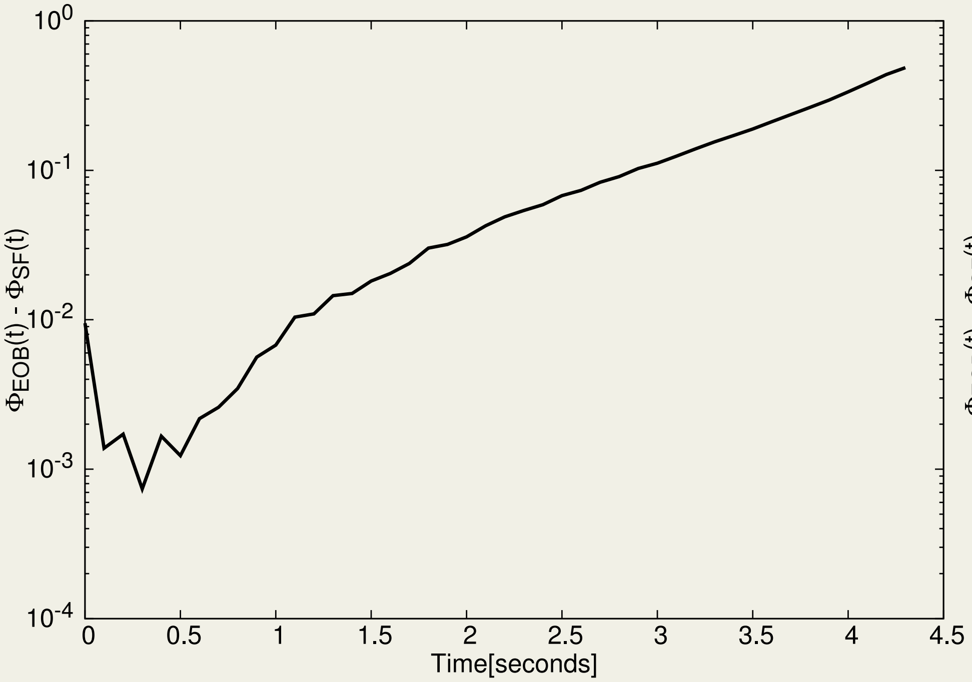
The orbital frequency saturates prior to reaching the light ring. Attachment of quasinormal modes is then consistent



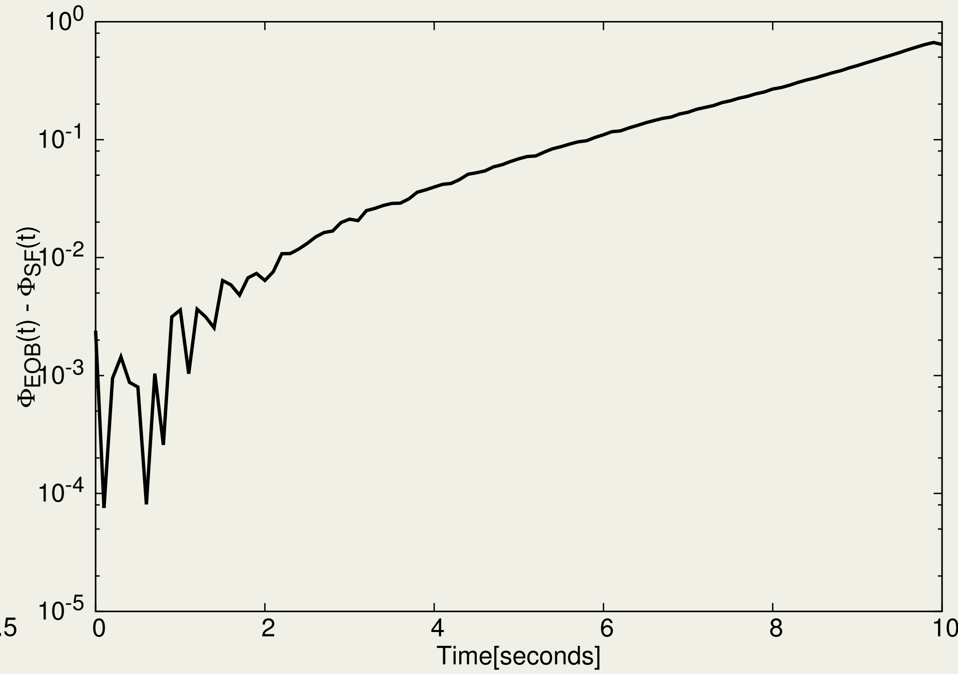
Phase evolution

Comparison between self-forced model and EOBNRv2 from r=30M to the light ring. The phase discrepancy is less than 1rad at the light ring: numerical relativity simulations have numerical error ~0.5rad at the same point

$(1M_\odot, 6M_\odot)$



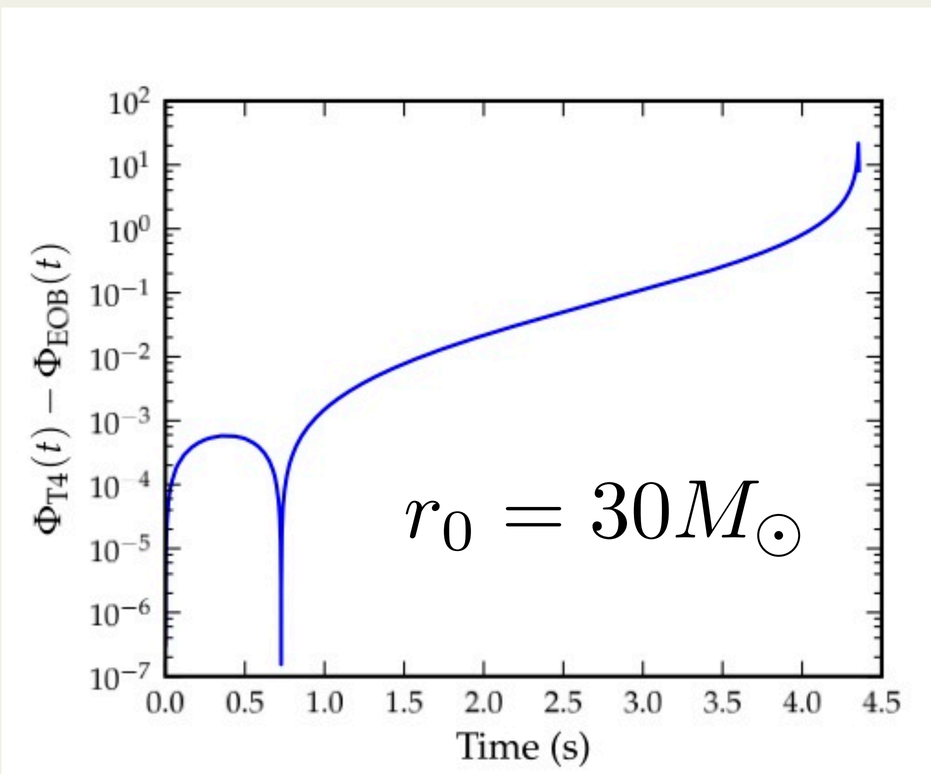
$(1M_\odot, 10M_\odot)$



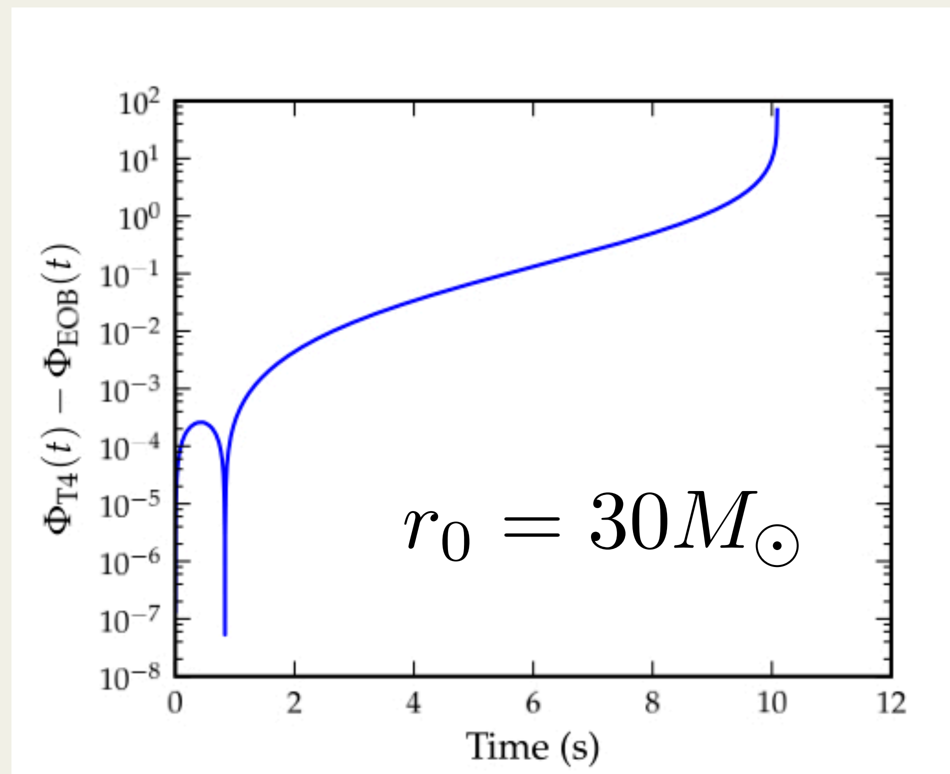
TaylorT4 vs EOBNRv2

PN expansions are not reliable for moderate and highly asymmetric mass-ratio systems

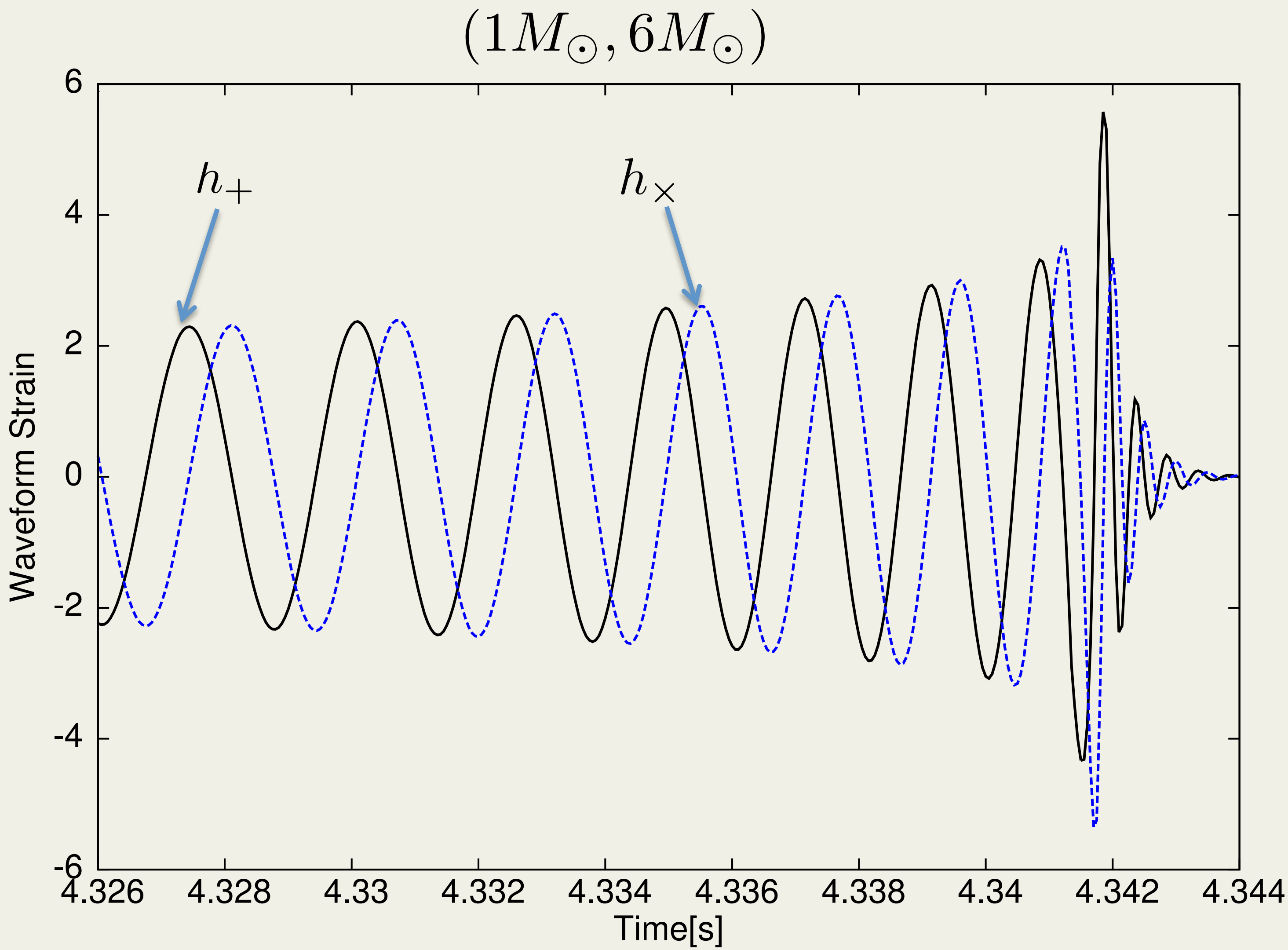
$(1M_\odot, 6M_\odot)$



$(1M_\odot, 10M_\odot)$

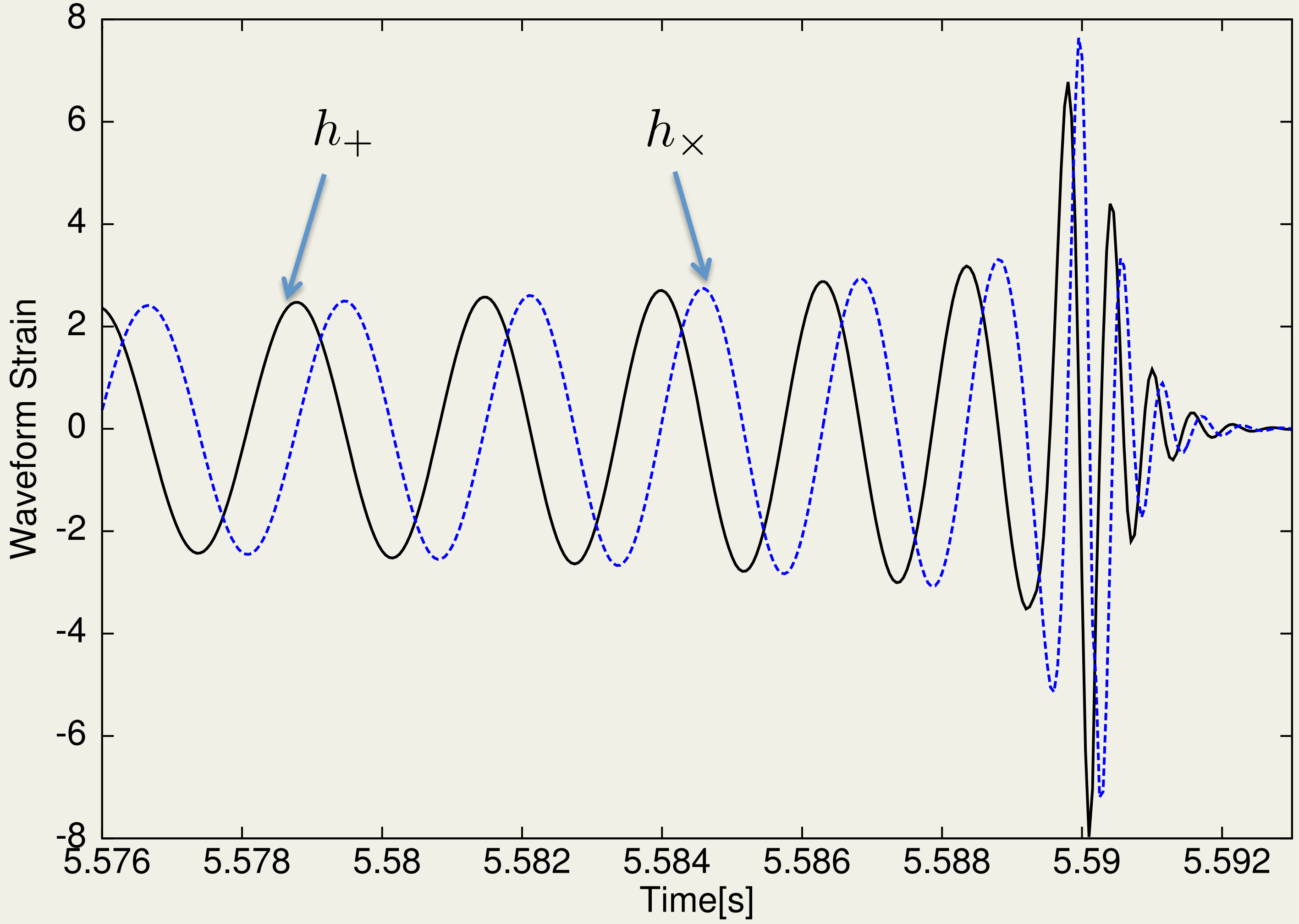


Complete waveforms



Complete waveforms

$(1M_\odot, 7M_\odot)$



Conclusions

- The self-force formalism provides the appropriate framework to model in a unified way the inspiral, merger and ringdown of compact binaries with extreme to comparable mass-ratios

- We have presented results that suggest that formally computing the second order corrections to the metric pertubation $h_{\alpha\beta}$ from the relation

$$g_{\alpha\beta} \to \eta_{\alpha\beta} + h_{\alpha\beta}$$

will enable us to accurately describe the gravitational radiation emitted by sources that may be detected by second and third generation ground-based GW detectors