A new crystalline phase in magnetar crusts

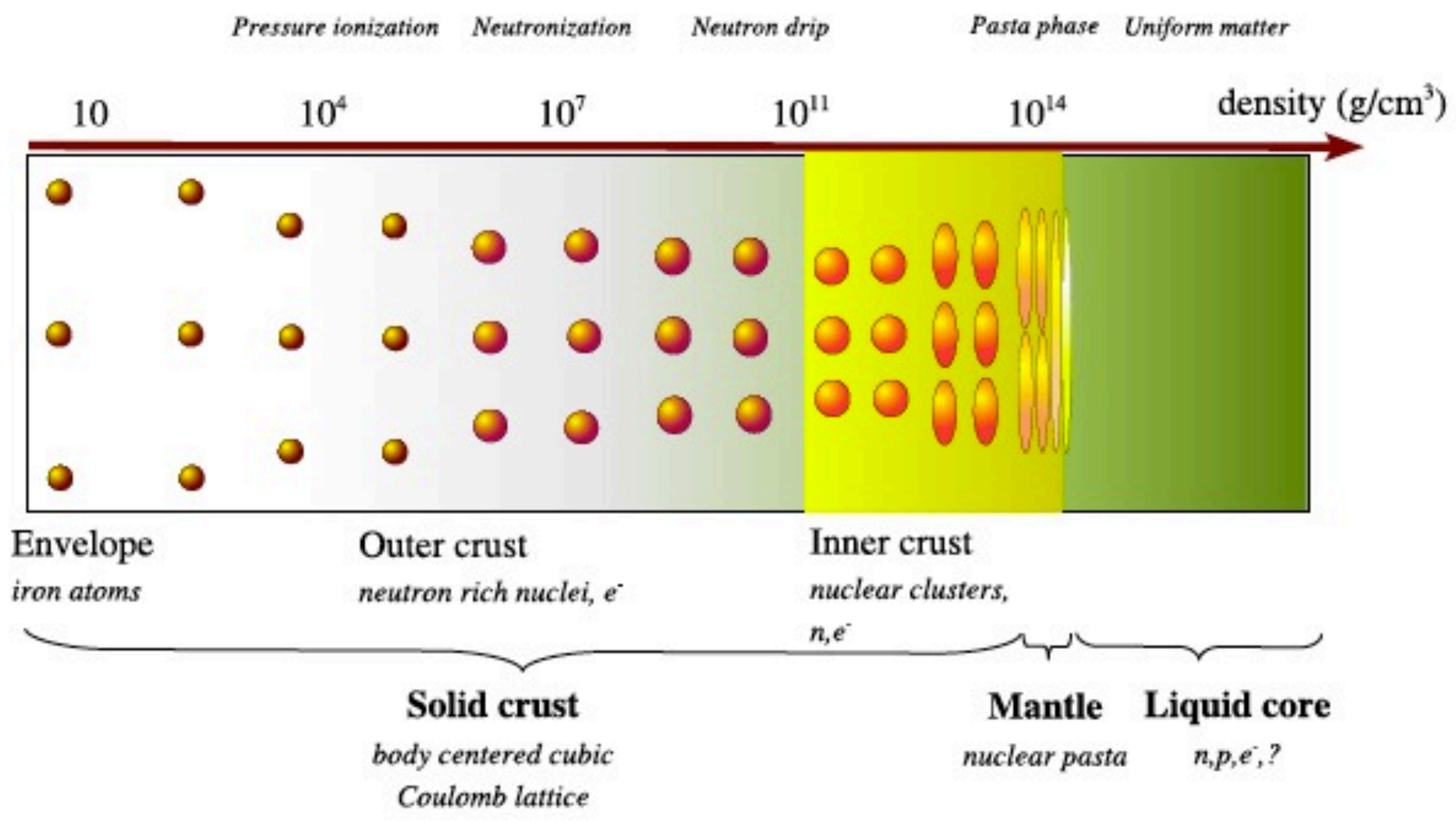
Simin Mahmoodifar
University of Maryland

Dec. 2013

Bedaque, Mahmoodifar, Sen, PRC 88, 055801 (2013)

Bedaque, Mahmoodifar, Ng, Sen [arXiv:1312.0591 (astro-ph.HE)]

Schematic picture of the ground state structure of neutron stars



- Large magnetic fields can change the structure and properties of matter

- Magnetars are highly magnetized neutron stars with $\mathrm{B}\sim 10^{15}$ G

- Electrons motion perpendicular to the magnetic field will be quantized into Landau orbitals

Single electron energy levels:

$$E_n = \sqrt{k_z^2 + 2neB + m_e^2}$$

$$n_e = (eB/2\pi^2)k_e$$

We consider low enough densities where only the lowest landau level is occupied.

$$\rho \lesssim \frac{AM}{\sqrt{2}\pi^2 Z}(eB)^{3/2} \approx 5.2 \times 10^8 \mathrm{g/cm}^3 \ \left(\frac{A_{66}}{Z_{28}}\right) B_{15}^{3/2}$$

Screening of the ion-ion potential by electrons

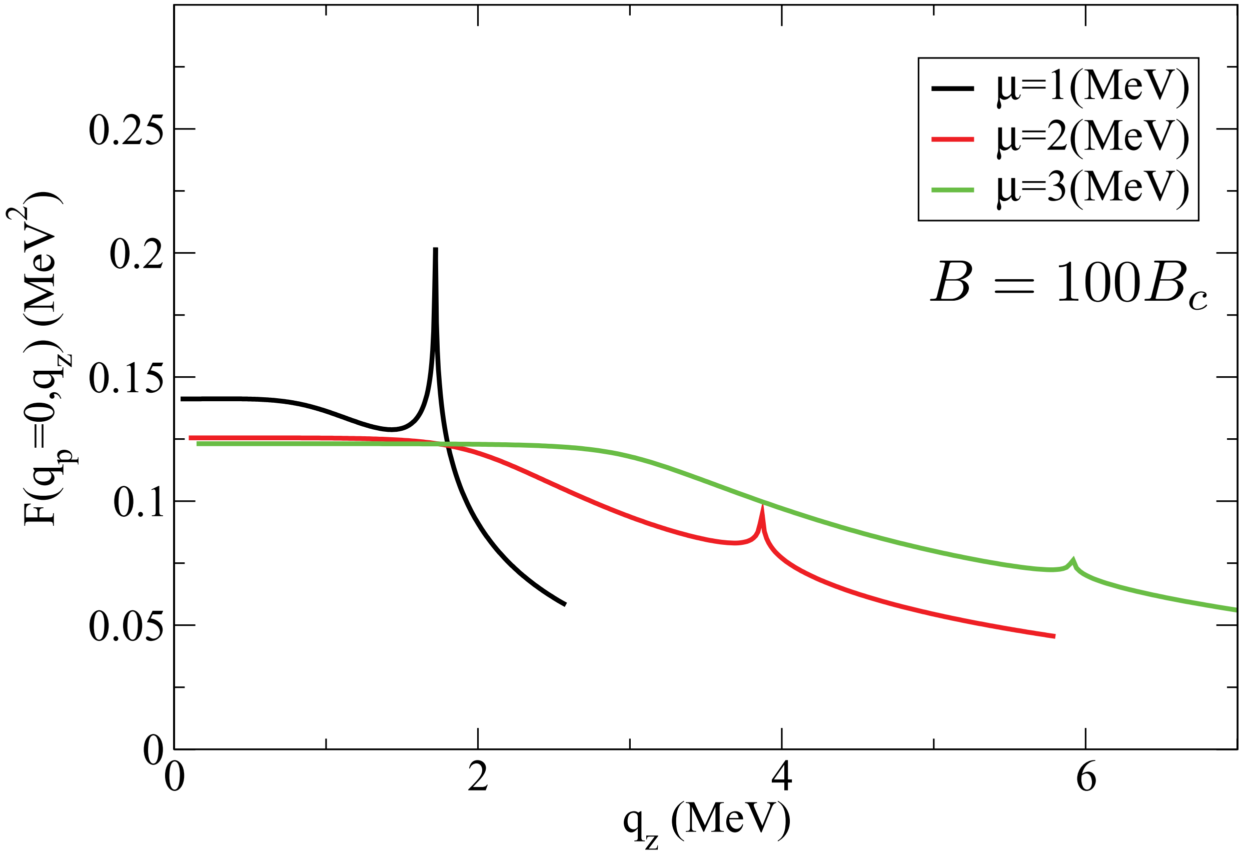
Sharma and Reddy (2011) computed the effect of one loop electron-hole polarization function on the ion-ion potential.

$$V(\mathbf{q}) = \frac{Z_1 Z_2 e^2}{q^2 - e^2 \Pi(\mathbf{q})}$$

$$V(\mathbf{r}) = \frac{Z_1 Z_2 e^2}{r} g(\mathbf{r})$$

$$g(\mathbf{r}) = 4\pi r \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q}\cdot\mathbf{r}}}{q^2 + F(\mathbf{q})}$$

Due to a sharp Fermi surface, $\Pi(\mathbf{q})$ shows non-analytic behavior at $q_z = \pm 2k_e$, which causes a long-range potential in the position space.

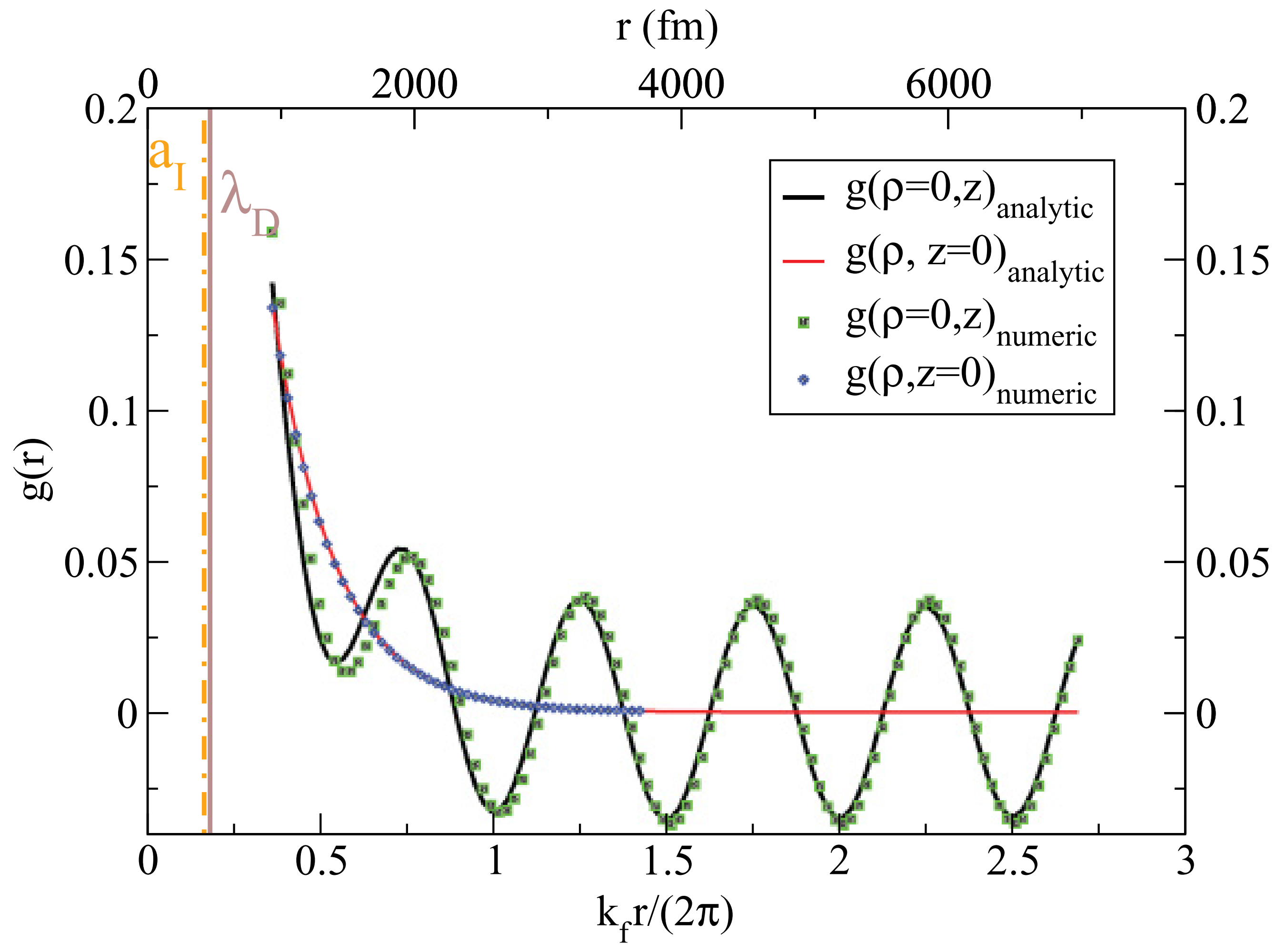


Sharma & Reddy, PRC **83**, 025803 (2011)

Only important when electrons are non-relativistic.

Friedel Oscillations

$$V(r_\perp,z) = Z^2\alpha\left[\frac{e^{-m_D\sqrt{r_\perp^2+z^2}}}{\sqrt{r_\perp^2+z^2}} - \frac{m_D^2 e^{-z/\lambda_T}}{4z}\frac{\cos(2k_e z)r_\perp}{\sqrt{4k_e^2+\frac{m_D^2}{2}\ln(4k_e z)}}K_1\left(r_\perp\sqrt{4k_e^2+\frac{m_D^2}{2}\ln(4k_e z)}\right)\right]$$



Sharma & Reddy, PRC **83**, 025803 (2011)

$$V_F(0,z) \approx -\frac{Z^2 \alpha m_D^2}{4} \frac{e^{-z/\lambda_T} \cos(2k_e z)}{z} \frac{1}{4k_e^2 + \frac{m_D^2}{2} \log(4k_e z)} f\left(\frac{2k_e^2 + \frac{m_D^2}{4} \log(4k_e z)}{eB}\right)$$

$$f(x) = 1 + x e^x E_i(-x),$$

$$V(\rho = 0, z) = Z^2 \alpha \left[\frac{e^{-m_D z}}{z} - \frac{m_D^2 e^{-z/\lambda_T}}{4z} \frac{\cos(2k_e z)}{4k_e^2 + \frac{m_D^2}{2} \ln(4k_e z)} \right]$$

$$\lambda_T = \frac{2\pi k_e}{mT} \approx (1.6 \times 10^{-7}\ \mathrm{cm}) \left(\frac{Z_{28}}{A_{66}}\right) \left(\frac{\rho_8}{B_{15} T_1}\right),$$

$$\frac{1}{m_D} \approx (1.3 \times 10^{-10}\ \mathrm{cm}) \sqrt{\frac{Z_{28} \rho_8}{A_{66}} \frac{1}{B_{15}}},$$

$$\frac{1}{k_e} \approx (3.0 \times 10^{-11}\ \mathrm{cm}) \left(\frac{A_{66}}{Z_{28}}\right) \left(\frac{B_{15}}{\rho_8}\right),$$

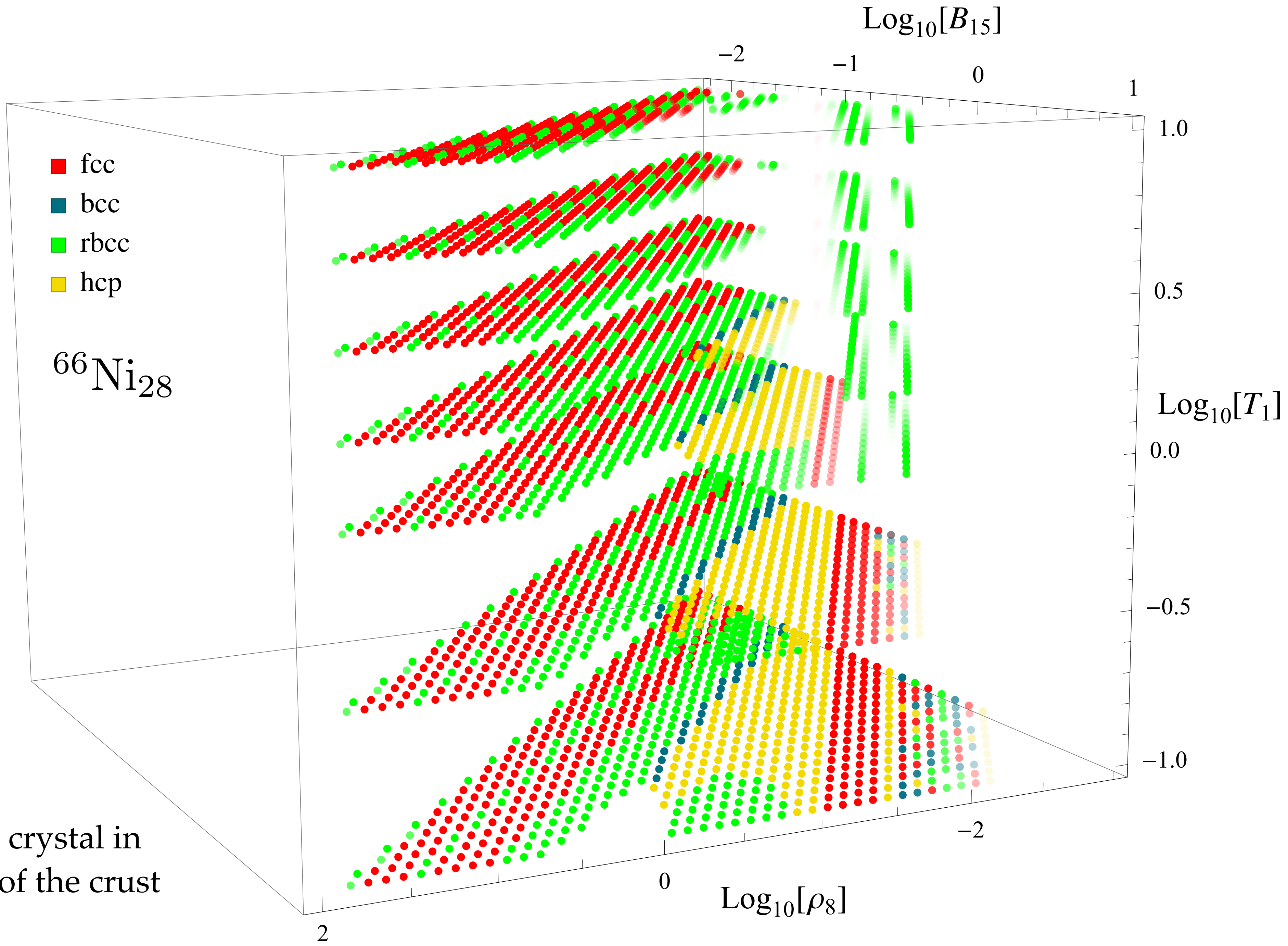
$$\frac{1}{\sqrt{eB}} \approx (8.1 \times 10^{-12}\ \mathrm{cm}) \frac{1}{\sqrt{B_{15}}}.$$

Bedaque, Mahmoodifar, Sen, PRC 88, 055801 (2013)

Parameter Space of "Friedel Crystals"

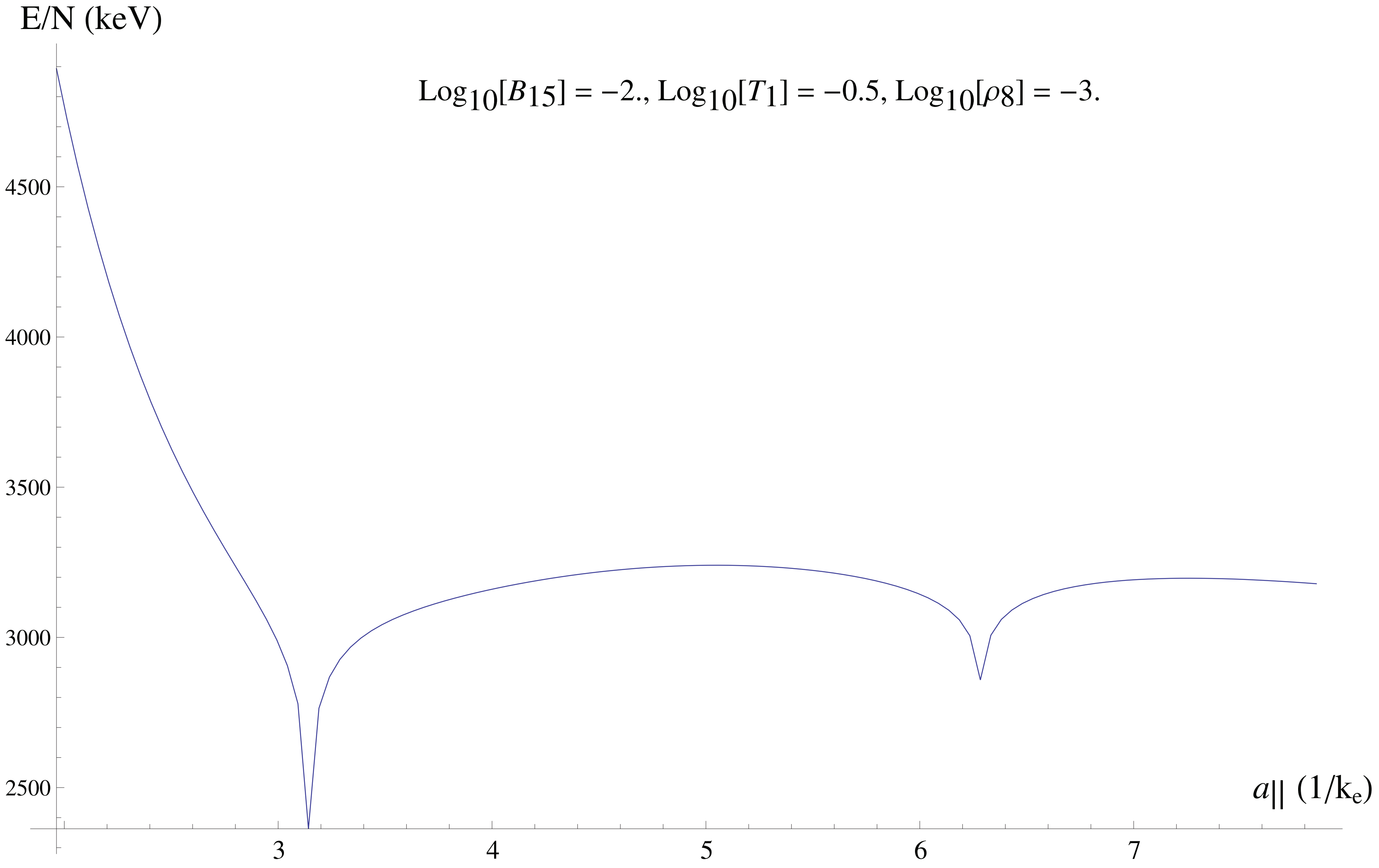
$$a_{\|} \sim n \times \frac{\pi}{k_e}$$

To keep the density fixed, $a_\perp$ varies with $a_{\|}$.



Normal bcc crystal in other parts of the crust

Finite temperature effects:



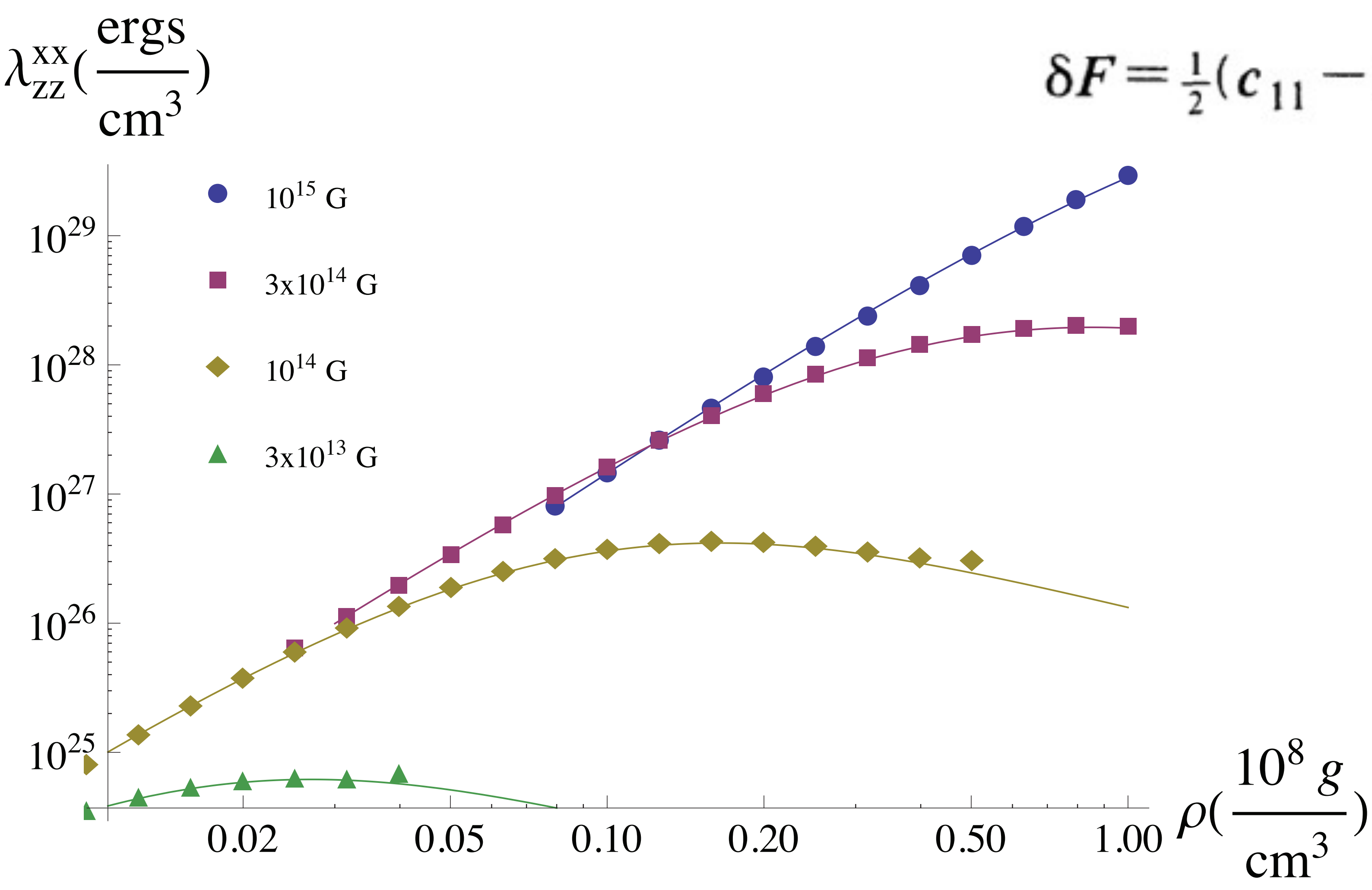
$$\frac{E}{N} = \frac{1}{2}\sum_{i\neq 0} V(r_{\perp i}, z_i)$$

λ^{xx}_{zz}

The energy change of the lattice due to a small displacement in the x direction when the wave is propagating in the z direction.

$$\Delta U \approx \frac{1}{2}\int d^3x\lambda^{ij}_{kl}\partial_k\xi^i\partial_l\xi^j$$

$$\delta F = \tfrac{1}{2}(c_{11} - c_{12})u_{ii}^2 + c_{44}u_{ik}u_{ik} \quad (i \neq k)$$

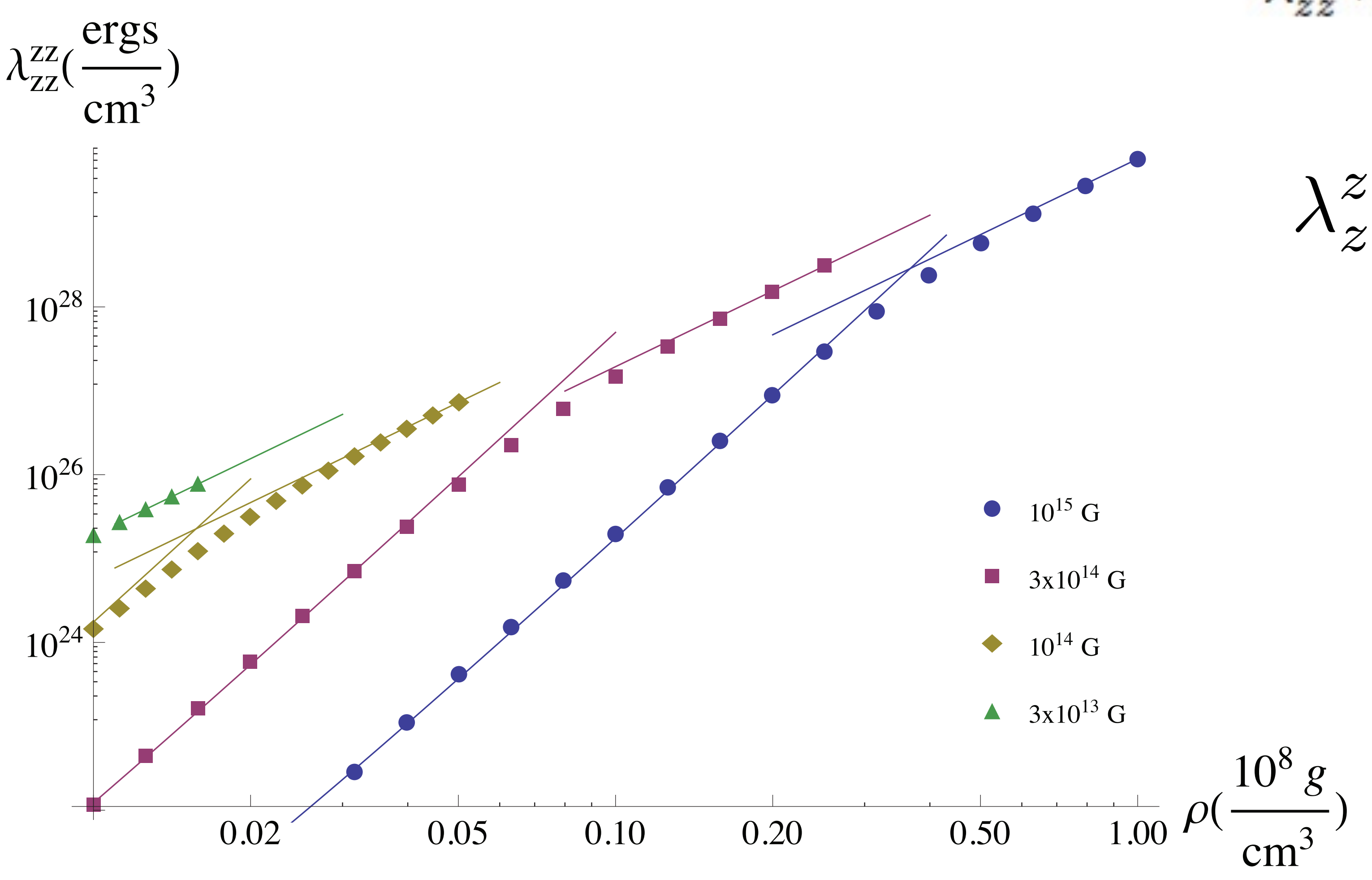


$T \sim 10^7 MeV$ $\quad$ $^{66}\mathrm{Ni}_{28}$

λ_{zz}^{zz}

The energy change of the lattice due to a small displacement in the z direction when the wave is also propagating in the z direction.

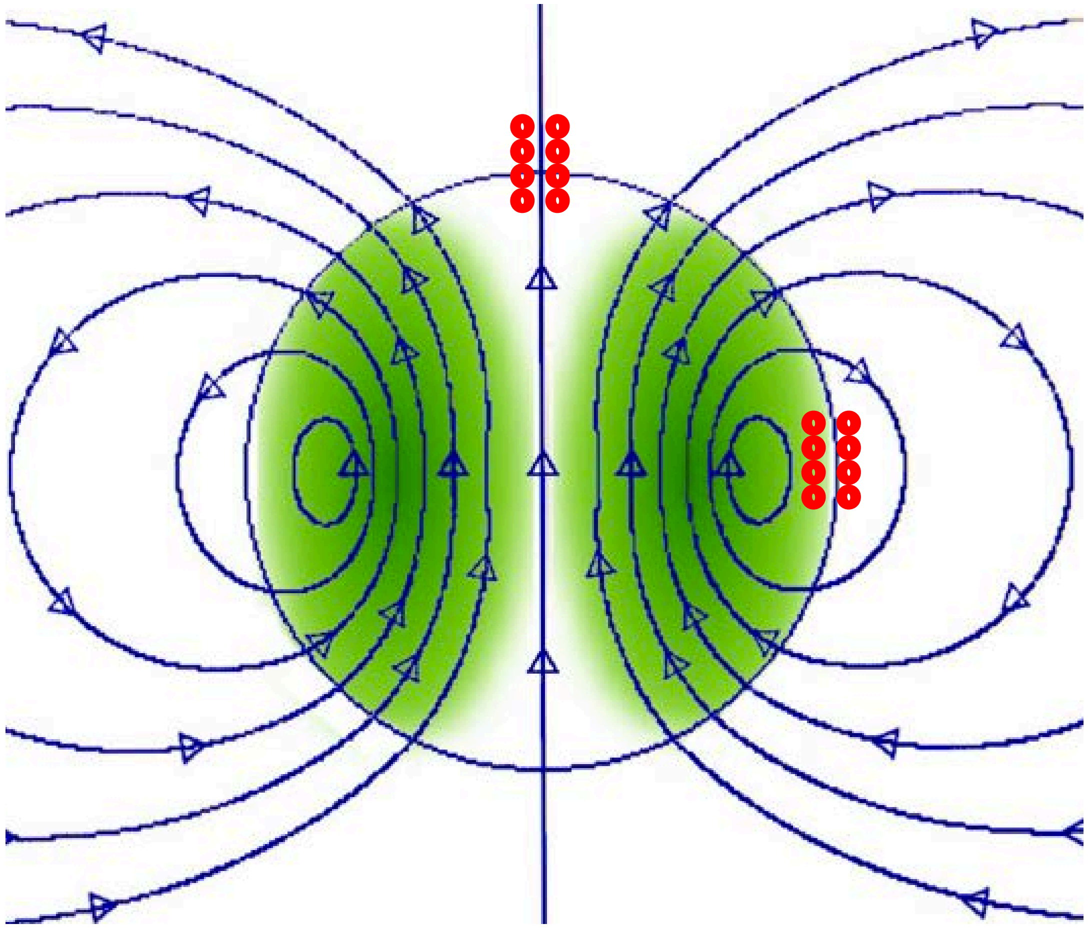
$$\lambda_{zz}^{zz} \approx 5.9 \times 10^{29} \frac{ergs}{cm^3} \left(\frac{Z_{28}^4 \rho_8^3}{A_{66}^3 B_{15}} \frac{1}{T_1^2} \right)$$



$$\lambda_{zz}^{zz} \sim 10^4 \times (c_{11} - c_{12})$$

$T \sim 10^7 MeV$ $\quad$ $^{66}\text{Ni}_{28}$

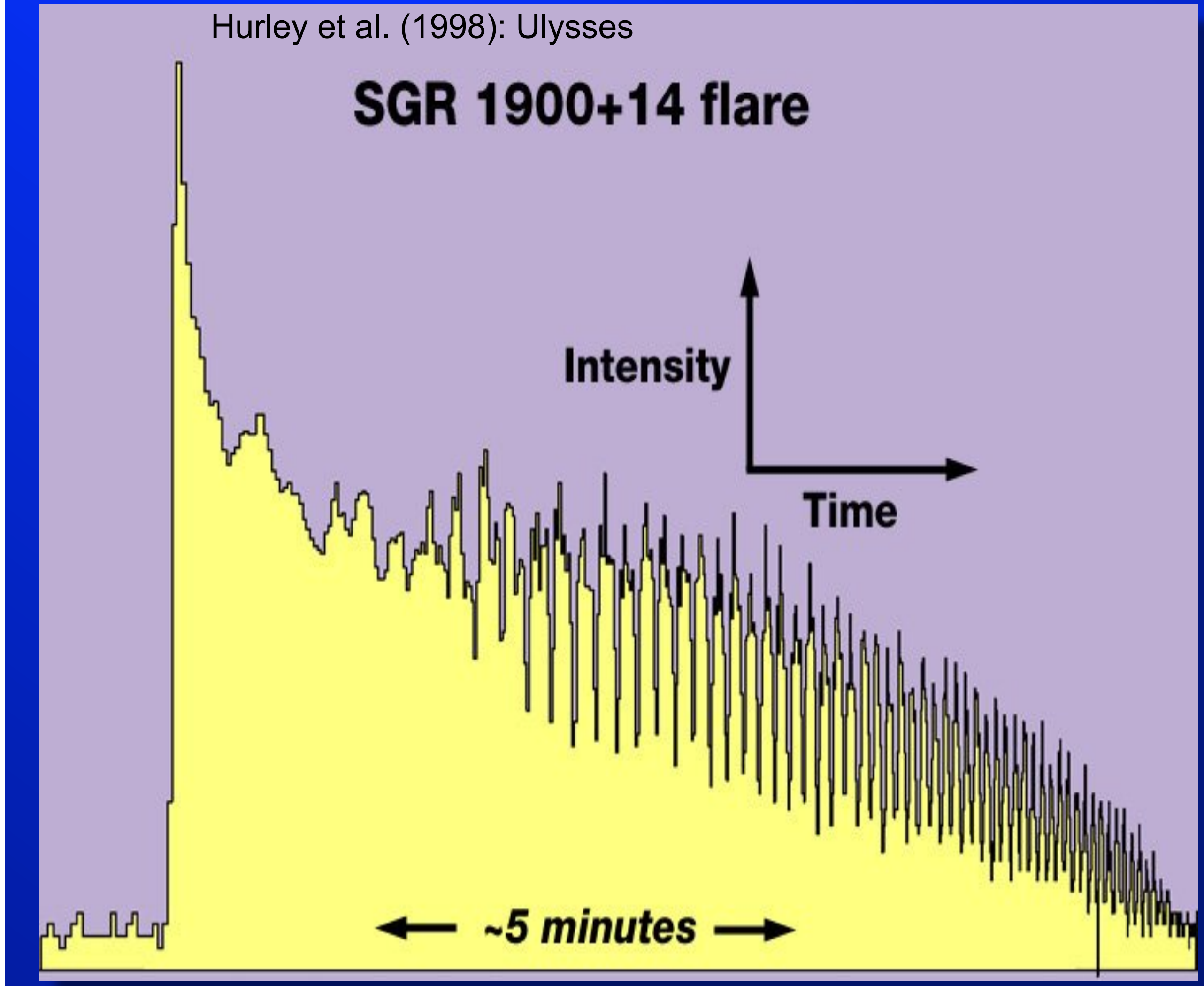
Elastic constants that are dominated by the longitudinal structure of the lattice are significantly larger than that of a bcc Coulomb crystal of comparable densities.



Potentially interesting implications for the X-ray oscillations seen from magnetars during their giant flares.

Goddard Space
Flight Center

Anatomy of a Hyperflare



- 10^{15} G magnetic fields implied (Thompson & Duncan 95)

Three events to date:

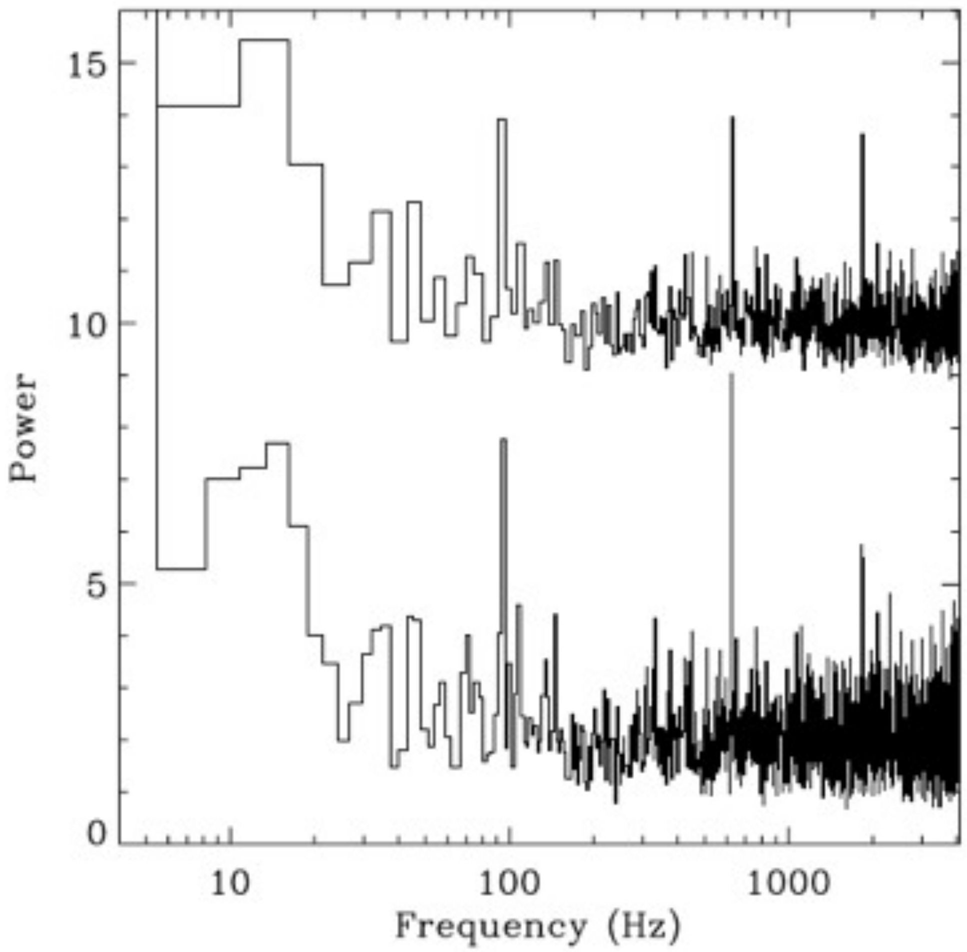
- March 5th 1979: SGR 0526-66
- August 27th 1998: SGR 1900+14
- December 27th 2004: SGR 1806-20

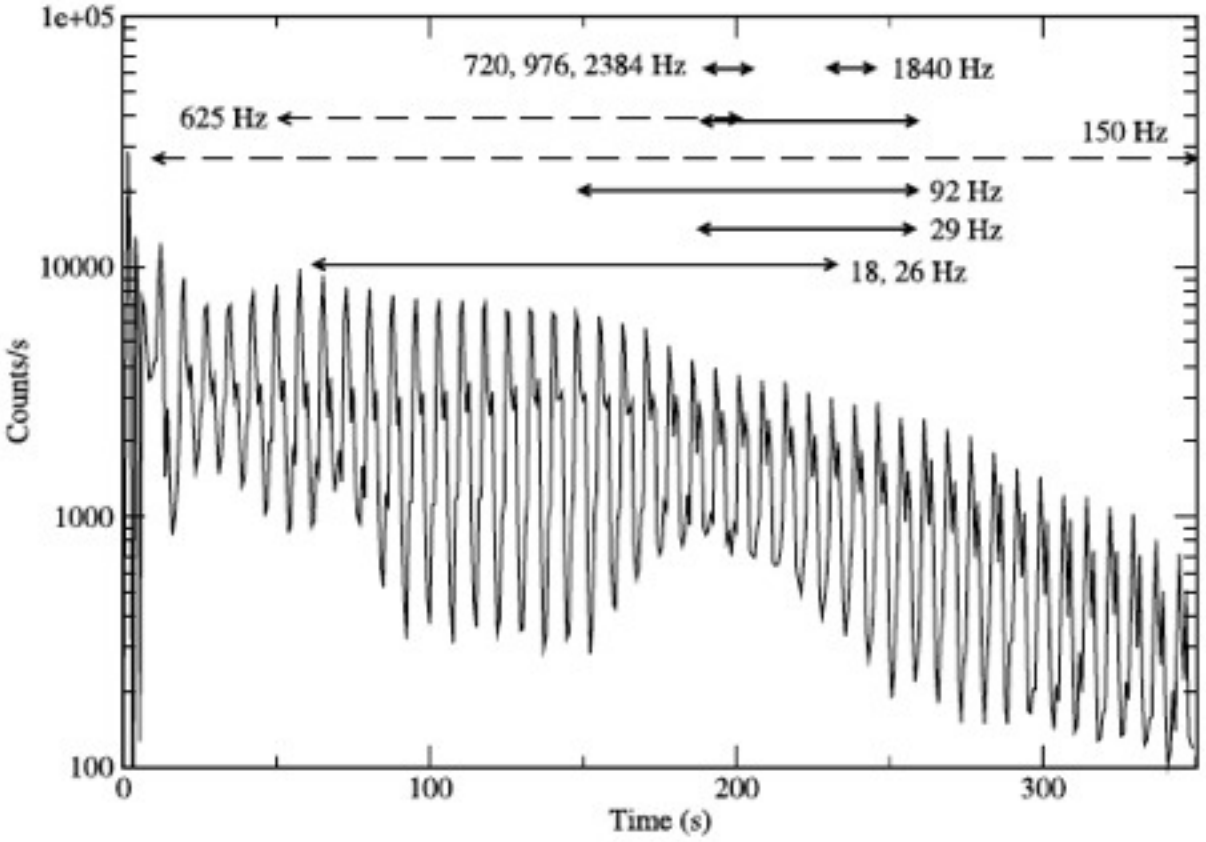
Powered by global magnetic instability (reconfiguration), crust fracturing.

- Short, hard, luminous initial pulse.
- Softer X-ray tail persists for minutes, and reveals neutron star spin period.
- Emission from a magnetically confined plasma.

Thanks to Tod strohmayer!

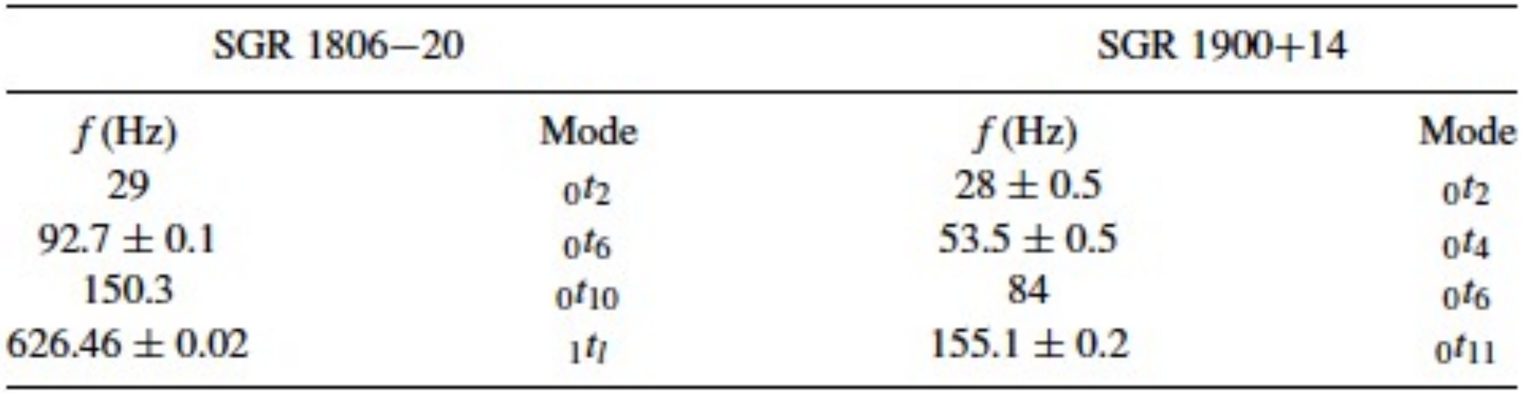
Dec. 2004 hyperflare from SGR 1806-20





T. Strohmayer and A. Watts 2006

The observations of **global oscillations** of neutron stars can provide a powerful **probe of their interior properties**, similar to the field of helioseismology.



SGR 1806−20		SGR 1900+14	
f (Hz)	Mode	f (Hz)	Mode
29	${}_0t_2$	28 ± 0.5	${}_0t_2$
92.7 ± 0.1	${}_0t_6$	53.5 ± 0.5	${}_0t_4$
150.3	${}_0t_{10}$	84	${}_0t_6$
626.46 ± 0.02	${}_1t_l$	155.1 ± 0.2	${}_0t_{11}$

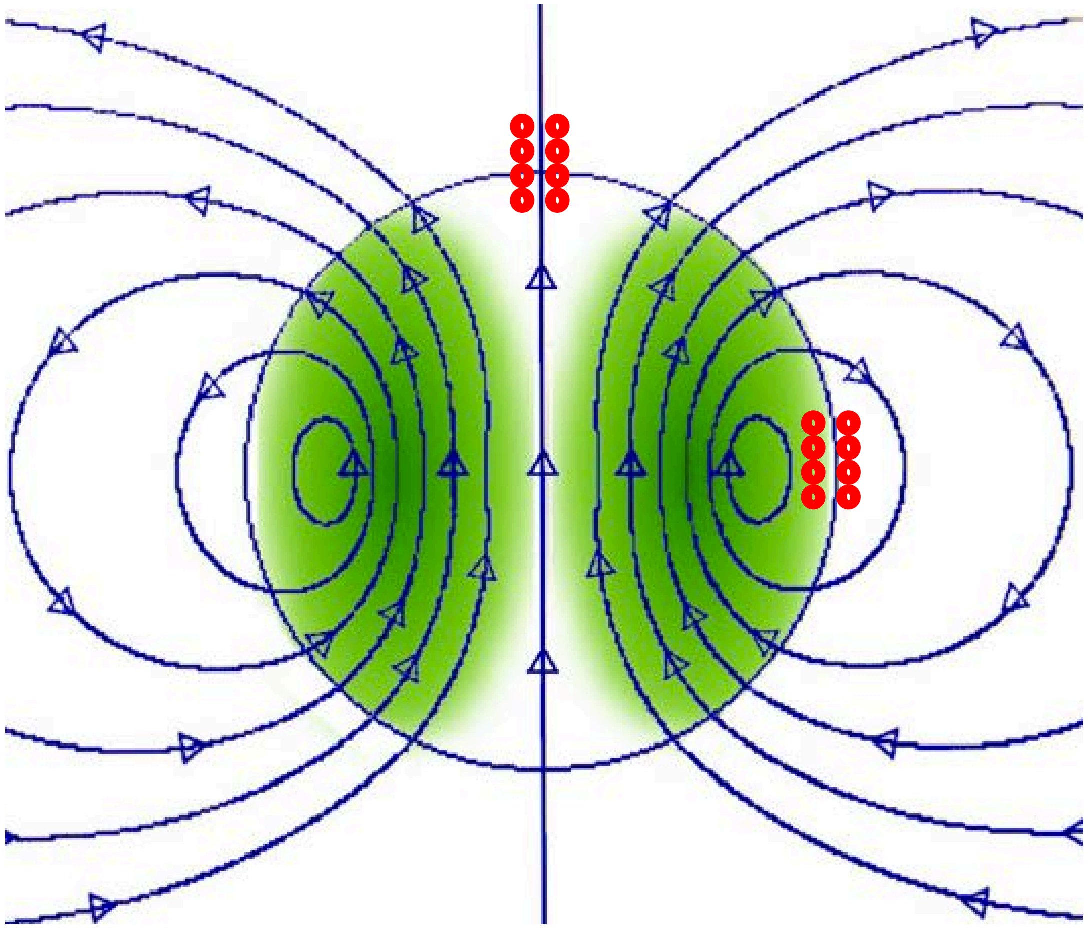
$$\omega_{0,l}^2 \propto \frac{v_t^2 l(l+1)}{R^2}$$

$$v_t = \sqrt{\mu/\rho}$$

Core Alfven modes?

Crust shear modes?

Effect of an anisotropic outer crust on the oscillation frequencies of the crustal modes?



$$\mu_{eff} = 0.1194\frac{n_i Z^2 e^2}{a}$$

Ogata et al. 1990, Strohmayer et al.

$$\mu_{eff} = 0.1108\frac{n_i Z^2 e^2}{a}$$

Horowitz and Hughto

Conclusion

- Long range oscillations in the ion-ion potential along the magnetic field due to anisotropic screening of the Coulomb force by electrons in the presence of strong magnetic fields

- The long-ranged potential forces the ions to organize themselves into strongly coupled filaments along the magnetic field.

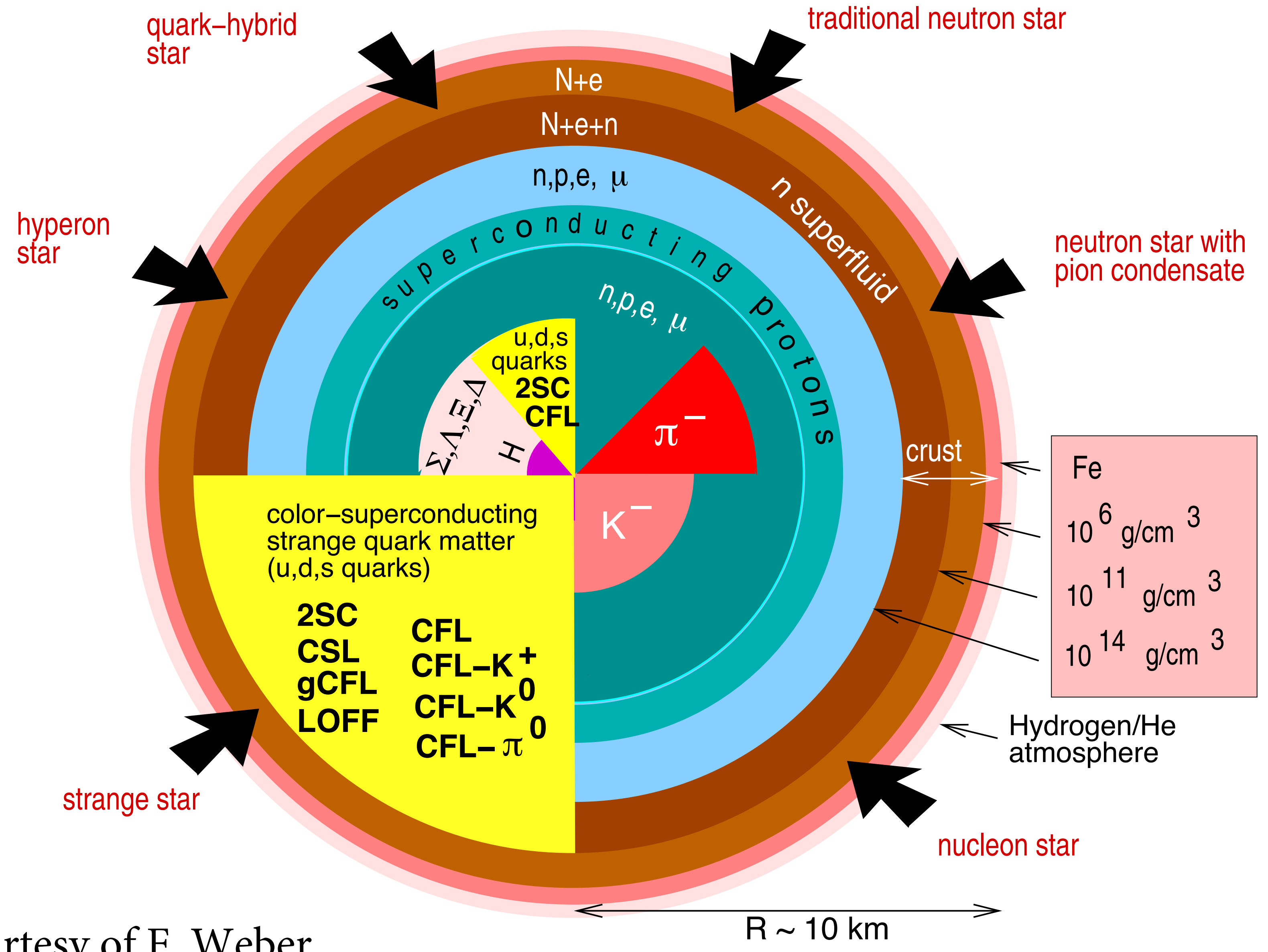
- Friedel crystals form in the outer crust of magnetars

- Large elastic constants in the longitudinal direction

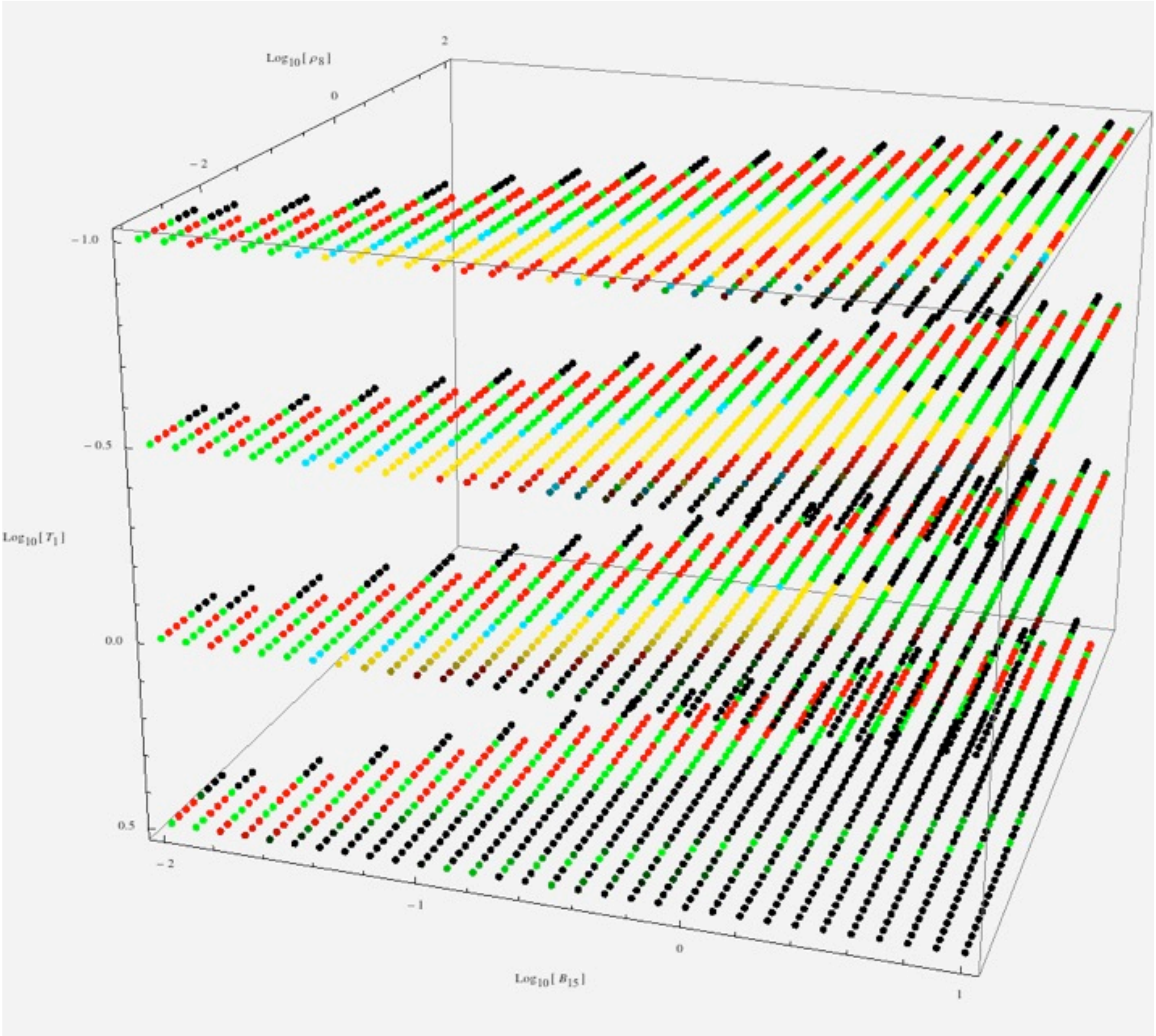
- Implications for QPOs (shear mode frequencies) and GWs (breaking strain)

Compact Stars: The only “laboratory” for the study of cold ultra-dense matter

They may contain exotic form of matter.



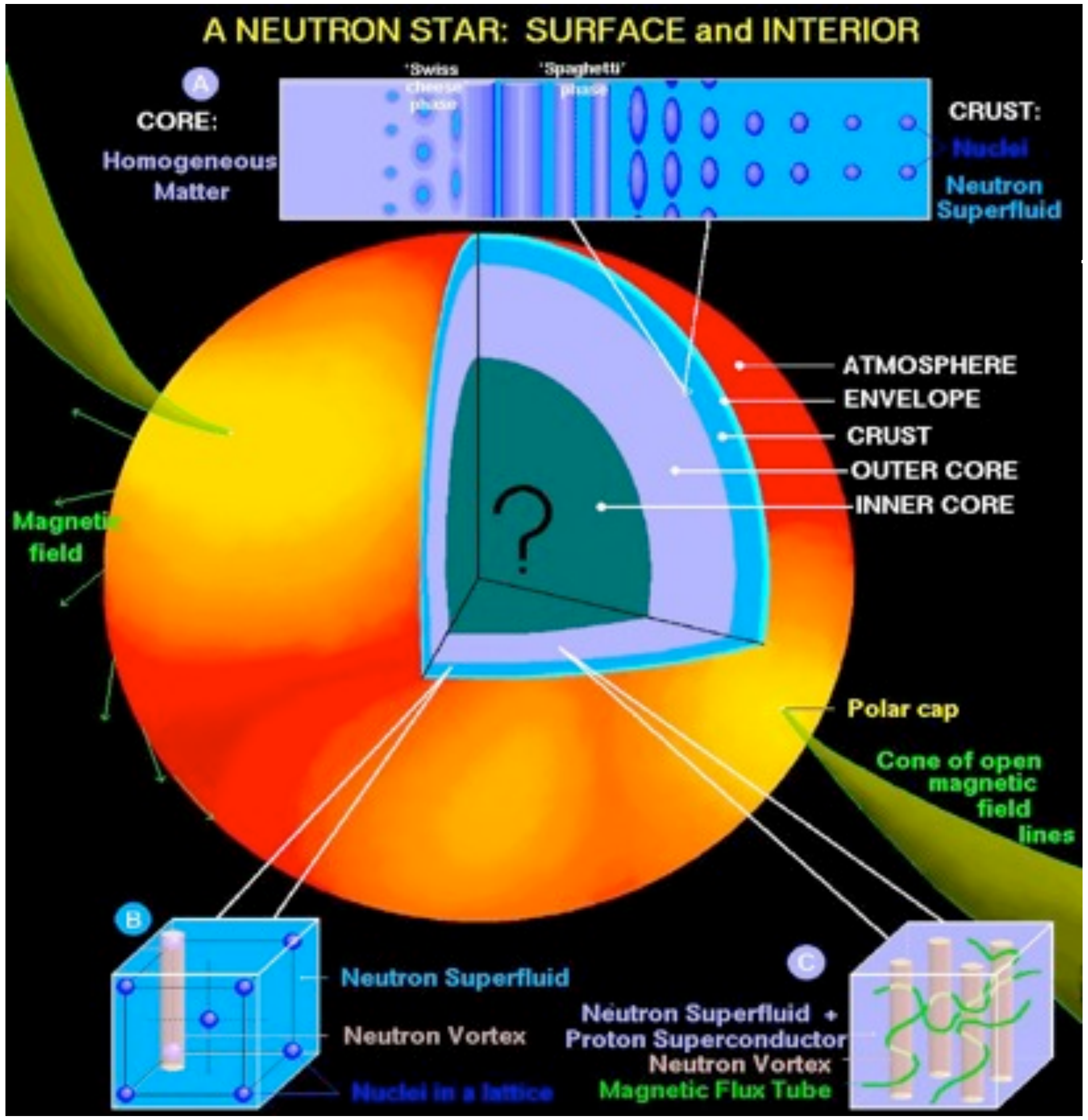
Courtesy of F. Weber



Nathan Ng

Compact Stars

The only "laboratory" for the study of cold ultra-dense matter



http://www.astroscu.unam.mx/neutrones/NS-Picture/NStar/NStar.html

Mass ~ $1.4M_\odot$
Radius ~ O(10 km)
Density > $\rho_{nuclear}$
T < 1 MeV

They may contain exotic form of matter.