Nuclear Pasta

A.S. Schneider[1], C.M. Briggs[1], C.J. Horowitz[1]
J. Hughto[1], D.K. Berry[2]

[1] Department of Physics and Nuclear Theory Center
Indiana University

[2] University Information Technology Center
Indiana University

Dallas, TX - December 2013

Nuclear Pasta

Nuclear Physics: determine the equation of state of nuclear matter. It is well established that

- Low densities ($n \ll n_0$) ⇒ isolated nuclei;
- High densities ($n \gtrsim n_0$) ⇒ uniform matter.

So what happens to matter between these two extremes?

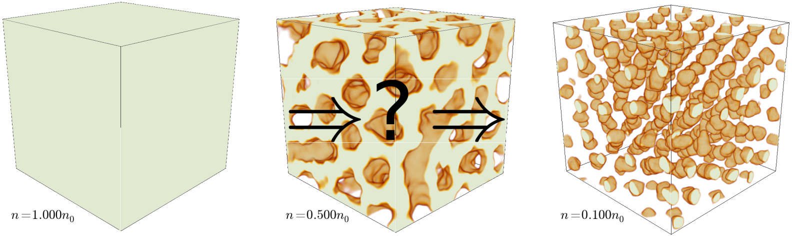


Figure: Matter at nuclear saturation density n_0 (left) and one tenth of nuclear saturation density $n_0/10$ (right).

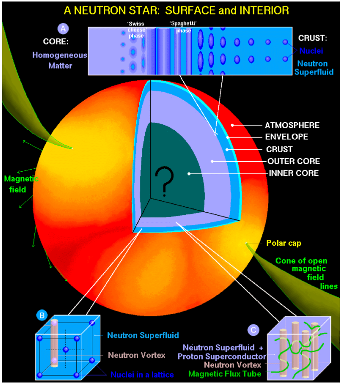
Nuclear Pasta

Astrophysics relevance:

- Present in core-collapse supernovae and inner crust of neutron stars.
- Important for the structure, evolution and properties of compact stars.

Not accessible to laboratory experiments.

- High density;
- Low temperatures;
- Large isospin assymetry.



Nuclear Pasta

Need to know phases and properties of matter at a large range of densities, proton fractions and temperatures.
Different approaches:

- Liquid drop model
- Thomas-Fermi approximation
- Hartree-Fock
- Density functional theory
- Molecular Dynamics (MD)
- Quantum Molecular Dynamics (QMD)

In our case, we use MD to simulate large systems for long times to calculate complex observables.

Formalism

System of protons and neutrons immersed in a background electron gas.
Nucleons interact through a potential:

$$V_{np}(r_{ij}) = a\mathrm{e}^{-r_{ij}^2/\Lambda} + [b - c]\mathrm{e}^{-r_{ij}^2/2\Lambda}$$

$$V_{nn}(r_{ij}) = a\mathrm{e}^{-r_{ij}^2/\Lambda} + [b + c]\mathrm{e}^{-r_{ij}^2/2\Lambda}$$

$$V_{pp}(r_{ij}) = a\mathrm{e}^{-r_{ij}^2/\Lambda} + [b + c]\mathrm{e}^{-r_{ij}^2/2\Lambda} + \frac{\alpha}{r_{ij}}\mathrm{e}^{-r_{ij}/\lambda}$$

$\lambda = \frac{1}{2k_F}\sqrt{\frac{\pi}{\alpha}}$ is the Thomas-Fermi screening length for relativistic electrons.
$k_F = (3\pi^2 n_e)^{1/3}$ is the Fermi momentum and n_e the e^- density.

a	b	c	Λ	λ
110 MeV	-26 MeV	24 MeV	1.25 fm^2	10 fm

Table: Parameters of the model. λ was arbitrarily decreased to 10 fm.

Formalism

At low densities model predicts reasonable results for binding energies of finite nuclei.

Nucleus	Monte-Carlo $\langle V_{tot} \rangle$ (MeV)	Experiment (MeV)
^{16}O	-7.56±0.01	-7.98
^{40}Ca	-8.75±0.03	-8.45
^{90}Zr	-9.13±0.03	-8.66
^{208}Pb	-8.2 ± 0.1	-7.86

Table: Binding energies per nucleon in MeV from parameters defined above from Phys Rev C 69 405804

Formalism

At high densities model predicts that:

- Neutron matter is unbound;
- symmetric nuclear matter saturates ar the correct density $n = 0.16\,\mathrm{fm}^{-3}$;
- energy per nucleon is about $-17\,\mathrm{MeV}$.

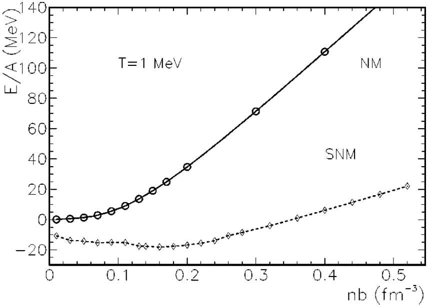


Figure: Energy per nucleon for symmetric (dashed) and pure-neutron (solid) matter *vs* baryon density *n* at $T = 1\,\mathrm{MeV}$. From Phys Rev C 69 405804.

Simulations

- Number of particles $N = 51\,200$;
- Proton fraction $Y_p = 0.40$: 30 720 neutrons and 20 480 protons;
- Temperature of $1\,\mathrm{MeV}$ (approximate infall phase of a SN);
- Cubic box with periodic boundary conditions;
- Start from random at a density of $0.16\,\mathrm{fm}^{-3}$ (box side is $68\,\mathrm{fm}$);
- Expand the system at different rates $\dot{\xi}$;
- After expansion starts, side of the box at time t:

$$l(t) = l_0(1 + \dot{\xi} t);$$

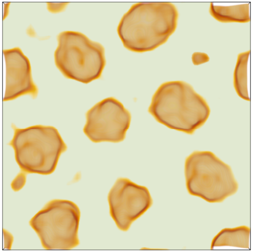
- Compare topology the systems stretched at different rates.

Simulations

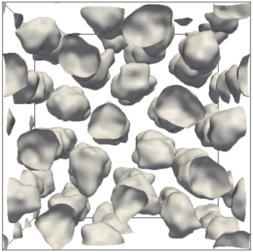
Simulations

Configuration of system at density of $n = 0.100\,\mathrm{fm}^{-3}$.

- Golden (left), white (right) isosurfaces with $n_{\mathrm{ch}} = 0.030\,\mathrm{fm}^{-3}$
- Cream: regions of charge density $n_{\mathrm{ch}} > 0.030\,\mathrm{fm}^{-3}$



$n = 0.625 n_0$

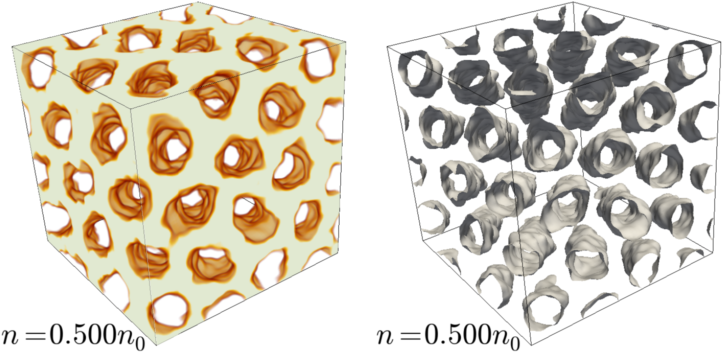


$n = 0.625 n_0$

Simulations

Configuration of system at density of $n = 0.080\,\mathrm{fm}^{-3}$.

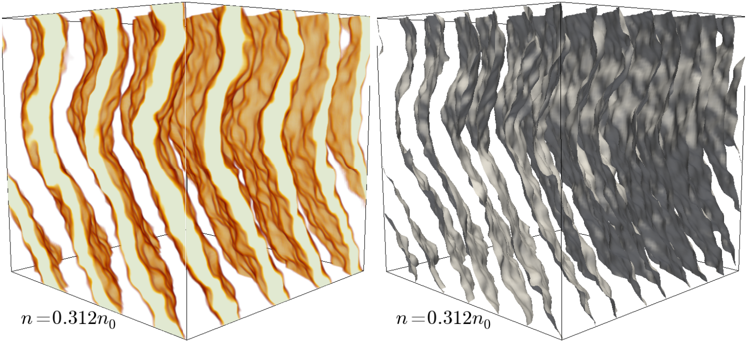
- Golden (left), white (right) isosurfaces with $n_{\mathrm{ch}} = 0.030\,\mathrm{fm}^{-3}$
- Cream: regions of charge density $n_{\mathrm{ch}} > 0.030\,\mathrm{fm}^{-3}$



Simulations

Configuration of system at density of $n = 0.050\,\mathrm{fm}^{-3}$.

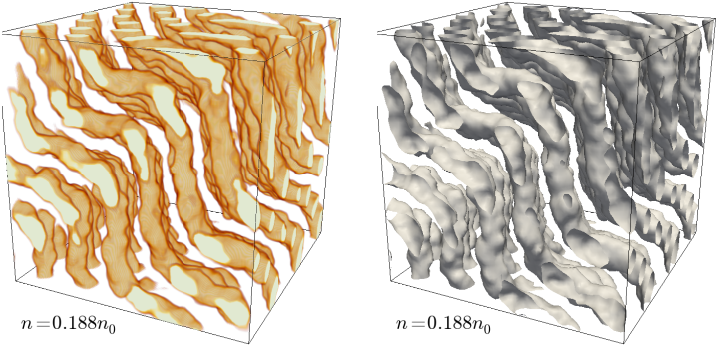
- Golden (left), white (right) isosurfaces with $n_{\mathrm{ch}} = 0.030\,\mathrm{fm}^{-3}$
- Cream: regions of charge density $n_{\mathrm{ch}} > 0.030\,\mathrm{fm}^{-3}$



Simulations

Configuration of system at density of $n = 0.030\,\mathrm{fm}^{-3}$.

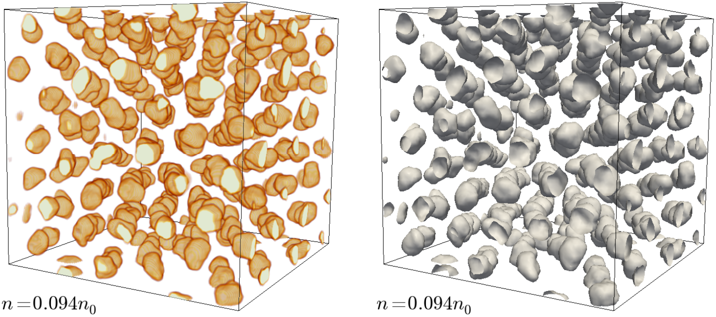
- Golden (left), white (right) isosurfaces with $n_{\mathrm{ch}} = 0.030\,\mathrm{fm}^{-3}$
- Cream: regions of charge density $n_{\mathrm{ch}} > 0.030\,\mathrm{fm}^{-3}$



Simulations

Configuration of system at density of $n = 0.015\,\mathrm{fm}^{-3}$.

- Golden (left), white (right) isosurfaces with $n_{\mathrm{ch}} = 0.030\,\mathrm{fm}^{-3}$
- Cream: regions of charge density $n_{\mathrm{ch}} > 0.030\,\mathrm{fm}^{-3}$



Simulations

Shapes at intermediate densities, $n_0/10 \lesssim n \lesssim n_0/2$, are collectively known as *nuclear pasta*.

What gives rise to the richeness of pasta shapes?

- *Frustration*, *i.e.*, energy scale of nuclear forces and Coulomb forces is comparable;
- Competition between the two forces makes nucleons cluster in complex shapes.

Topological Characterization

Minkowski functionals

$W_1 \propto V$	Volume V;
$W_2 \propto \int_{\partial K} dA$	Surface area A;
$W_3 \propto \int_{\partial K} \left(\frac{\kappa_1+\kappa_2}{2}\right) dA$	Mean breadth B;
$W_4 \propto \int_{\partial K} (\kappa_1 \cdot \kappa_2)\, dA$	Euler characteristic χ.

κ_1 and κ_2 are the principal curvatures on ∂K the bounding surface of K.

$\chi =$ (# isolated regions) - (# tunnels) + (# cavities)

Use B/A and χ/A as measures to compare systems.

Results

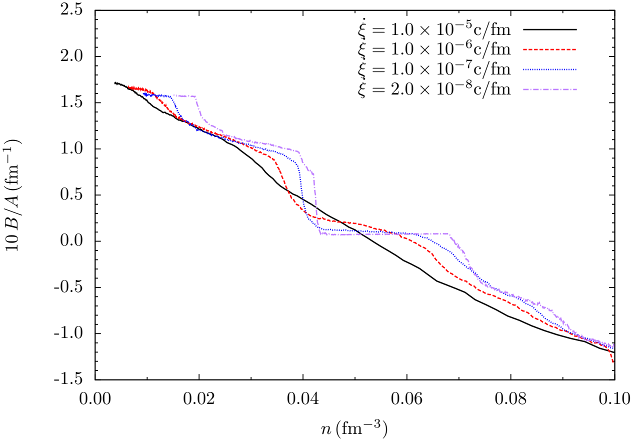


Figure: Normalized mean breadth B/A as a function of density n for different stretch rates. The simulations contain 51 200 nucleons with $Y_p = 0.40$ at 1 MeV.

Results

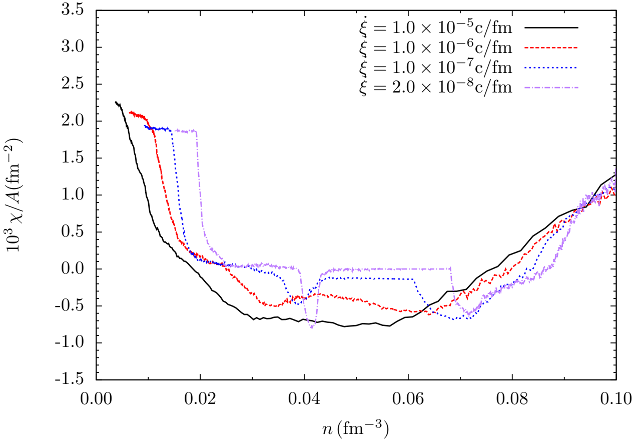


Figure: Normalized Euler characteristic χ/A as a function of density n for different stretch rates. The simulations contain 51 200 nucleons with $Y_p = 0.40$ at 1 MeV.

Results

Static structure factor for different structures of the pasta phase.

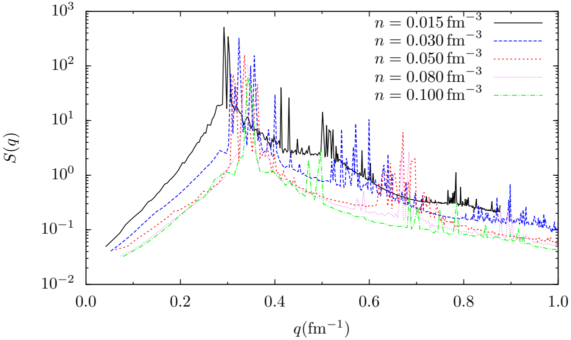


Figure: Static structure factor $S(q)$ of protons.

Results

Static structure factor for different structures of the pasta phase.

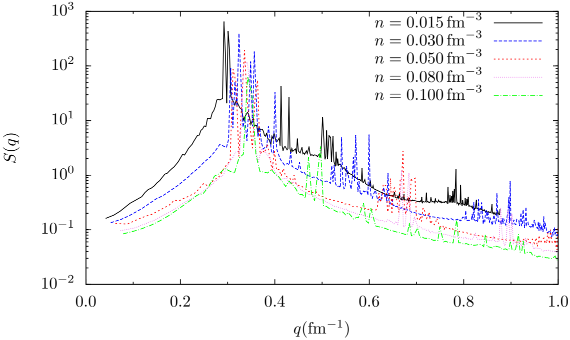
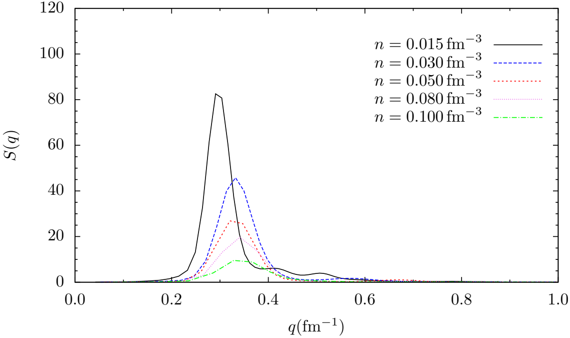


Figure: Static structure factor $S(q)$ of neutrons.

Results

Static structure factor for different structures of the pasta phase.



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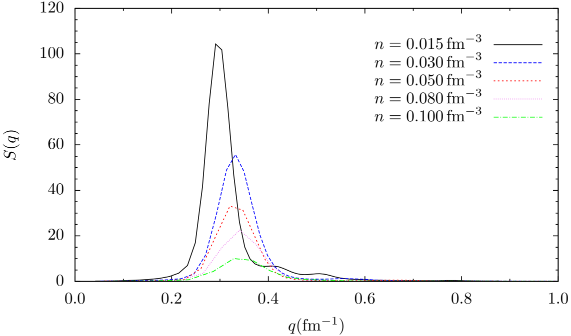

Figure: Static structure factor $S(q)$ of neutrons.

Prospects

Recent developments:

- Bring finite size effects under control (larger simulations).
- Started exploring parameter space using other (lower) proton fractions.
- Obtain static structure factors of pasta phases.

Future:

- Obtain shear viscosity, bulk viscosity, shear modulus and breaking strain of different pasta structures.
- Add other parameters and/or momentum and spin dependence to effective potential.

Observables

From Jose Pons *et al.* . *Nature Physics* 9 431 (2013)

- Observations of isolated X-ray pulsars.
- NS with $|\mathbf{B}| \gtrsim 10^{13}$ G and $P \lesssim 12$ s.
- Problem:
  - Pulsars should spin down rapidly.
  - $P \sim 100$ s in about $T \lesssim 10\,000$ years.
  - Such pulsars are not observed.

- High resistive layer in the inner crust of a NS limits spin period to about $10 - 20$ s.
- This may be the first observational evidence for an amorphous inner crust. Possibly due to the existence of "nuclear pasta".

Observables

- Decay of **magnetic fields** (Jose Pons *et al.* .)
- **Thermal conductivity** and **electrical conductivity**.
  - Depends on coherent e^--pasta scattering.
  - Important for NS crust properties.
- **Shear modulus**
  - Response to small deformations of simulation volume.
  - Determines NS oscillation frequencies.
- **Shear viscosity** and **bulk viscosity**
  - Depends on hysteresis of pasta shapes with density changes.
  - May be important for damping of NS r-mode oscillations.
- **Breaking strain**
  - Response to large deformations of simulation volume.
  - Important for star quakes, magnetar giant flares and mountain heights.
- ν-**oppacity**
  - Depends on coherent ν-pasta scattering.
  - Important for SN simulations as $\lambda_\nu \sim$ pasta sizes.