Mean field approach to the structure and properties of neutron star matter

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Matter of neutron stars

Density

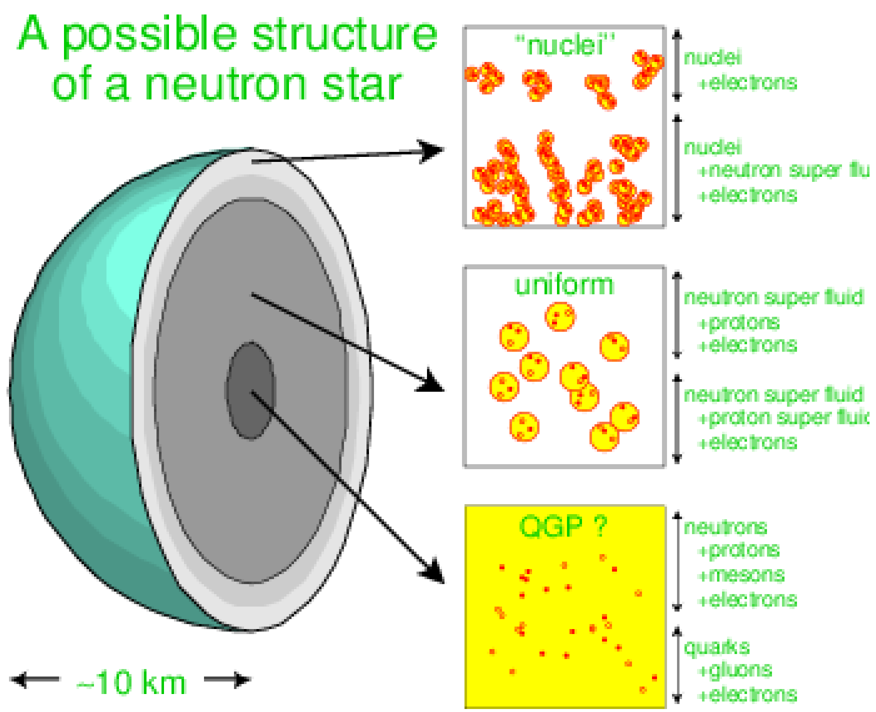
$\approx 0 \cdots \approx 10\rho_0$

Composition

- Nucleons + leptons
- . . . + mesons, hyperons
- quarks + gluons + leptons

Structure & correlation

- uniform
- crystal
- pasta
- amorphous
- pairing
- …..

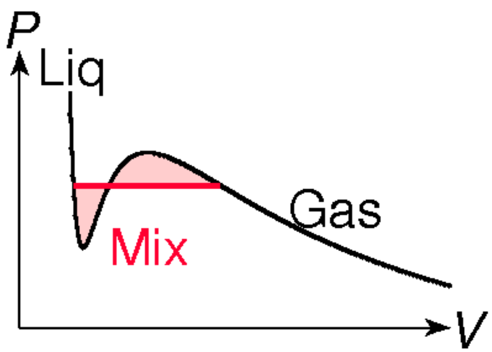


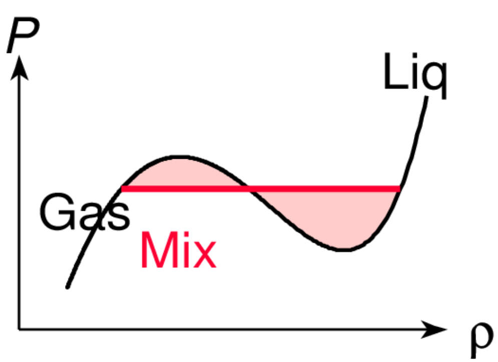
First-order phase transition and EOS

- Single component **_congruent_**

(e.g. water)

Maxwell construction **satisfies** the Gibbs cond. $T^{I}=T^{II}$, $P^{I}=P^{II}$, $\mu^{I}=\mu^{II}$.





- Many components **_non-congruent_**

(e.g. water+ethanol)

Gibbs cond. $T^{I}=T^{II}$, $P_i^{I}=P_i^{II}$, $\mu_i^{I}=\mu_i^{II}$.

No Maxwell construction *!*

- Many charged components

(nuclear matter)

Gibbs cond. $T^{I}=T^{II}$, $\mu_i^{I}=\mu_i^{II}$.

No Maxwell construction *!*

No constant *pressure !* Microscopic structure affects

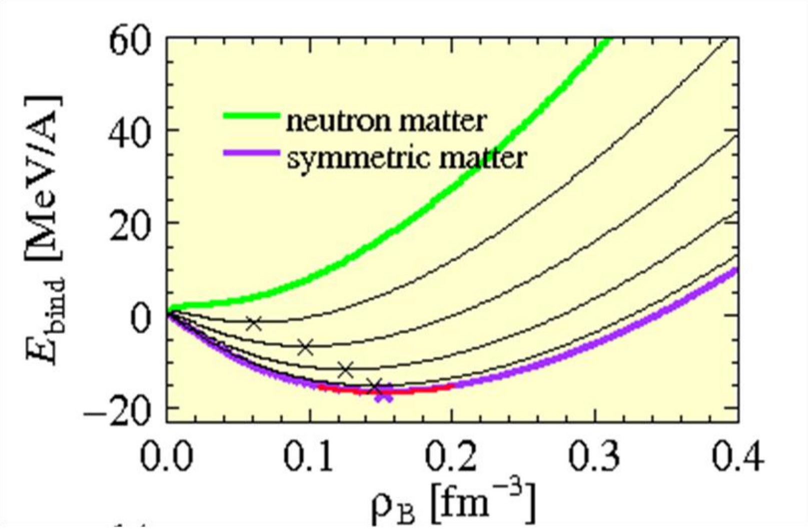
$$\frac{dP_i}{dr} = -\frac{\partial U_i(\rho_i; r)}{\partial r}$$

This is the case for nuclear matter !

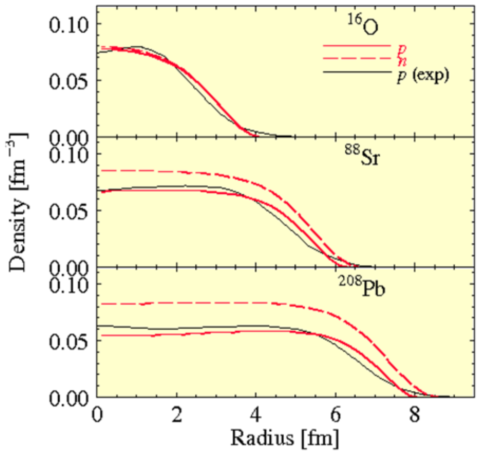
(1) Low-density nuclear matter

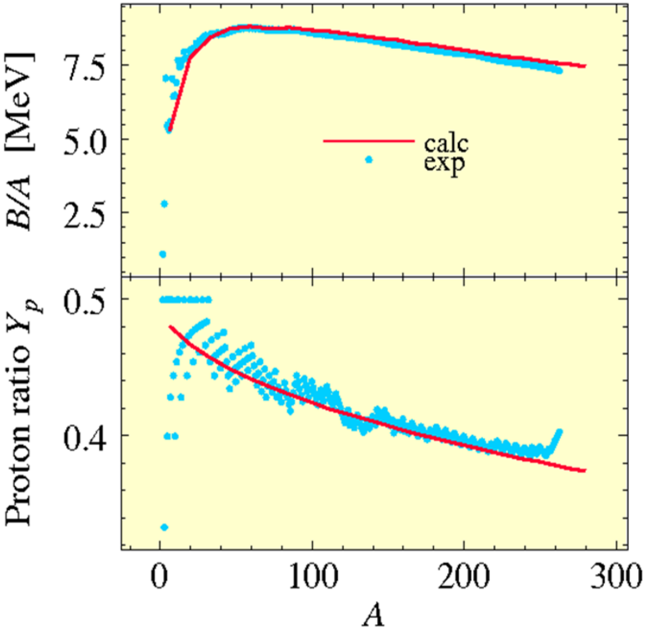
RMF + Thomas-Fermi model

Nucleons interact with each other via coupling with σ, ω, ρ mesons. Simple but realistic enough.



Binding energies ↓, proton fractions ↓, and density profiles → of nuclei are well reproduced.



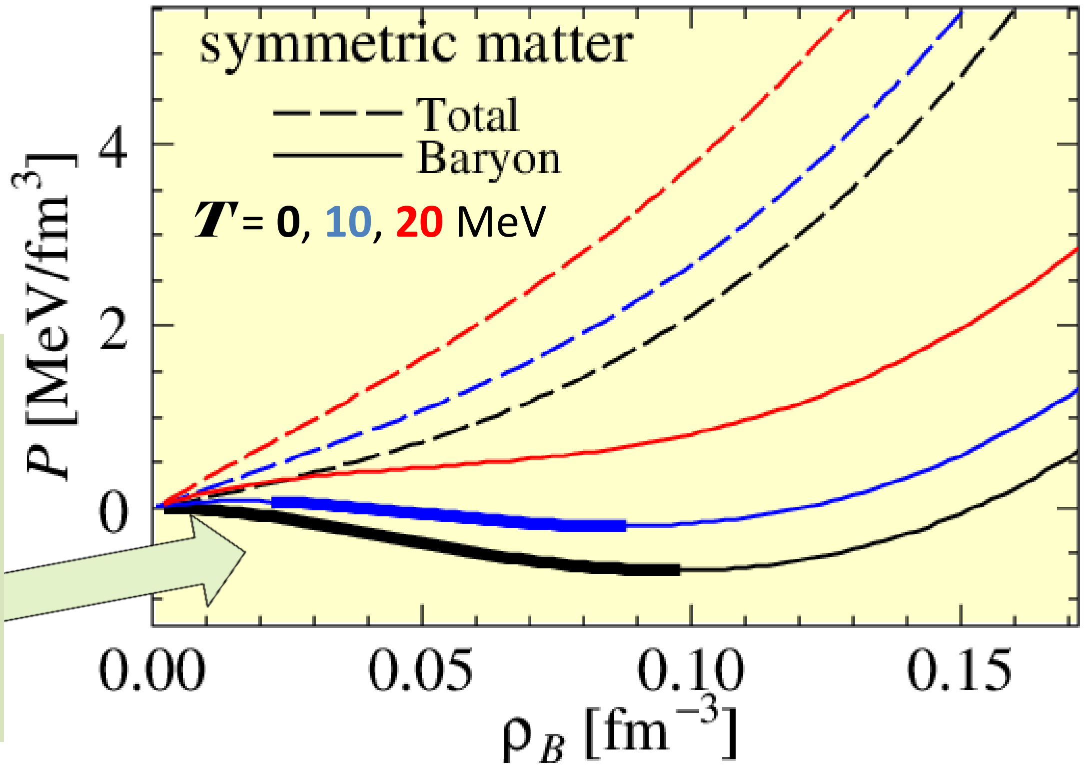


Saturation property of symmetric nuclear matter : minimum energy $E/A \approx -16$ MeV at $\rho_B \approx 0.16\ \text{fm}^{-3}$.

Pressure vs density of uniform matter

[Phys.Rev. C72 (2005) 015802]

• Total pressure is positive. Monotonically increases with density and temperature.

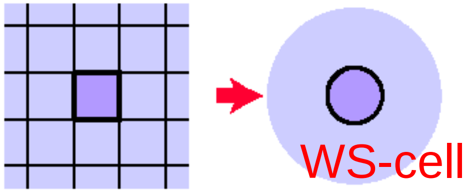


• Baryon partial pressure has ***van der Waals*** behavior
→ mechanically unstable region ($dP/d\rho < 0$).
→ First order phase transition.
→ **Inhomogeneous**

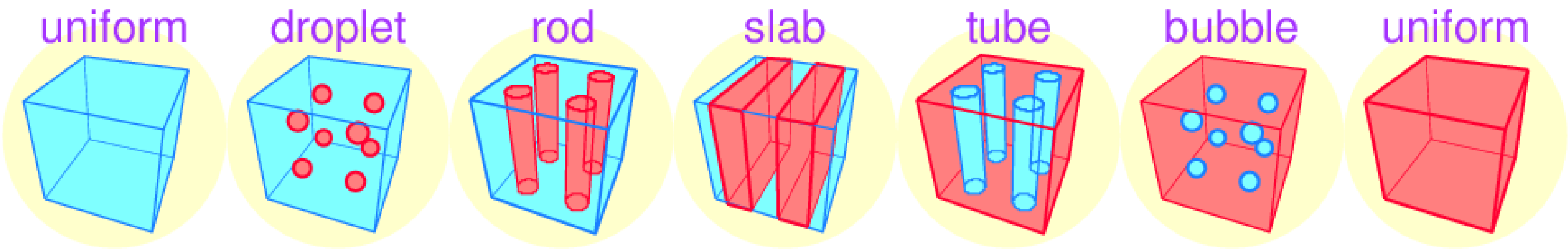
Numerical calculation of mixed-phase structure

• Assume regularity in structure: divide whole space into equivalent and neutral cells with a geometrical symmetry (3D: sphere, 2D : cylinder, 1D: plate).

→ Wigner-Seitz approx.



• Give a baryon density ρ_B and a geometry (Unif/Dropl/Rod/...).
• Solve the field equations numerically. Optimize the cell size (choose free-energy-minimum).
• Choose the free-energy-minimum geometry among 7 cases (Unif (I), Droplet, Rod, Slab, Tube, Bubble, Unif (II)),



which are called "**pasta**" structures.

RMF + Thomas-Fermi model

Lagrangian

$$L = L_N + L_M + L_e,$$

$$L_N = \bar{\Psi}\left[i\gamma^\mu \partial_\mu - m_N^* - g_{\omega N}\gamma^\mu \omega_\mu - g_{\rho N}\gamma^\mu \vec{\tau}\vec{b}_\mu - e\frac{1+\tau_3}{2}\gamma^\mu V_\mu\right]\Psi$$

$$L_M = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{R}_\mu\vec{R}^\mu,$$

$$L_e = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \bar{\Psi}_e\left[i\gamma^\mu\partial_\mu - m_e + e\gamma^\mu V_\mu\right]\Psi_e, \qquad (F_{\mu\nu} \equiv \partial_\mu F_\nu - \partial_\nu F_\mu)$$

$$m_N^* = m_N - g_{\sigma N}\sigma, \qquad U(\sigma) = \frac{1}{3}bm_N(g_{\sigma N}\sigma)^3 + \frac{1}{4}c(g_{\sigma N}\sigma)^4$$

Nucleons interact with each other via coupling with σ, ω, ρ mesons. Simple but feasible!

From $\partial_\mu\left[\partial L/\partial(\partial_\mu\phi)\right] - \partial L/\partial\phi = 0,$

$$(\phi = \sigma, \omega_\mu, R_\mu, V_\mu, \Psi),$$

$$-\nabla^2\sigma(\mathbf{r}) + m_\sigma^2\sigma(\mathbf{r}) = g_{\sigma N}(\rho_n^{(s)}(\mathbf{r}) + \rho_p^{(s)}(\mathbf{r})) - \frac{dU}{d\sigma}(\mathbf{r}),$$

$$-\nabla^2\omega_0(\mathbf{r}) + m_\omega^2\omega_0(\mathbf{r}) = g_{\omega N}(\rho_p(\mathbf{r}) + \rho_n(\mathbf{r})),$$

$$-\nabla^2 R_0(\mathbf{r}) + m_\rho^2 R_0(\mathbf{r}) = g_{\rho N}(\rho_p(\mathbf{r}) - \rho_n(\mathbf{r})),$$

$$\nabla^2 V_C(\mathbf{r}) = 4\pi e^2\rho_{\rm ch}(\mathbf{r}),$$

↑ Interaction between fermions

For fermions, we employ Thomas-Fermi approx. with finite T

$$f_{i=n,p}(\mathbf{r};\mathbf{p},\mu_i) = \left(1 + \exp\left[\left(\sqrt{p^2 + m_i^*(\mathbf{r})^2} - \sqrt{p_{Fi}(\mathbf{r})^2 + m_i^*(\mathbf{r})^2}\right)\Big/T\right]\right)^{-1},$$

$$f_e(\mathbf{r};\mathbf{p},\mu_e) = \left(1 + \exp\left[\left(p - (\mu_e - V_C(\mathbf{r}))\right)/T\right]\right)^{-1},$$

$$\rho_{i=p,n,e,\nu}(\mathbf{r}) = 2\int_0^\infty \frac{d^3p}{(2\pi)^3} f_i(\mathbf{r};\mathbf{p},\mu_i),$$

$$\mu_n = \sqrt{p_{Fn}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) - g_{\rho N}R_0(\mathbf{r}), \qquad \mu_n = \mu_p + \mu_e,$$

$$\mu_p = \sqrt{p_{Fp}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) + g_{\rho N}R_0(\mathbf{r}) - V_C(\mathbf{r}),$$

$$\int_V d^3r\left[\rho_p(\mathbf{r}) + \rho_n(\mathbf{r})\right] = \text{const}, \qquad \int_V d^3r\,\rho_p(\mathbf{r}) = \int_V d^3r\,\rho_e(\mathbf{r}),$$

Condition of a fermion →

Numerical procedure

Try to **equilibrate** $\mu_i(r)$ **in** $\boldsymbol{r}$ and **among species** $\boldsymbol{i}$

remove r-dependence $\mu_i(r) = \mu_i$

satisfy chemical balances $\mu_n = \mu_p + \mu_e$

} #

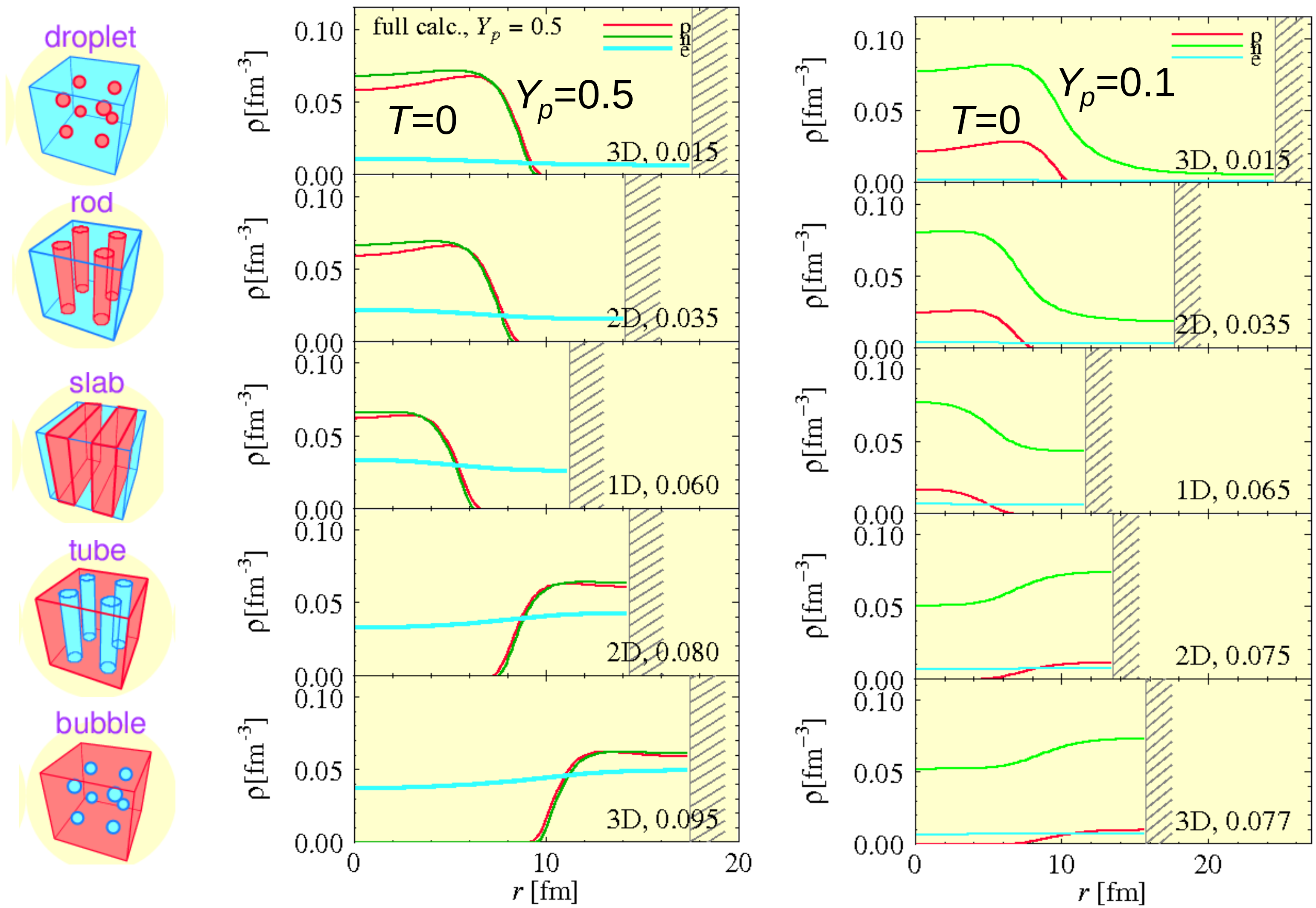
- Distribute fermions (p, n, e etc) with $\int d^3r\, \rho_i(r) = \text{given}$
- Solve field equations for $\sigma(r), \omega(r), \rho(r), V_{\text{Coul}}(r)$
- Calculate chemical potentials of fermions $\mu_i(r)$
- Adjust densities $\rho_i(r)$ as

  $\mu_i(r) > \mu_i(r') \rightarrow \rho_i(r) \downarrow, \rho_i(r') \uparrow$

  $\mu_n(r) > \mu_p(r) + \mu_e \rightarrow \rho_n(r) \downarrow, \quad \rho_p(r) \uparrow$
- repeat until #

Density profiles in WS cells

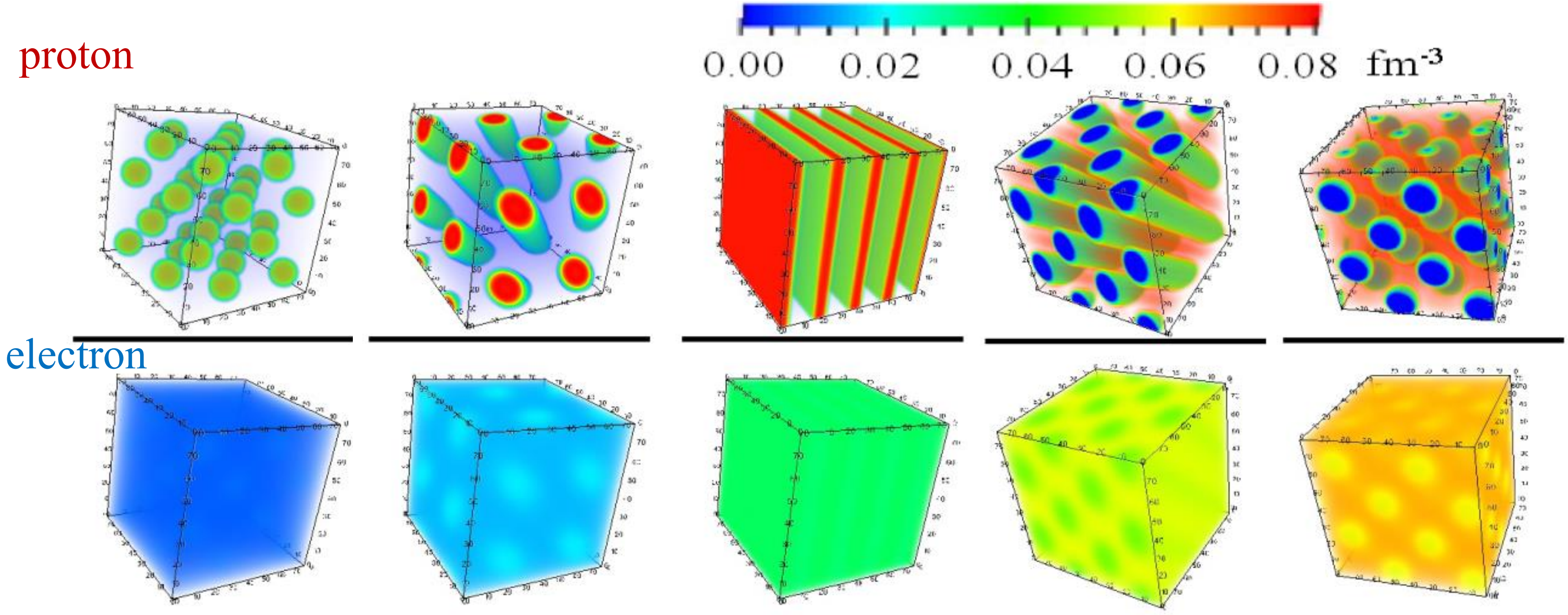
Wigner Seitz approx:
- assume geometrical symmetry.
- fast computation.



Fully 3D RMF+ThomasFermi calculations

[**Phys.Lett. B713 (2012) 284**]

$Y_p = Z/A = 0.5$



“droplet” [fcc] ρ_B = 0.012 fm^{-3}

“rod” [honeycomb] 0.024 fm^{-3}

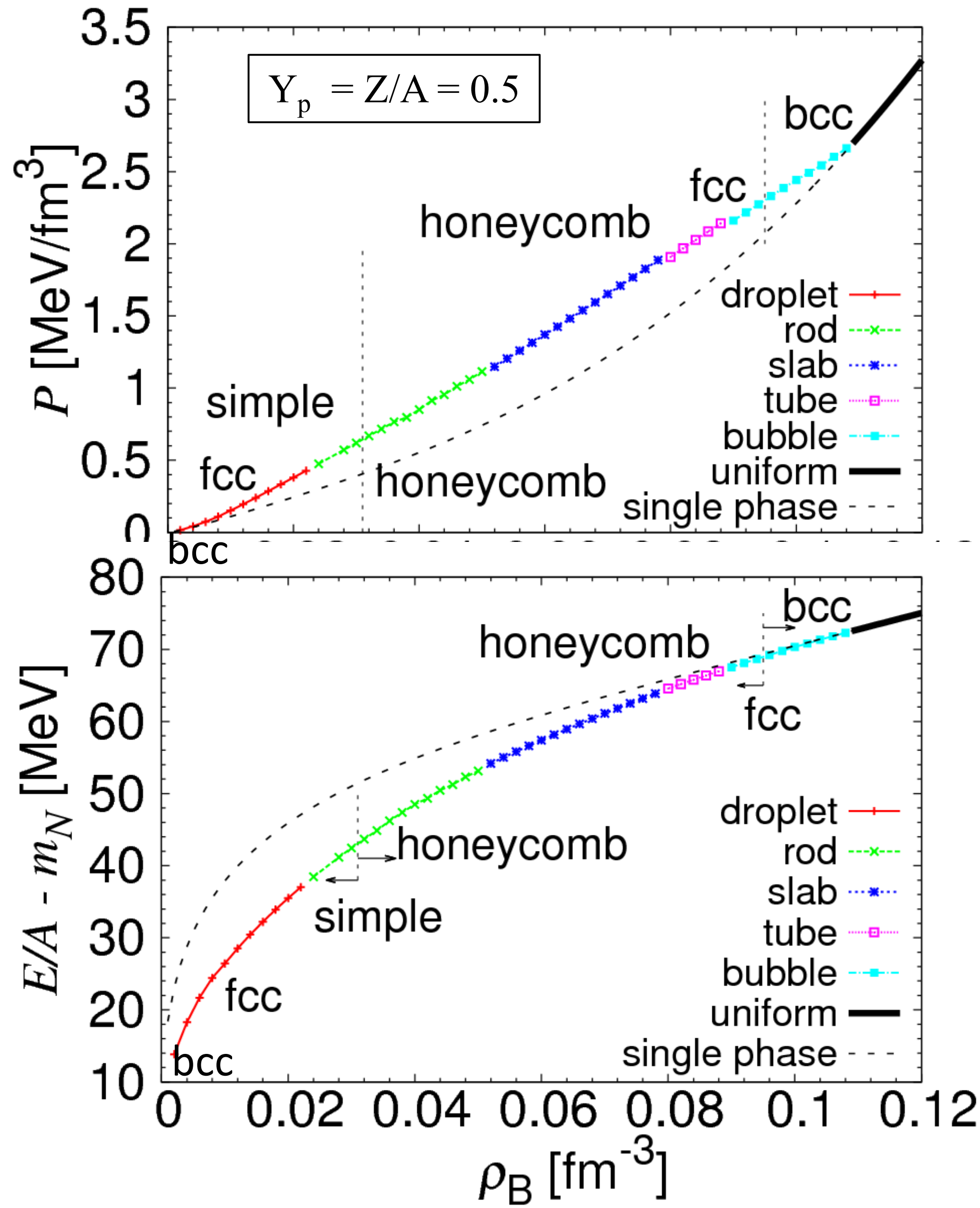
“slab” 0.05 fm^{-3}

“tube” [honeycomb] 0.08 fm^{-3}

“bubble” [fcc] 0.094 fm^{-3}

EOS has a similar behavior to that of the conventional studies.

Novelty:
fcc lattice of droplets can be the ground state at some density.
← Not the Coulomb interaction among "point particles" but the change of the droplet size is relevant.



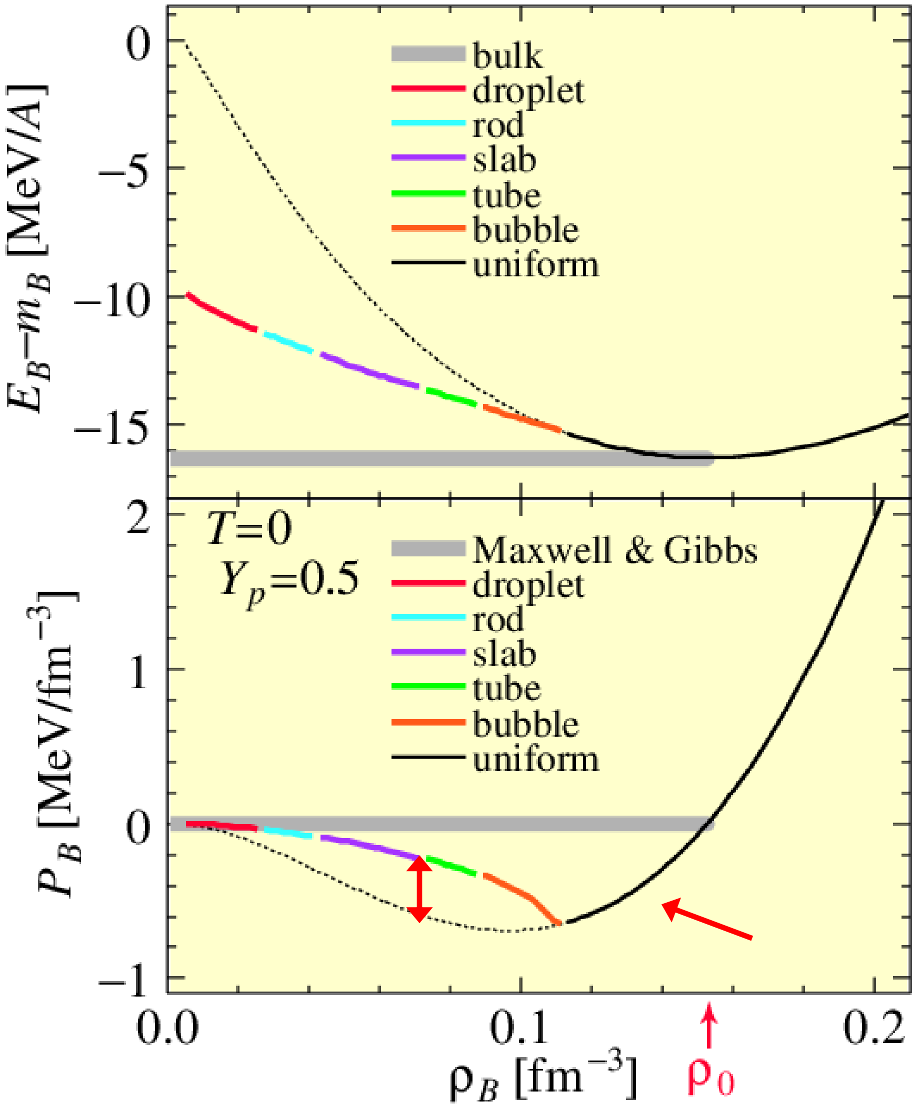
EOS of nuclear matter

Before and after clusterization

Since baryons and electron are **not congruent**, baryon partial system can clusterized.
Negative pressure (at $\rho_B < \rho_0$) is not favored.
→ mixed phase (clustering)

With pasta structures, baryon partial pressure increases. However, at some density just below ρ_0, uniform matter is favored due to **finite size effects** (surface and Coulomb).

Low-density nuclear matter is a mixed phase of liquid-gas, which exhibits pasta structures.

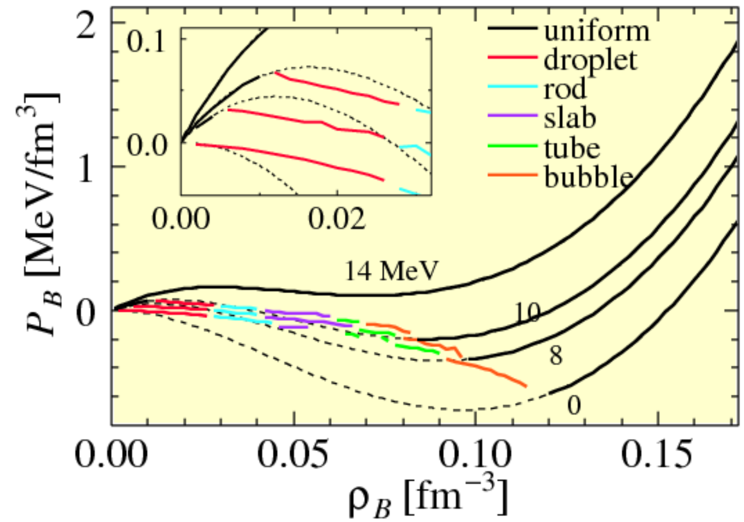


EOS with pasta structures in nuclear matter at $T \geq 0$

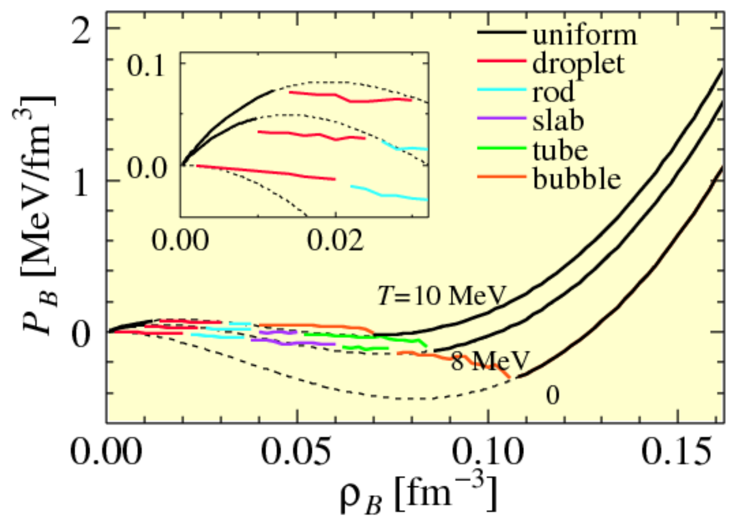
Pasta structures appear at $T \leq 10$ MeV

coexistence region (Maxwell for Y_p=0.5 and bulk Gibbs for Y_p<0.5) is meta-stable.
Uniform matter is allowed in some coexistence region due to finite-size effects.

Symmetric matter Y_p=0.5

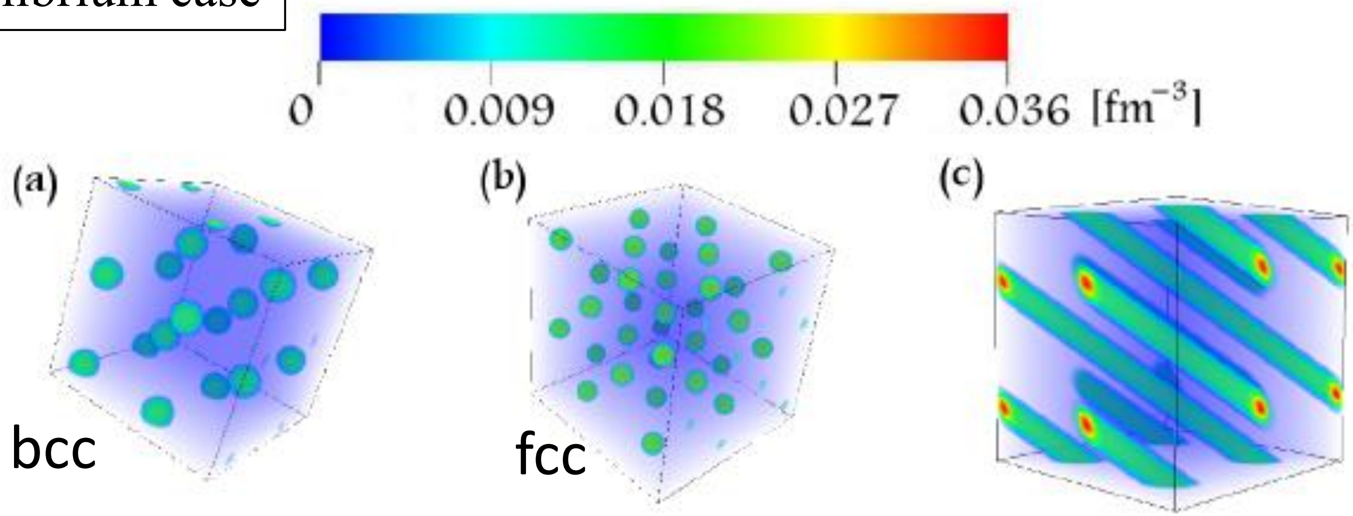


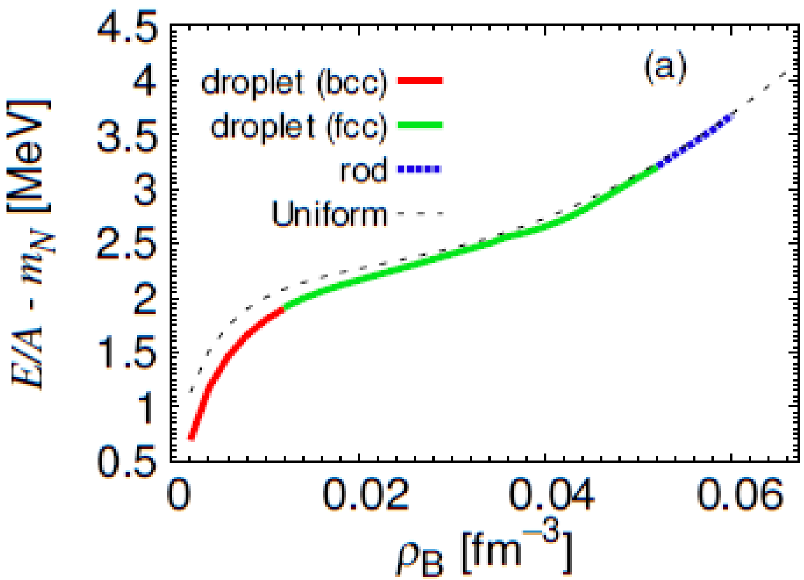
Asymmetric matter Y_p=0.3



Beta equilibrium case

[Phys. Rev. C **88**, 025801]





Slightly different from the WS approx.
Deoplets with bcc & fcc crystal.
Rod phase appears.

(2) Kaon condensation

[Phys. Rev. C **73**, 035802]

K single particle energy (model-independent form)

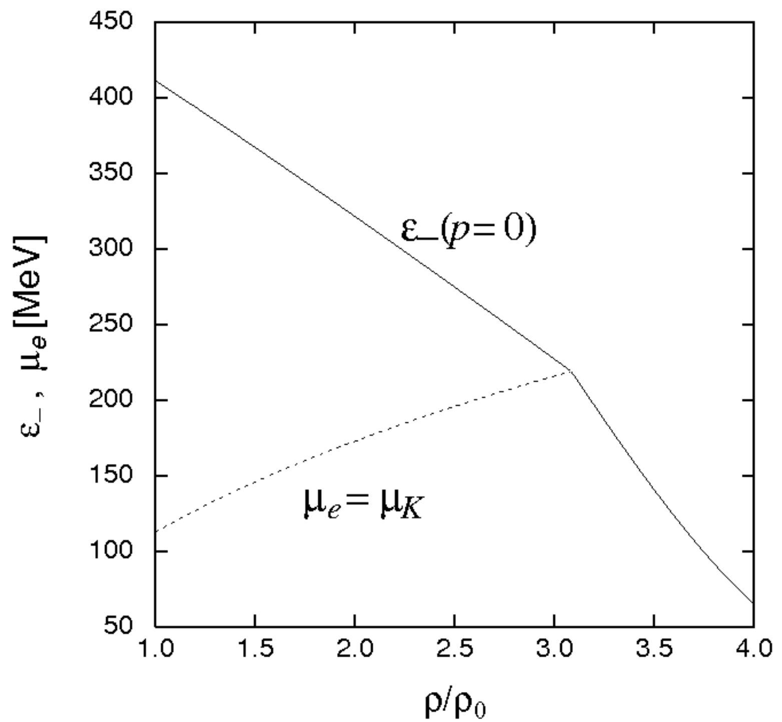
$$\varepsilon_{\pm}(\mathbf{p}) = \sqrt{p^2 + m_K^{*\,2} + \left(\left(\rho_n + 2\rho_p\right)/4f^2\right)^2} \pm \left(\rho_n + 2\rho_p\right)/4f^2,$$

From a Lagrangian with chiral symmetry

$$m_K^{*\,2} = m_K^{\,2} - \Sigma_{KN}\left(\rho_n + 2\rho_p\right)/4f^2,$$

$$\mu_K = \varepsilon_-\left(p = 0\right) = \mu_n - \mu_p = \mu_e$$

Threshold condition of condensation



EOM for fields (RMF model)

Kaon field $K(\mathbf{r})$ added.

$$\nabla^2 \sigma = m_\sigma^2 + \frac{dU_\sigma}{d\sigma} - g_{\sigma N}\left(\rho_p^s + \rho_n^s\right) - 4 g_{\sigma K} m_K f_K^2 K^2$$

$$\nabla^2 \omega_0 = m_\omega^2 \omega_0 - g_{\omega N}\left(\rho_n + \rho_p\right) - 2 g_{\omega K} m_K f_K^2 K^2 \left(\mu_K - V_C + g_{\omega K}\omega_0 + g_{\rho K} R_0\right)$$

$$\nabla^2 R_0 = m_\rho^2 R_0 - g_{\rho N}\left(\rho_n - \rho_p\right) - 2 g_{\rho K} m_K f_K^2 K^2 \left(\mu_K - V_C + g_{\omega K}\omega_0 + g_{\rho K} R_0\right)$$

$$\nabla^2 K = \left[m_K^{*\,2} - \left(\mu_K - V_C + g_{\omega K}\omega_0 + g_{\rho K} R_0\right) \right] K$$

$$\nabla^2 V_C = 4\pi e^2 \rho_{\mathrm{ch}}, \qquad \rho_{\mathrm{ch}} = \rho_p - \rho_e - \rho_K$$

$$\rho_K = 2\left(\mu_K - V_C + g_{\omega K}\omega_0 + g_{\rho K} R_0\right) K^2$$

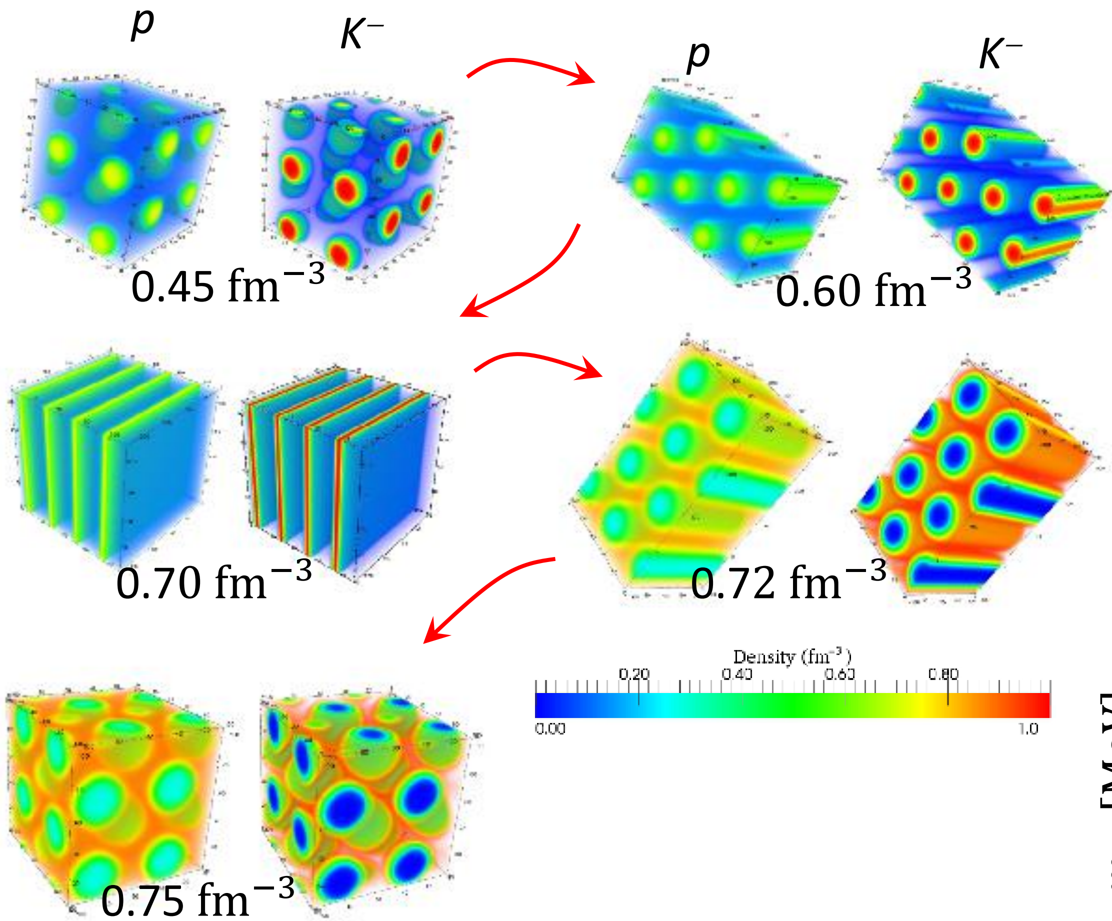
$$\mu_e = \left(3\pi\rho_e\right)^{1/3} + V_C$$

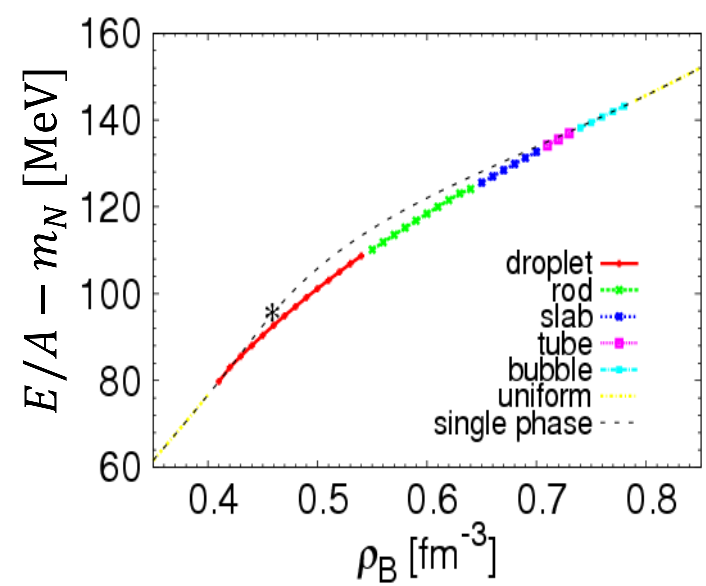
$$\mu_n = \sqrt{k_{Fn}^{\ 2} + m_N^{*\,2}} + g_{\omega N}\omega_0 - g_{\rho N} R_0$$

$$\mu_p = \sqrt{k_{Fp}^{\ 2} + m_N^{*\,2}} + g_{\omega N}\omega_0 + g_{\rho N} R_0 - V_C$$

[unpublished yet]

Fully 3D calculation



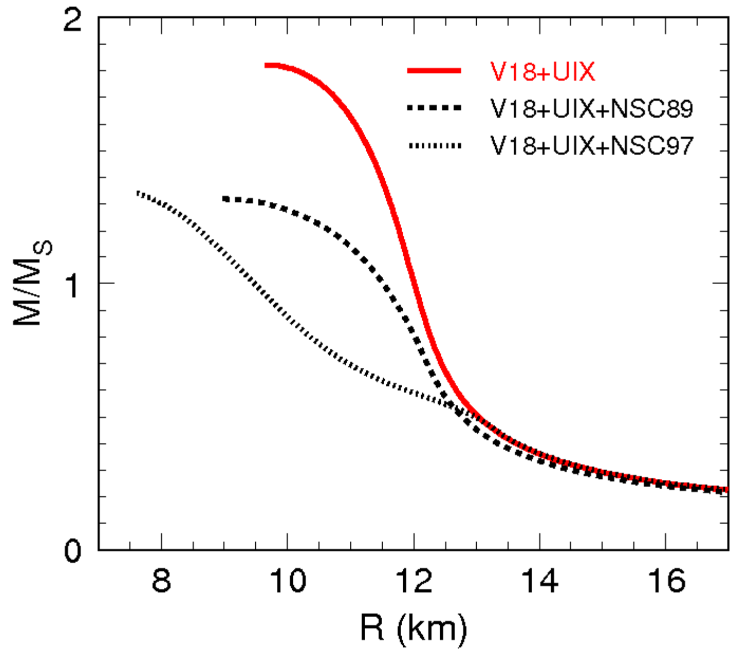


(3) Hadron-quark phase transition

[Phys. Rev. D **76**, 123015]

At 2--3ρ_0, hyperons are expected to appear.
→ Softening of EOS
→ Maximum mass of neutron star becomes less than 1.4 solar mass and far from 2 .0 solar mass.
→ Contradicts the obs >1.5 M_{sol}

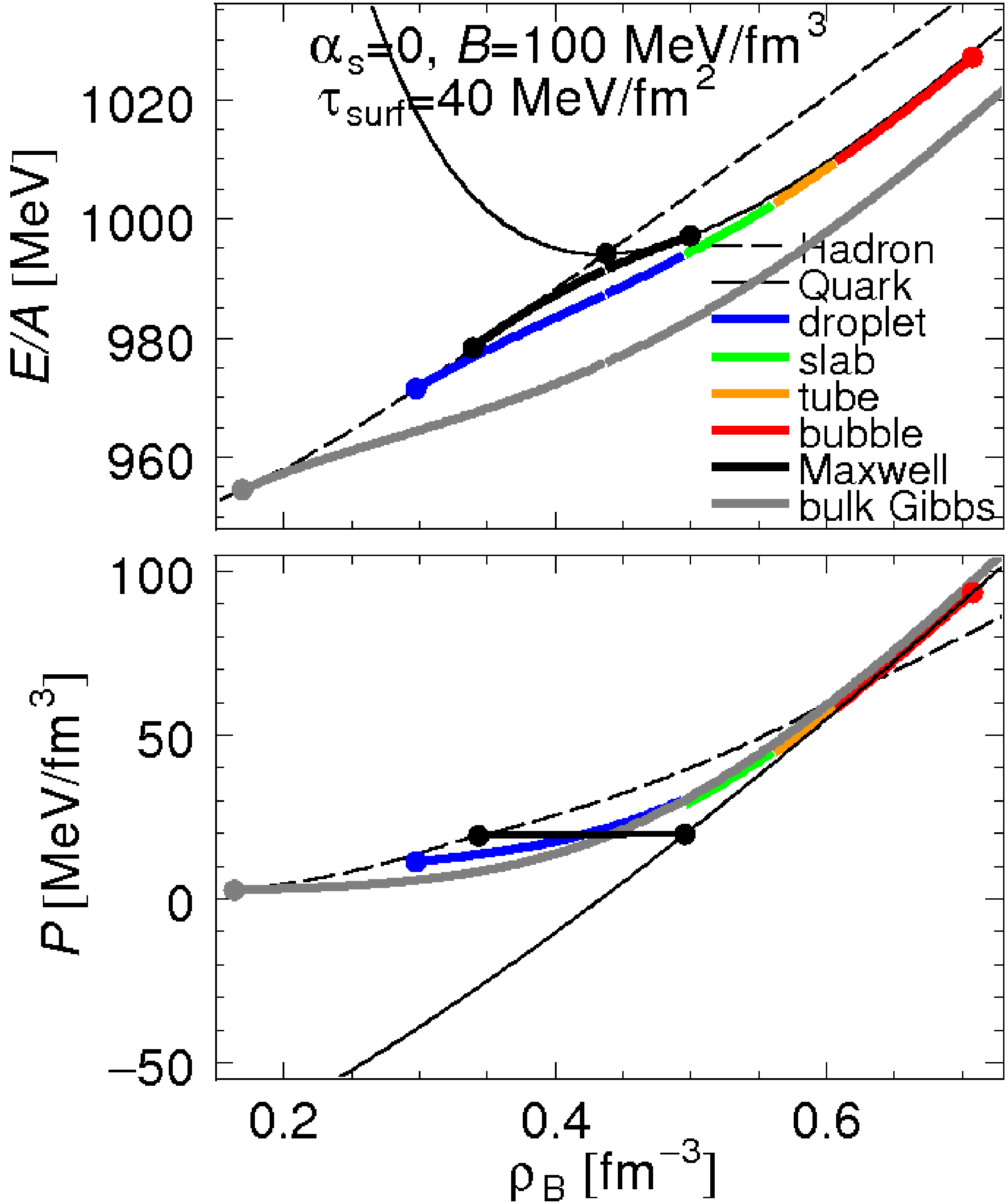
Schulze et al, PRC73 (2006) 058801



EOS of matter

Full calculation is between the Maxwell construction (local charge neutral) and the bulk Gibbs calculation (neglects the surface and Coulomb).

Closer to the Maxwell.



Structure of compact stars

TOV equation

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2}\left(1+\frac{4\pi r^3 P}{m}\right)\left(1+\frac{P}{\rho}\right)\left(1-\frac{2Gm}{r}\right)^{-1}$$

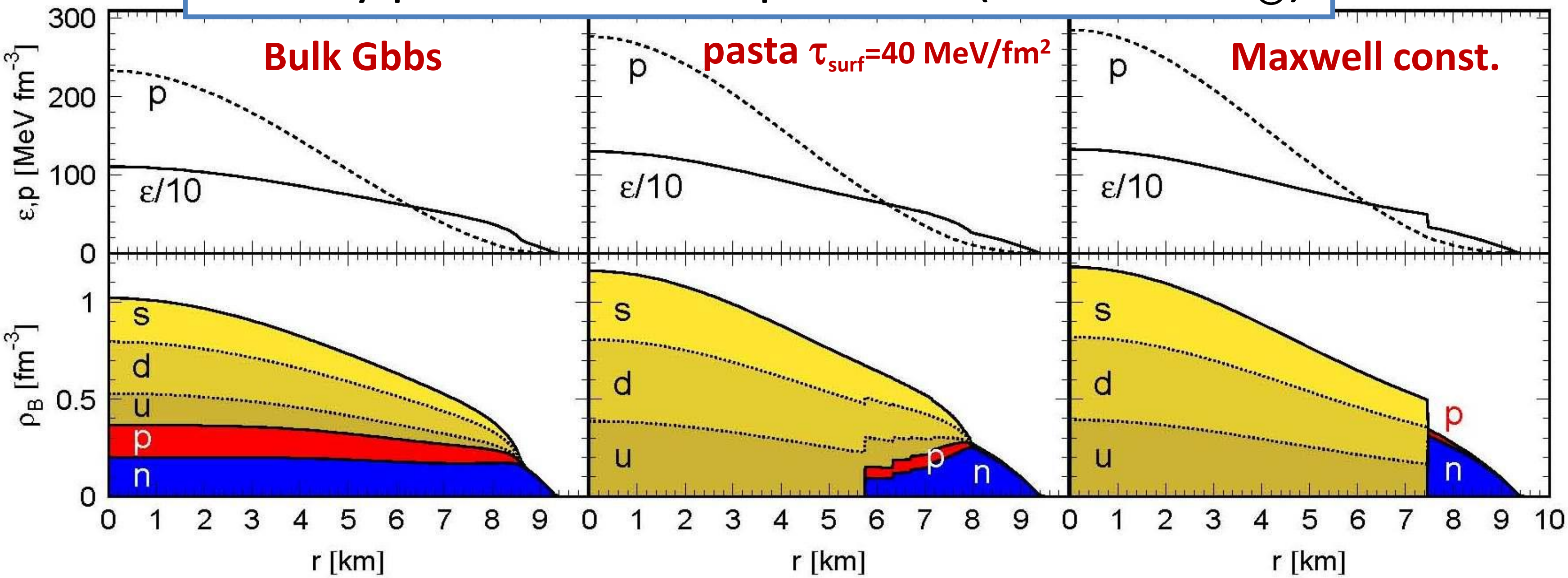
$P = P(\rho)$ Pressure (input of TOV eq.)

$\rho = \rho(r)$ Density at position r

$m = m(r) = \int_0^r 4\pi s^2 \rho(s) ds$ mass inside the position r

$M = m(R), \quad R = R(\rho \approx 0)$ total mass and radius.

Density profile of a compact star ($M = 1.4\ M_\odot$)

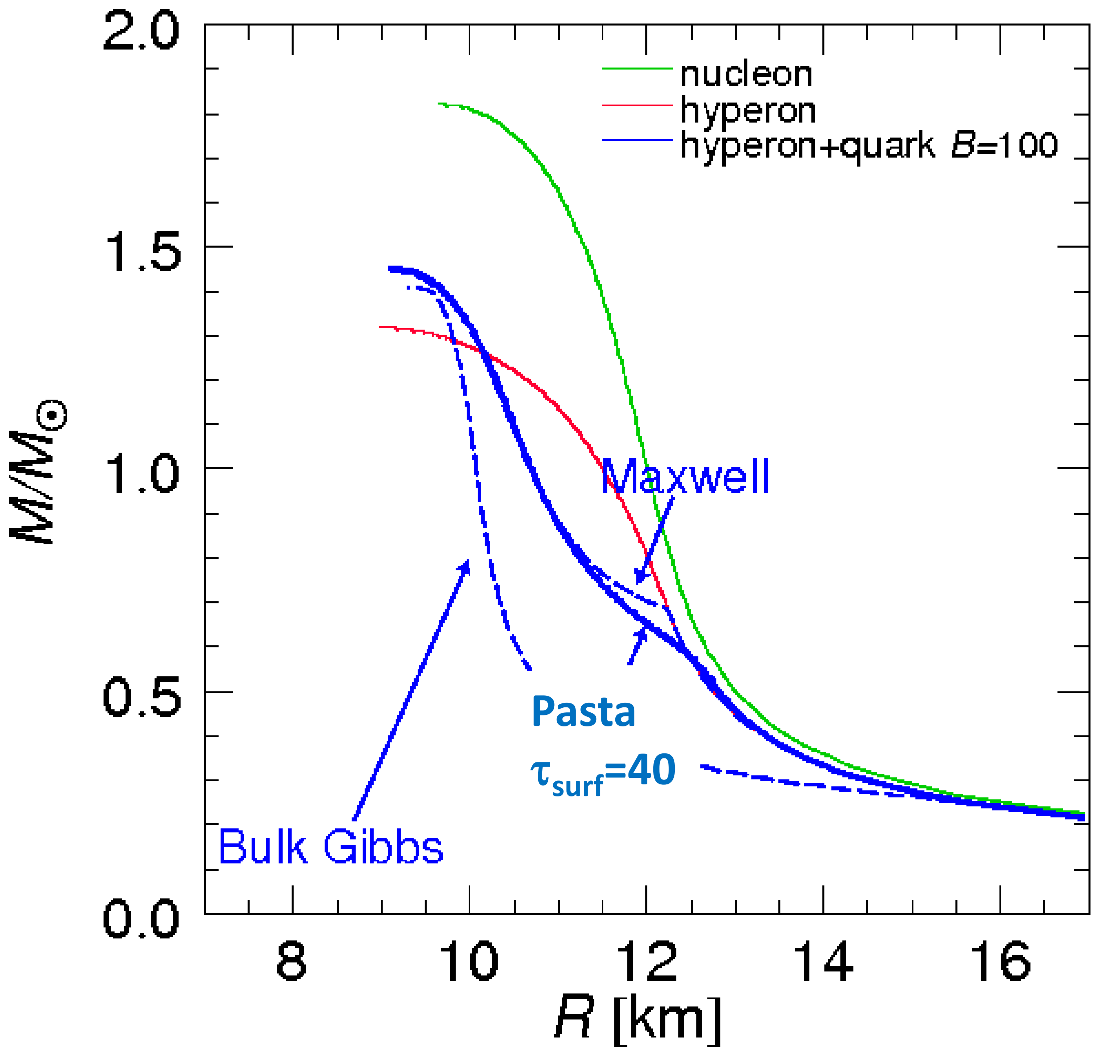


Mass-radius relation of a cold neutron star

Full calculation with pasta structures yields similar result to the Maxwell construction.

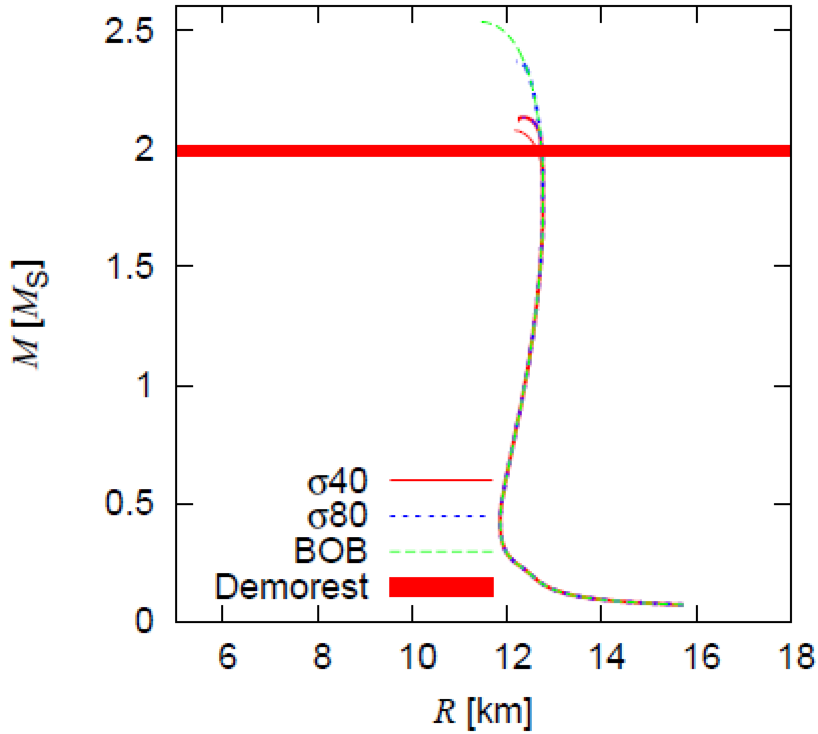
Maximum masses are almost the same for 3 cases.

We need to improve largely the quark EOS or hadron EOS to get $\sim 2M_{\odot}$



Another model for quark phase:
Schwinger-Dyson eq.

[arXiv:1309.1954]



[Yasutake, etal, arXiv1309:1954]

Summary

You should take into account the existence of mixed phases.

Low-density matter:

Mixed phase from the beginning (from low density).
Does not affect the structure and mass of neutron stars.
But important for neutrino emission and crust properties.

Kaon condensation:

crucial for the structure and mass of neutron stars.
Depends on the kaons potential.

Hadron-quark mixed phase:

crucial for the structure and mass of neutron stars.
Surface tension is unknown.

strong surface tension → larger size → charge neutrality → closer to Maxwell

weak surface tension → smaller size → Coulomb ineffective → closer to bulk Gibbs