Astrophysical Tests of Gravity

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JS arXiv: 1309.0495
Jain, Vikram, JS arXiv: 1204.6044
Davis, Lim, JS, Shaw arXiv: 1102.5278
see also Chang & Hui 1011.4107

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Modified Gravity

Modified Gravity manifests itself through the presence of additional or *fifth*-forces, we can use these as probes of new theories.

General Relativity has been tested to incredibly high precision over the last 100 years or so! So well that any modification is usually rendered irrelevant once the bounds are imposed e.g. Cassini:

$$\frac{F_5}{F_{\mathrm{N}}} < 10^{-5} \quad (10^{-13} \quad \text{for WEP violations})$$

Can we still have Interesting Theories?

Two different classes:

1. Gravity: Theories are tuned to satisfy the local bounds.

   Interesting effects in the strong gravity regime e.g. Spontaneous Scalarization Damour & Espositio-Farese 93.

2. Cosmology: Some sort of screening mechanism giving GR in high-density environments.

   Show novel effects in less dense systems e.g. new cosmological features.

This talk will be concerned with the second class.

Scalar-Tensor Screening

Scalar field is conformally coupled to matter via the "coupling function" $A(\phi)$ in the Einstein Frame:

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2R}{2}-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi-V(\phi)\right]+S_{\rm m}\left[\psi_{\rm i};A^2(\phi)g_{\mu\nu}\right]$$

Matter moves on geodesics of the Jordan Frame metric $\tilde{g}_{\mu\nu}=A^2(\phi)g_{\mu\nu}\Rightarrow$ observers in the Einstein frame infer a *fifth-force*

$$\vec{F}_\phi=\frac{\beta(\phi)}{M_{\rm pl}}\vec{\nabla}\phi \quad \beta(\phi)=M_{\rm pl}\frac{\mathrm{d}\ln(A)}{\mathrm{d}\phi}$$

Scalar-Tensor Screening

The equations of motion imply an effective potential for ϕ:

$$V_{\rm eff}(\phi) = V(\phi) + \rho A(\phi).$$

If this has a minimum then we can move the field value around as a function of the local density and can screen the fifth-force in dense environments if either:

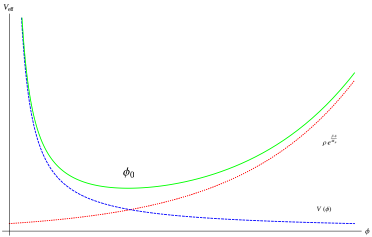
- $m_{\rm eff}(\phi_{\rm min})R \gg 1$ - the force is sub-mm (Chameleon).
- $\beta(\phi) \ll 1$ - the coupling to matter is negligible (Symmetron, Dilaton).

$$m_{\rm eff} = V_{\rm eff}(\phi_{\rm min})_{,\phi\phi} \quad \vec{F}_\phi = \frac{\beta(\phi)}{M_{\rm pl}}\vec{\nabla}\phi$$

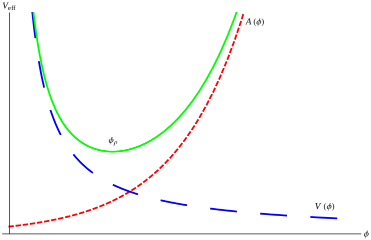
Chameleon Screening

Khoury & Weltman 03

$$V(\phi) = \frac{M^{4+n}}{\phi^n} \quad A(\phi) = e^{\beta \frac{\phi}{M_{\rm pl}}}$$



Low ρ

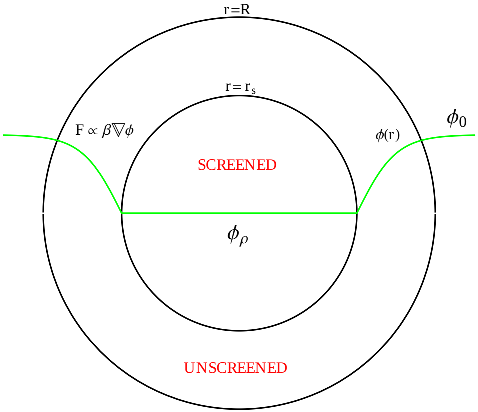


High ρ

$$m_{\rm eff}(\phi_\rho) \gg m_{\rm eff}(\phi_0)$$

$f(R)$ gravity is a chameleon under some circumstances.

Spherical Screening



Screening Parameterisation

There is a model-independent parameterisation perfect for small-scale tests:

$\alpha \equiv 2\beta(\phi_0)^2 \quad G \to G(1+\alpha)$ when the object is fully unscreened.

$f(R)$ gravity has $\alpha = 1/3$.

$$\chi_0 \equiv \frac{\phi_0}{2M_{\rm pl}\beta(\phi_0)} \text{ - the self-screening parameter.}$$

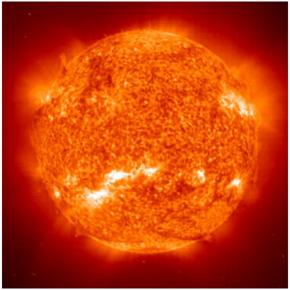
Rule of thumb: an object is partially unscreened if

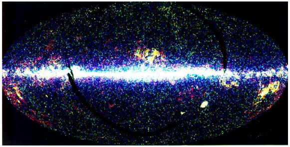
$$\chi_0 > \Phi_{\rm N}.$$

$\Phi_{\rm N}$ is the Newtonian Potential.

Previous Constraints

$$\chi_0 \sim 10^{-6}$$





The Sun and Milky Way have Newtonian potential $\Phi_N \sim 10^{-6}$ so we impose $\chi_0 < 10^{-6}$.

Stellar Structure Tests

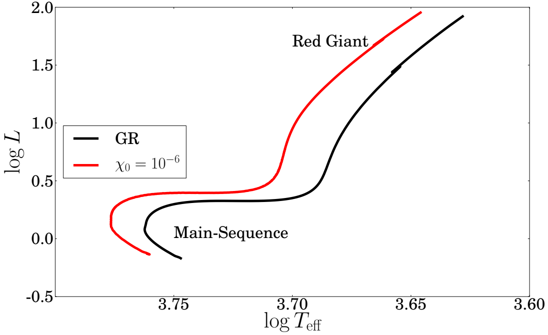
Stars in modified gravity:

- Need to burn more fuel per unit time to stave off gravitational collapse.
- More compact
- Hotter
- Brighter

We have a modified stellar structure code, MESA, which can simulate real stars including modified gravity.

Modified Gravity with MESA

$$M = 1M_\odot \quad \alpha = 1/3$$



Davis, Lim, JS & Shaw 2011

Cepheid Distance Indicator Tests

We exploit these effects by comparing distances from screened and unscreened indicators.

The period of Cepheid pulsations changes in modified gravity:

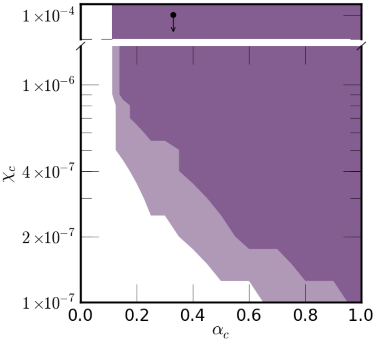
$$\log d \propto \log \tau$$

$$\tau \propto G^{-\frac{1}{2}}$$

$$\frac{d_{\rm MG} - d_{\rm GR}}{d_{\rm GR}} \approx -0.3\frac{\Delta G}{G}$$

We compute $\frac{\Delta G}{G}$ using MESA profiles and compare with tip of the red giant branch distances (insensitive to modified gravity).

Constraints



Jain, Vikram & JS 2012

$$\chi_0 \lesssim 4 \times 10^{-7} \quad (f(R)\ \text{Gravity})$$

These are currently the strongest constraints in the literature.

Motivation

$$\frac{d_{\rm MG}-d_{\rm GR}}{d_{\rm GR}}\approx -0.3\frac{\Delta G}{G}$$

is an approximation - need full hydrodynamic perturbation theory.

$$\ddot{\vec{r}} = -\frac{1}{\rho}\nabla P + \underbrace{\vec{f}}_{\text{force per unit mass}}$$

$$\vec{f} = -\frac{GM(r)}{r^2} - \frac{\beta(\phi)}{M_{\rm pl}}\nabla\phi$$

Modified Linear Adiabatic Wave Equation

$$\frac{\delta r}{r} = \xi(r) e^{i\omega t} \quad \Gamma_{1,0} = \frac{\partial \ln P_0}{\partial \ln \rho_0}$$

General Relativity:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^4 \Gamma_{1,0} P_0 \frac{\mathrm{d}\xi}{\mathrm{d}r}\right) + r^3 \frac{\mathrm{d}}{\mathrm{d}r}\left[\left(3\Gamma_{1,0} - 4\right) P_0\right] \xi + r^4 \rho_0 \omega^2 \xi = 0$$

This is a Sturm-Liouville problem.

The eigenfrequencies give the periods of oscillation.

Modified Linear Adiabatic Wave Equation

$$\frac{\delta r}{r} = \xi(r) e^{i\omega t} \quad \Gamma_{1,0} = \frac{\partial \ln P_0}{\partial \ln \rho_0}$$

Modified Gravity:

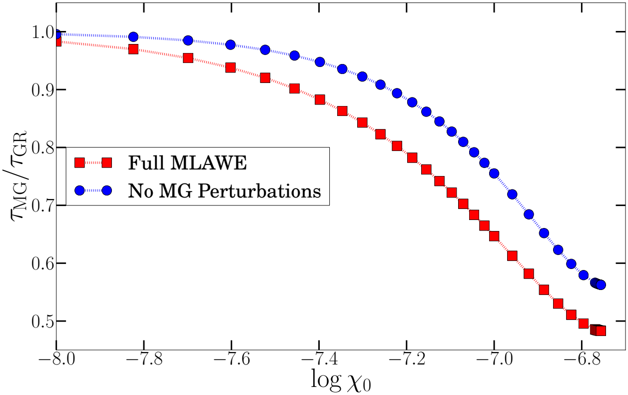
$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^4 \Gamma_{1,0} P_0 \frac{\mathrm{d}\xi}{\mathrm{d}r}\right) + r^3 \frac{\mathrm{d}}{\mathrm{d}r}\left[\left(3\Gamma_{1,0} - 4\right) P_0\right]\xi - 4\pi\alpha G r^4 \rho_0^2 \xi + r^4 \rho_0 \omega^2 \xi = 0$$

This is a Sturm-Liouville problem.

The eigenfrequencies give the modified periods of oscillation.

Example: Lane-Emden Models

Semi-analytic model (Davis, Lim, JS & Shaw 2011) with $P = K\rho^{\frac{5}{3}}$:



$$\frac{GM}{R} = 10^{-7}$$

Cepheid Distances

α	χ_0	$\Delta d/d$ (approx)	$\Delta d/d$ (LAWE)	$\Delta d/d$ (MLAWE)
$1/3$	4×10^{-7}	-0.03	-0.04	-0.12
$1/2$	4×10^{-7}	-0.05	-0.06	-0.16
1	2×10^{-7}	-0.06	-0.07	-0.19

Full hydrodynamic prediction is 3 times larger than the equilibrium prediction!

Can improve the constraints using the same data sets.

Future Constraints

Vary χ_0 until $\Delta d/d$ matches the approximation at fixed α:

α	χ_0
$1/3$	9×10^{-8}
$1/2$	7×10^{-8}
1	3×10^{-8}

Table: JS, 2013

The constraints can be greatly improved. This is work in progress

Summary

- Astrophysical tests can constrain chameleon gravity to levels unreachable by other means.
- Using an approximation we have placed the tightest constraints to date.
- We have investigated this by deriving the full equations governing perturbations in modified gravity hydrodynamics.
- We find that the period of Cepheid oscillations could be three times shorter than we previously predicted.
- This means we can place even tighter constraints with the same data sets.

Code Comparisons

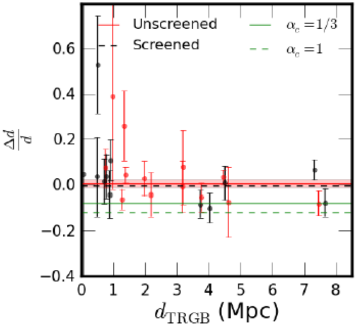
We compare our GR dimensionless frequencies

$$\tilde{\omega}^2 = \frac{(n+1)\omega^2}{4\pi G\rho_c}$$

to those found by Hurley, Roberts & Wright 1966:

n	Hurley, Roberts & Wright	Me
0.5	0.37071	0.370714029
1.0	0.38331	0.38331184243
1.5	0.37640	0.376399032288
2.0	0.35087	0.350866992807
2.5	0.30389	0.303893585012
3.0	0.22774	0.227742000109
3.25	0.17731	0.177307835186
3.5	0.12404	0.124042556661
4.0	0.04056	0.0405613874985

Distance Comparisons



Systematics