Gravitational lensing with $f(\chi) = \chi^{3/2}$ gravity.

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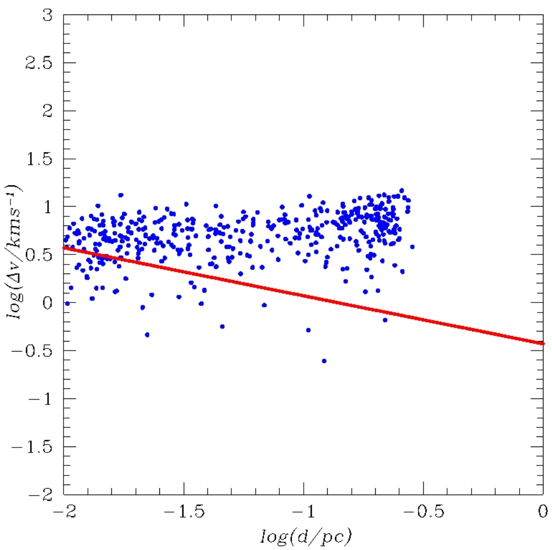
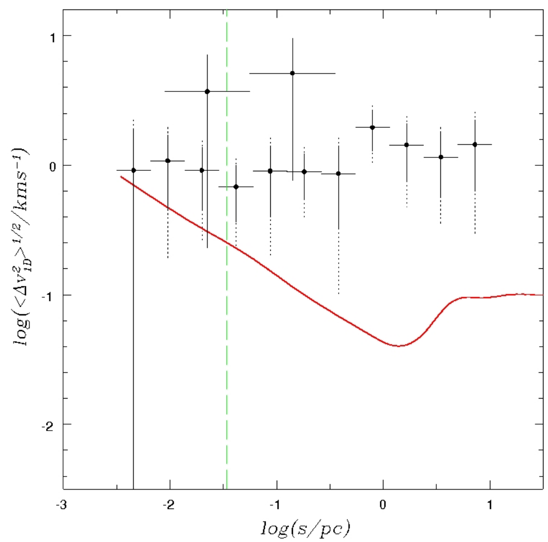
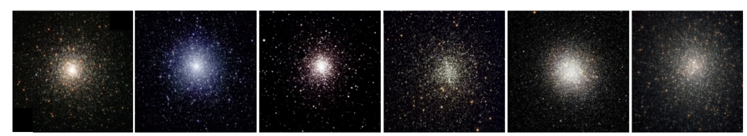
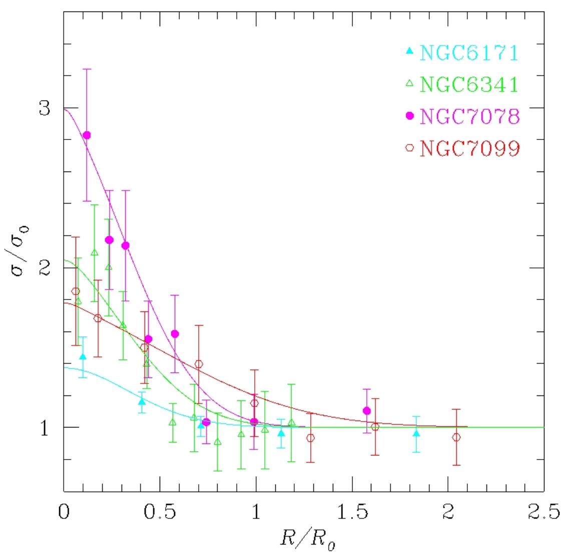
The relative velocities of observed wide binaries are inconsistent with full Newtonian Gravity models even

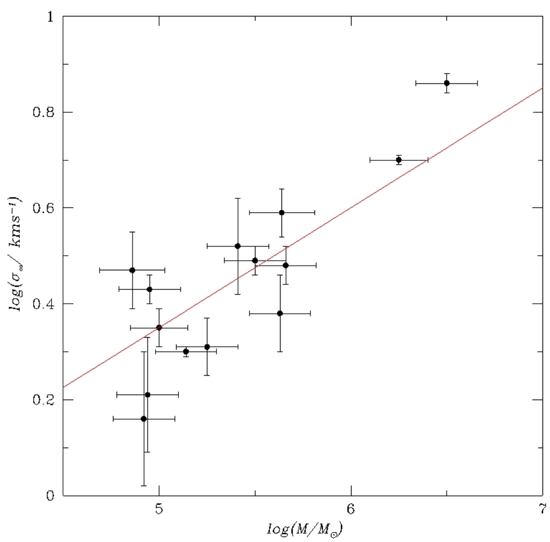
including 10Gyr of evolution in the Galactic environment, tidal fields, ambient perturbers. The inconsistency appears precisely on crossing the a_0 threshold.











(Hernandez et al 2012a,b) Outer velocity dispersions of globular clusters become flat and show the same galactic $v = (GMa_0)^{1/4}$ TF scaling.

(see X. Hernandez talk on Thursday).

Newtonian gravity

- Rotation curves (Kepler's third law):

$$v \propto \frac{M^{1/2}}{r^{1/2}}.$$

- Centrifugal balance $a \propto v^2/r$.
- Acceleration force is then:

$$a = -G_{\rm N} \frac{M}{r^2}.$$

- Callibrate with observations:

$$G_{\rm N} = 6.67 \times 10^{-11}\,{\rm m}^3\,{\rm s}^{-2}\,{\rm kg}^{-1}.$$

Extended gravity

- Rotation curves (Tully-Fisher law):

$$v \propto M^{1/4}.$$

- Centrifugal balance $a \propto v^2/r$.
- Acceleration force is then:

$$a = -G_{\rm M} \frac{M^{1/2}}{r}.$$

- Callibrate with observations:

$$G_{\rm M} \approx 8.94 \times 10^{-11}\,{\rm m}^2\,{\rm s}^{-2}\,{\rm kg}^{-1/2}.$$

- Simplest form of MOND found since[a]

$$a_0 := \frac{G_{\rm M}^2}{G_{\rm N}}.$$

[a] Mendoza et al. (2011) built an extended non-relativistic theory of gravity using a_0 as a fundamental theory of gravity. This approach fits observations of solar system, globular clusters, fundamental plane of elliptical galaxies, dSph galaxies, globular clusters.

Relativistic Kepler's 3rd law

- Weakfield geodesic motion (massive particles):

$$g_{00} = 1 + \frac{2\phi}{c^2} = 1 + \frac{2G_{\rm N}M}{rc^2}.$$

- Isotropic coordinates:

$$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 - \left(1 + 2\gamma\phi/rc^2\right)\delta_{kl}\mathrm{d}x^k\mathrm{d}x^l.$$

- Spherical coordinates:

$$g_{rr} = -1 - \frac{2\gamma G_{\rm N}M}{rc^2}.$$

Lensing observations imply $\gamma = 1$. Schwarzschild solution of Einstein's field equations also imply $\gamma = 1$.

Relativistic Tully-Fisher law

- Weakfield geodesic motion (massive particles):

$$g_{00} = 1 + \frac{2\phi}{c^2} = 1 - \frac{2G_{\rm M}M^{1/2}}{c^2}\ln\left(\frac{r}{r_\star}\right).$$

- Isotropic coordinates:

$$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 - \left(1 + 2\gamma\phi/rc^2\right)\delta_{kl}\mathrm{d}x^k\mathrm{d}x^l.$$

- Spherical coordinates:

$$g_{rr} = -1 - \frac{2\gamma G_{\rm M}M^{1/2}}{c^2}$$

Lensing observations imply $\gamma = 1$.
$f(\chi) = \chi^{3/2}$ also imply $\gamma = 1$.
Mendoza et al. (2013)
Mendoza & Olmo (2013)

- Lensing on elliptical, spiral and galaxy groups can be modelled using total matter distributions with isothermal profiles ($M_{\mathrm{T}} = v^2 r/G$ and DM profiles obey the same Tully-Fisher relation of baryonic matter of spirals: $v \propto M_{\mathrm{b}}$.
- Take GR -Schwarzschild- + DM.

$$g_{00\mathrm{S}} = 1/g_{11\mathrm{S}} = 1 - \frac{2r_{\mathrm{g}}}{r} = 1 - \frac{2GM_{\mathrm{T}}(r)}{c^2 r} = 1 - 2\left(\frac{v}{c}\right)^2. \tag{1}$$

- The deflection angle $\beta_{GR} = F(g_{00\mathrm{S}}, g_{11\mathrm{S}}, r_i)$ can thus be calculated.
- This deflection angle is THE SAME for any metric theory of gravity and so $\beta_{GE} = \beta Ext.$
- Las relation is valid for all r_i and so, it is possible to find $g_{11\mathrm{Ext}}$ at $\mathcal{O}(2)$.

In short, Tully-Fisher law and lensing observations, at perturbation order $\mathcal{O}(2)$, the extended spherically symmetric metric is:

$$g_{00} = 1 + \frac{2\phi}{c^2} = 1 - \frac{2G_{\mathrm{M}}M^{1/2}}{c^2} \ln\left(\frac{r}{r_\star}\right),$$

$$g_{rr} = -1 - \frac{2G_{\mathrm{M}}M^{1/2}}{c^2}.$$

Mendoza et al. (2013), Mendoza & Olmo (2013)

Metric $f(\chi)$ extension

$$S_{\mathrm{H}} = -\frac{c^3}{16\pi G}\int \frac{f(\chi)}{L^2}\sqrt{-g}\,\mathrm{d}^4x, \qquad \chi := L^2 R, \tag{2}$$

$$L = \frac{2\sqrt{2}}{9}\left(r_{\mathrm{g}}\, l_M\right)^{1/2}, \qquad r_{\mathrm{g}} := \frac{GM}{c^2}, \qquad l_M := \left(\frac{GM}{a_0}\right)^{1/2}. \tag{3}$$

$$f(\chi) = \begin{cases} \chi & \Longrightarrow \quad \text{general relativity} \\ \chi^{3/2} & \Longrightarrow \quad \text{extended relativistic Tully-Fisher} \end{cases}$$

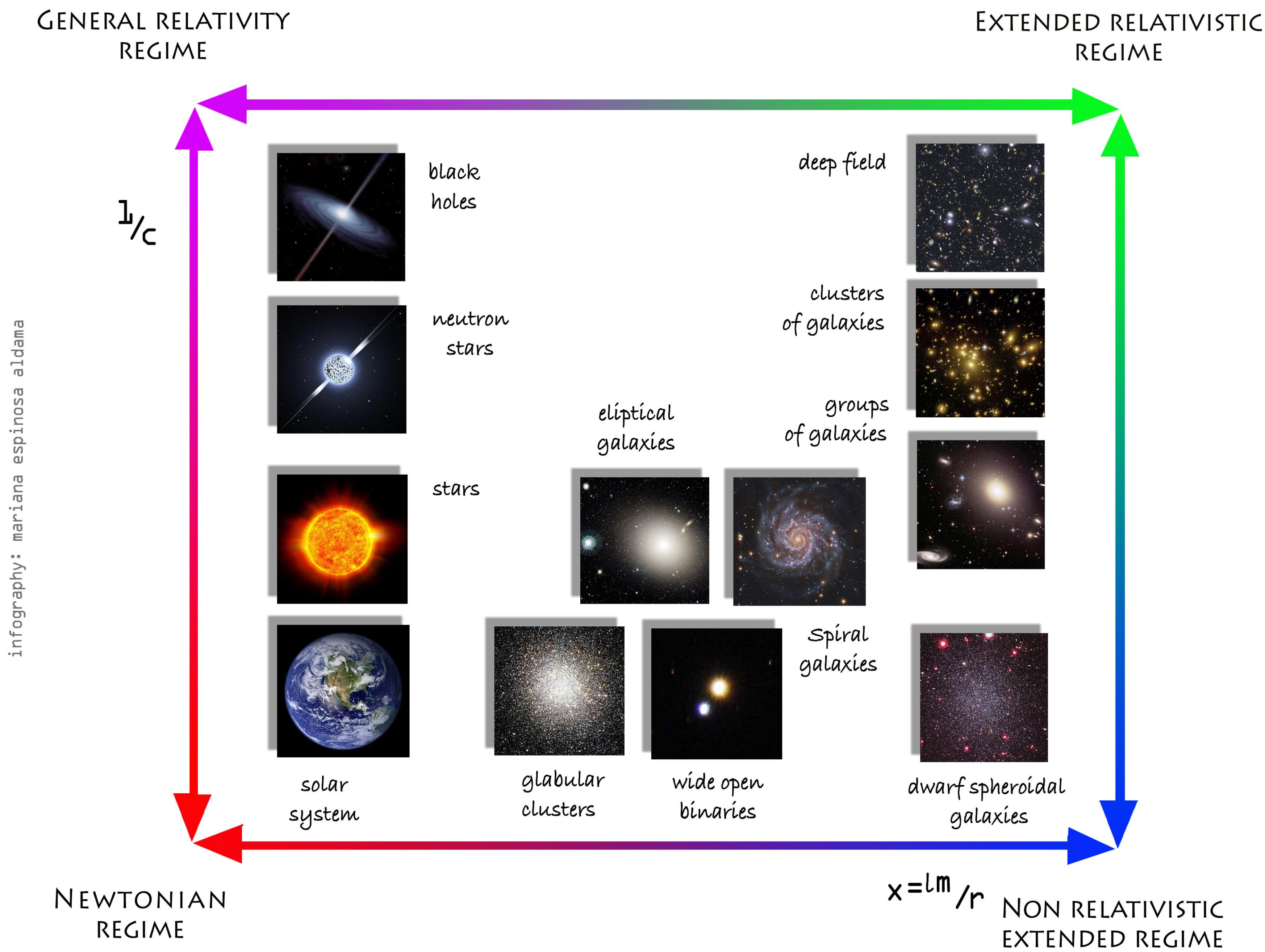
Connection with Harko et al. (2012) approach for “symmetrical” systems:

$$F(R,T) := f(\chi)/L^2, \qquad T := T^{\mu}{}_{\mu}, \qquad M := 4\pi \int \rho r^2 \mathrm{d}^3x = \frac{4\pi}{c^2}\int T \mathrm{d}^3x$$

Mendoza (2012), Carranza et al. (2013).

$f(\chi) = \chi^{3/2}$

- Bernal et al. (2012) showed that $f(R) = R^{3/2}$ and $f(\chi) = \chi^{3/2}$ yield Tully-Fisher behaviour in the weak-field limit at order $\mathcal{O}(2)$.
- Mendoza et al. (2013) showed that perturbations to order $\mathcal{O}(2)$ in spherical symmetry of $f(\chi) = \chi^{3/2}$ DOES yield correct lensing observations with $\gamma = 1$, but $f(R) = R^{3/2}$ DOES NOT.
- Working with $f(R)$ perturbations? Check `mendozza.org/sergio/mexicas`. A GPL'd open source code. Metric EXtended-gravity Incorporated through a Computer Algebra System (MEXICAS) Copyright © 2013 T. Bernal, S. Mendoza & L.A. Torres.
- Carranza et al. (2013) and Mendoza (2012) showed a possible generalisation with Harko et al. (2011) theory: $F(R, T) = f(\chi)/L^2$ and check if an $f(\chi) = \chi^b$ could explain current accelerated expansion of the universe. Their results (see DA Carranza's poster) show that $b \approx 1.5$ can do it and so *NO* DM and *NO* DE are required.
- Fourth order $\mathcal{O}(4)$ corrections to the dynamics of clusters of galaxies can be fitted with the theory (Bernal & Mendoza -in preparation).


GENERAL RELATIVITY REGIME
EXTENDED RELATIVISTIC REGIME
1/c
black holes
neutron stars
stars
solar system
deep field
clusters of galaxies
groups of galaxies
eliptical galaxies
spiral galaxies
glabular clusters
wide open binaries
dwarf spheroidal galaxies
infography: mariana espinosa aldama
NEWTONIAN REGIME
x = lm /r
NON RELATIVISTIC EXTENDED REGIME