Testing modified gravity at large distances with rotation curves

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The Grumiller's Theory

In Grumiller's modified gravity theory, the spacetime is described by the EH action with the spherically symmetric metric

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta + \Phi^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where the two dimensional metric $g_{\alpha\beta}(x^\gamma)$ and the surface radius $\Phi(x^\gamma)$ depends only on the coordinates $x^\gamma = \{t, r\}$. The 4-dimensional spherical spacetime manifold M can be decomposed to $M = L \otimes S$, L: radial, S:angular. The "spherical reduction" process simplifies the 4-dimensional EH action to a specific 2-dimensional dilation gravity model, demanding the IR-theory to be power-counting renormalizable and non-singularity of curvature at large Φ. One obtains

$$S = -\int d^2x\sqrt{-g}[f(\Phi)R + 2(\partial\Phi)^2 - 2V(\Phi)], \tag{2}$$

where

$$V(\Phi) \equiv 3\Lambda\Phi^2 - 4a\Phi - 1. \tag{3}$$

The Grumiller's Theory

The solutions, back in 4-d, are:

$$g_{\mu\nu} = \mathrm{diag}(-K^2, 1/K^2, r^2, r^2 \sin^2\theta) \quad \text{with} \quad K^2 \equiv 1 - \frac{2M}{r} - \Lambda r^2 + 2ar. \tag{4}$$

For a point particle with energy E and angular momentum ℓ moving along a geodesic in the plane $\theta = \pi/2$ in the background of the metric Eq.(4) one has:

$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 = \frac{1}{2}E^2 - V^{\text{eff}}, \tag{5}$$

where

$$V^{\text{eff}} \equiv \frac{K^2}{2}\left(1 + \frac{\ell^2}{r^2}\right) = -\frac{M}{r} + \frac{\ell^2}{2r^2} - \frac{M\ell^2}{r^3} - \frac{\Lambda r^2}{2} + ar(1 + \frac{\ell^2}{r^2}) + \ldots \tag{6}$$

is the effective potential.

Modified Newtonian Gravity

In Grumiller's approach the effective potential of a point mass (M) is:

$$\phi_i(r) = -G\frac{M}{r} + a\,r, \tag{7}$$

where *a* is a universal constant, the Rindler acceleration.
For a smooth matter distribution with spherical symmetry one has:

$$\phi(r) = -G\int \frac{\rho(r\prime)}{|\vec{r}-\vec{r\prime}|}\,\mathrm{d}^3r\prime + a\,|\vec{r}|, \tag{8}$$

where $\rho(r)$ is the density profile at radius r. The corresponding rotation velocity yields

$$v^2(r) = v^2_{\mathrm{N}}(r) + v^2_R(r), \tag{9}$$

"N" is the Newtonian contribution and the Rindler velocity is

$$v^2_R(r) \equiv a\,|\vec{r}| \qquad \text{or} \qquad v^2_{GR}(r) \equiv a|\vec{r}|^n \tag{10}$$

The components

Bringing together all contributions to the total rotation curve, with stars ($\star$), gas (G), modified gravity *a la* Grumiller (GR),

$$v_T^2(r) = \Upsilon_\star v_\star^2 + v_G^2 + v_{GR}^2(r)\,, \tag{11}$$

where $\Upsilon_\star$ is the mass-to-light ratio.

Note that there is no dark matter in the model !!

Stars

The Newtonian contribution from stars ($\star$) and gas (G) can be thought as given by a spherized disk.
The contribution from stars is then given the Freeman-disk:

$$\rho_\star(r) = \frac{M_d}{2\pi r_d^2}\, e^{-r/r_d}, \tag{12}$$

where M_d is the mass of the disk and r_d its radius. Thus, the rotation curve contribution from stars with a standard Newtonian dynamics, yields

$$v_\star^2(r) = \frac{GM_d}{2r_d}\left(\frac{r}{r_d}\right)^2\left[\mathrm{I}_0\left(\frac{r}{2r_d}\right)\mathrm{K}_0\left(\frac{r}{2r_d}\right) - \mathrm{I}_1\left(\frac{r}{2r_d}\right)\mathrm{K}_1\left(\frac{r}{2r_d}\right)\right]. \tag{13}$$

On the other hand, the gas contribution v_G^2 is computed by integrating its surface brightness as in standard Newtonian lore.

Stars' mass models $\Upsilon_\star$

Kroupa Studies of the stellar population in the Milky Way suggest that the Kroupa IMF produces low disk masses that we consider as the minimal limit for the stellar disk.

diet-Salpeter Here $\Upsilon_\star$ is set to be a constant value based on the diet-Salpeter IMF, in which the stellar population synthesis model has proven to give a maximum stellar disk for a given photometric constraint.

Free $\Upsilon_\star$ Here we ignore a priori any knowledge of the IMF and treat $\Upsilon_\star$ as an extra free parameter in the model, and we let the program to choose the best-fitted value for $\Upsilon_\star$.

The Sample

THINGS DATA

Galaxy (1)	Distance (2)	R_d (3)	DIET-SALPETER $\log_{10} M_{star}$ (4)	KROUPA $\log_{10} M_{star}$ (5)
N 925	9.2	3.30	10.01	9.86
N 2366	3.4	1.76	8.41	8.26
N 2403	3.2	1.81	9.67	9.52
N 2841	14.1	4.22	11.04	10.88
N 2903	8.9	2.40	10.15	10.0
N 2976	3.6	0.91	9.25	9.10
N 3031	3.6	1.93	10.84	10.69
N 3198	13.8	3.06	10.45	10.30
N 3521	10.7	3.09	11.09	10.94
N 3621	6.6	2.61	10.29	10.14
N 4736	4.7	1.99	10.27	10.12
N 5055	10.1	3.68	11.09	10.94
N 6946	5.9	2.97	10.77	10.62
N 7331	14.7	2.41	11.22	11.07
N 7793	3.9	1.25	9.44	9.29
I 2574	4.0	2.56	9.02	8.87
D 154	4.3	0.72	7.42	7.27

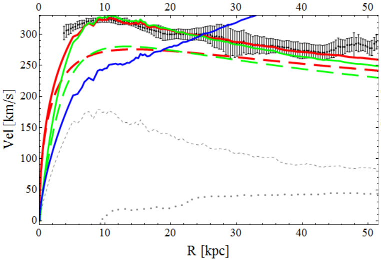
Fitting Procedure

We make use of the observed rotation curve, stellar, and gas components as an input for the numerical code, in order to obtain the Rindler parameters (a, n). To fit the observational velocity curve with the theoretical model we employ the χ^2 goodness-of-fit test (χ^2 test)

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{v_{\mathrm{obs}_i} - v_{T_i}(r; a, n)}{\sigma_i} \right)^2 , \tag{14}$$

where σ is the standard deviation, and n is the number of observations. One defines the reduced $\chi^2_{\mathrm{red}} \equiv \chi^2/(\mathrm{n} - p - 1)$, in which n is the number of observations and p is the number of fitted parameters.

Results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300
250
200
150
100
50
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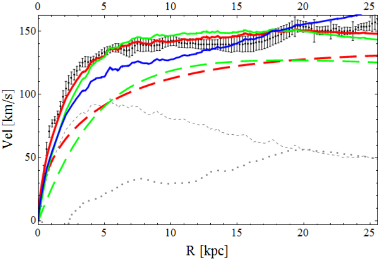
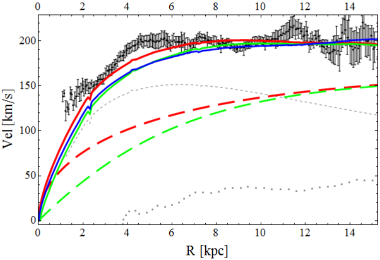
Vel [km/s]
R [kpc]


0 5 10 15 20 25
150
100
50
0
Vel [km/s]
R [kpc]


0 2 4 6 8 10 12 14
200
150
100
50
0
Vel [km/s]
R [kpc]

Results

Rindler $n = 1$

Galaxy	Kroupa a	Kroupa $a\left[\frac{\text{cm}}{\text{s}^2}\right]$	Kroupa χ^2_{red}	diet-Salpeter a	diet-Salpeter $a\left[\frac{\text{cm}}{\text{s}^2}\right]$	diet-Salpeter χ^2_{red}
D 154	$358.45^{+5.45}_{-5.49}$	$1.16^{+0.02}_{-0.02}$	2.10	$355.64^{+5.45}_{-5.48}$	$1.15^{+0.02}_{-0.02}$	2.03
I 2574	$297.35^{+6.91}_{-6.97}$	$0.96^{+0.02}_{-0.02}$	4.74	$272.61^{+6.91}_{-6.97}$	$0.88^{+0.02}_{-0.02}$	5.53
N 2366	$285.45^{+1.94}_{-134.51}$	$0.93^{+0.01}_{-0.44}$	3.48	$285.45^{+2.66}_{-99.97}$	$0.93^{+0.01}_{-0.32}$	3.29
N 2403	$1258.65^{+5.77}_{-5.78}$	$4.08^{+0.02}_{-0.02}$	11.80	$1225.23^{+5.75}_{-5.76}$	$3.97^{+0.02}_{-0.02}$	9.29
N 2841	$2952.36^{+19.45}_{-19.50}$	$9.57^{+0.06}_{-0.06}$	76.10	$2405.18^{+18.91}_{-18.96}$	$7.79^{+0.06}_{-0.06}$	44.30
N 2903	$1985.81^{+13.91}_{-13.96}$	$6.44^{+0.05}_{-0.05}$	71.80	$1998.74^{+13.92}_{-13.97}$	$6.48^{+0.05}_{-0.05}$	74.10
N 2976	$1000.00^{+2.64}_{-723.12}$	$3.24^{+0.01}_{-2.34}$	8.05	$698.04^{+44.46}_{-44.80}$	$2.26^{+0.14}_{-0.15}$	10.70
N 3031	$2617.86^{+32.68}_{-32.77}$	$8.48^{+0.11}_{-0.11}$	9.54	$191.44^{+29.52}_{-29.62}$	$0.62^{+0.10}_{-0.10}$	22.80
N 3198	$632.49^{+5.85}_{-5.87}$	$2.05^{+0.02}_{-0.02}$	14.00	$567.72^{+5.78}_{-5.80}$	$1.84^{+0.02}_{-0.02}$	11.50
N 3521	$684.59^{+36.21}_{-36.59}$	$2.22^{+0.12}_{-0.12}$	6.14	$562.41^{+35.10}_{-35.50}$	$1.82^{+0.11}_{-0.12}$	7.54
N 3621	$889.79^{+6.98}_{-7.00}$	$2.88^{+0.02}_{-0.02}$	11.20	$777.46^{+6.86}_{-6.88}$	$2.52^{+0.02}_{-0.02}$	3.29
N 4736	$1005.43^{+27.96}_{-28.11}$	$3.26^{+0.09}_{-0.09}$	20.50	$420.13^{+27.07}_{-27.24}$	$1.36^{+0.09}_{-0.09}$	8.74
N 5055	$559.90^{+8.71}_{-8.74}$	$1.81^{+0.03}_{-0.03}$	5.09	$399.76^{+8.30}_{-8.34}$	$1.30^{+0.03}_{-0.03}$	15.80
N 6946	$1650.37^{+16.51}_{-16.55}$	$5.35^{+0.05}_{-0.05}$	1.29	$908.03^{+16.12}_{-16.17}$	$2.94^{+0.05}_{-0.05}$	4.42
N 7331	$1938.02^{+24.45}_{-24.55}$	$6.28^{+0.08}_{-0.08}$	8.65	$1505.92^{+23.90}_{-24.00}$	$4.88^{+0.08}_{-0.08}$	29.60
N 7793	$1720.65^{+1.57}_{-358.31}$	$5.58^{+0.01}_{-1.16}$	10.50	$1688.18^{+37.17}_{-15.34}$	$5.47^{+0.12}_{-0.05}$	5.52
N 925	$612.29^{+14.40}_{-14.39}$	$1.98^{+0.05}_{-0.05}$	3.74	$453.50^{+14.26}_{-14.36}$	$1.47^{+0.05}_{-0.05}$	5.72

Results

Rindler $n \neq 1$

	Kroupa			diet-Salpeter		
Galaxy	a	n	$\chi^2_{\rm red}$	a	n	$\chi^2_{\rm red}$
D 154	$429.25^{+6.52}_{-6.56}$	$0.87^{+0.010}_{-0.010}$	1.77	$420.27^{+6.43}_{-6.47}$	$0.88^{+0.010}_{-0.010}$	1.75
I 2574	$77.96^{+1.74}_{-1.75}$	$1.72^{+0.011}_{-0.011}$	0.98	$53.57^{+1.28}_{-1.29}$	$1.87^{+0.011}_{-0.011}$	1.32
N 2366	$340.76^{+16.15}_{-16.42}$	$1.02^{+0.029}_{-0.028}$	3.27	$285.45^{+18.13}_{-11.09}$	$1.13^{+0.100}_{-0.009}$	3.23
N 2403	$4667.30^{+20.97}_{-21.01}$	$0.48^{+0.002}_{-0.002}$	2.27	$4000.25^{+18.46}_{-18.50}$	$0.53^{+0.002}_{-0.002}$	2.55
N 2841	$59071.70^{+333.82}_{-334.49}$	$0.04^{+0.002}_{-0.002}$	0.90	$38957.50^{+262.73}_{-263.27}$	$0.13^{+0.002}_{-0.002}$	1.23
N 2903	$32294.30^{+211.39}_{-212.01}$	$0.03^{+0.002}_{-0.002}$	7.99	$33026.10^{+215.04}_{-215.67}$	$0.02^{+0.002}_{-0.002}$	8.34
N 2976	$796.89^{+25.89}_{-26.15}$	$2.14^{+0.045}_{-0.044}$	1.15	$386.08^{+336.71}_{-0.34}$	$3.43^{+2.196}_{-0.001}$	10.60
N 3031	$9840.04^{+116.91}_{-117.17}$	$0.36^{+0.006}_{-0.006}$	5.12	$2.12^{+0.12}_{-0.12}$	$3.34^{+0.024}_{-0.023}$	20.30
N 3198	$2830.13^{+25.46}_{-25.54}$	$0.54^{+0.003}_{-0.003}$	7.77	$1531.59^{+15.37}_{-15.42}$	$0.69^{+0.003}_{-0.003}$	9.54
N 3521	$195.76^{+10.31}_{-10.44}$	$1.42^{+0.017}_{-0.016}$	5.82	$48.00^{+2.85}_{-2.89}$	$1.83^{+0.019}_{-0.018}$	6.84
N 3621	$3728.60^{+28.09}_{-28.17}$	$0.50^{+0.003}_{-0.003}$	0.87	$1880.38^{+16.30}_{-16.34}$	$0.70^{+0.003}_{-0.003}$	1.24
N 4736	$8377.42^{+176.93}_{-177.79}$	< 0.002	5.76	$4216.82^{+176.24}_{-176.98}$	< 0.003	4.11
N 5055	$322.28^{+5.04}_{-5.06}$	$1.16^{+0.004}_{-0.004}$	4.96	$11.81^{+0.24}_{-0.24}$	$2.01^{+0.000}_{-0.005}$	13.50
N 6946	$2005.70^{+20.02}_{-20.07}$	$0.92^{+0.004}_{-0.004}$	1.25	$57.86^{+1.00}_{-1.00}$	$2.11^{+0.007}_{-0.007}$	2.47
N 7331	$876.99^{+11.11}_{-11.15}$	$1.27^{+0.004}_{-0.004}$	8.21	$80.47^{+1.26}_{-1.27}$	$1.99^{+0.005}_{-0.005}$	26.00
N 7793	$2850.27^{+35.74}_{-35.91}$	$0.69^{+0.009}_{-0.009}$	4.48	$1884.05^{+26.61}_{-26.74}$	$0.92^{+0.009}_{-0.009}$	5.31
N 925	$100.24^{+2.31}_{-2.32}$	$1.83^{+0.010}_{-0.010}$	1.35	$20.43^{+0.60}_{-0.60}$	$2.41^{+0.012}_{-0.012}$	2.30

Results

Free mass model

	$n = 1$					$n \neq 1$				
Galaxy	a	$a\ [\frac{\text{cm}}{\text{s}^2}]$	M_D	M_B	χ^2_{red}	a	n	M_D	M_B	χ^2_{red}
D 154	$332.91^{+5.42}_{-5.50}$	$1.08^{+0.02}_{-0.02}$	$7.96^{+7.06}_{-7.07}$		1.78	$378.75^{+6.01}_{-6.05}$	$0.93^{+0.010}_{-0.010}$	$7.80^{+7.06}_{-7.06}$		1.75
I 2574	$365.24^{+6.89}_{-6.95}$	$1.18^{+0.02}_{-0.02}$	<1		3.94	$136.85^{+2.63}_{-2.65}$	$1.52^{+0.009}_{-0.009}$	$8.26^{+7.76}_{-7.77}$		1.03
N 2366	$176.84^{+16.41}_{-16.69}$	$0.57^{+0.05}_{-0.05}$	$9.00^{+7.12}_{-8.69}$		2.30	<10	$1.11^{+1.00}_{-1.00}$	$9.31^{+9.26}_{-7.92}$		1.58
N 2403	$797.22^{+97.65}_{-0.32}$	$2.58^{+0.32}_{-0.30}$	$10.2^{+9.83}_{-6.36}$	$6.77^{+5.80}_{-5.72}$	4.88	$3070.1^{+16.1}_{-16.1}$	$0.59^{+0.002}_{-0.002}$	$9.85^{+8.07}_{-8.07}$	$7.70^{+6.65}_{-6.65}$	0.75
N 2841	$1182.6^{+16.57}_{-16.73}$	$3.83^{+0.05}_{-0.05}$	$11.4^{+9.08}_{-9.08}$		1.08	61227^{+358}_{-359}	$0.02^{+0.002}_{-0.002}$	$11.0^{+9.08}_{-9.08}$		0.14
N 2903	$965.84^{+12.81}_{-12.87}$	$3.13^{+0.04}_{-0.04}$	$10.8^{+8.74}_{-8.75}$	$7.43^{+6.39}_{-6.47}$	2.47	$4686.8^{+52.1}_{-52.3}$	$0.53^{+0.004}_{-0.004}$	$10.7^{+8.74}_{-8.75}$	$7.70^{+6.71}_{-6.77}$	1.82
N 2976	$2646.5^{+99.93}_{-21.12}$	$8.58^{+0.32}_{-0.07}$	$5.86^{+6.95}_{-4.95}$		2.32	$2130.7^{+41.2}_{-41.6}$	$1.30^{+0.030}_{-0.030}$	$8.38^{+7.59}_{-7.59}$		0.99
N 3031	$1602.5^{+31.69}_{-31.80}$	$5.19^{+0.10}_{-0.10}$	$10.8^{+8.50}_{-8.50}$	$7.83^{+6.79}_{-6.78}$	5.93	34102^{+233}_{-228}	<0.002	$10.5^{+8.50}_{-8.51}$	$5.86^{+4.88}_{-4.89}$	4.86
N 3198	$503.00^{+5.72}_{-5.74}$	$1.63^{+0.02}_{-0.02}$	$10.5^{+8.62}_{-8.62}$	$7.96^{+6.95}_{-6.97}$	5.11	$2144.6^{+21.0}_{-21.0}$	$0.60^{+0.003}_{-0.003}$	$10.4^{+8.62}_{-8.62}$	$8.65^{+7.64}_{-7.69}$	3.96
N 3521	$4580.0^{+7.67}_{-294}$	$14.80^{+0.02}_{-0.95}$	$9.00^{+6.88}_{-6.45}$		63.10	$120.08^{+6.75}_{-8.96}$	$1.51^{+0.017}_{-0.023}$	$11.0^{+8.96}_{-9.05}$		1.78
N 3621	$689.06^{+4.84}_{-9.49}$	$2.23^{+0.02}_{-0.03}$	$10.4^{+8.39}_{-8.32}$		0.67	$2015.0^{+17.3}_{-17.4}$	$0.67^{+0.003}_{-0.003}$	$10.3^{+8.35}_{-8.35}$		0.56
N 4736	<10	<0.1	$10.5^{+8.44}_{-8.44}$	$7.27^{+6.34}_{-6.24}$	53.0	19022^{+153}_{-154}	$0.10^{+0.001}_{-0.001}$	$7.08^{+7.90}_{-5.90}$	$6.69^{+5.77}_{-5.63}$	32.1
N 5055	$556.69^{+8.75}_{-8.82}$	$1.80^{+0.03}_{-0.03}$	$11.0^{+8.76}_{-8.77}$	<1	9.30	27370^{+175}_{-175}	$0.02^{+0.002}_{-0.002}$	$10.5^{+8.76}_{-8.76}$	$7.70^{+6.62}_{-6.75}$	0.86
N 6946	$1378.7^{+16.64}_{-16.30}$	$4.47^{+0.05}_{-0.05}$	$10.7^{+8.55}_{-8.54}$	<1	1.48	$7052.8^{+53.5}_{-53.6}$	$0.50^{+0.003}_{-0.003}$	$10.6^{+8.54}_{-8.54}$	$3.70^{+2.67}_{-2.70}$	1.22
N 7331	$1707.9^{+24.65}_{-24.73}$	$5.53^{+0.08}_{-0.08}$	$11.0^{+9.01}_{-9.01}$	$12.2^{+11.2}_{-11.1}$	0.26	$3348.2^{+45.7}_{-45.8}$	$0.79^{+0.005}_{-0.005}$	$10.9^{+9.01}_{-9.01}$	$7.70^{+6.67}_{-6.77}$	0.24
N 7793	$1523.1^{+23.79}_{-23.79}$	$4.94^{+0.08}_{-0.08}$	$9.47^{+7.85}_{-7.85}$		4.88	$2277.8^{+30.2}_{-30.4}$	$0.83^{+0.009}_{-0.009}$	$9.31^{+7.85}_{-7.85}$		4.61
N 925	$803.09^{+0.42}_{-450}$	$2.60^{+0.01}_{-1.46}$	$7.55^{+6.88}_{-6.88}$		4.07	$420.11^{+7.12}_{-7.17}$	$1.33^{+0.008}_{-0.008}$	$9.47^{+8.34}_{-8.34}$		1.12

Results

Goodness-of-fit comparison table

Galaxy	NFW	Burkert	Kroupa Rindler ($n=1$)	Rindler ($n \neq 1$)	NFW	Burkert	diet-Salpeter Rindler ($n=1$)	Rindler ($n \neq 1$)
D 154	1.06	0.43	2.10	1.77	1.09	0.39	2.03	1.75
I 2574	2.40	0.69	4.74	0.98	5.49	1.16	5.53	1.32
N 2366	3.00	1.34	3.48	3.27	2.94	1.34	3.29	3.23
N 2403	0.82	1.60	11.80	2.27	1.08	1.37	9.29	2.55
N 2841	1.29	3.02	76.10	0.90	0.49	0.93	44.30	1.23
N 2903	3.43	2.01	71.80	7.99	3.73	1.23	74.10	8.34
N 2976	6.31	1.00	8.05	1.15	10.49	5.11	10.70	10.60
N 3031	4.96	5.12	9.54	5.12	22.76	4.37	22.80	20.30
N 3198	4.75	2.49	14.00	7.77	7.86	0.93	11.50	9.54
N 3521	5.66	5.21	6.14	5.82	12.48	11.5	7.54	6.84
N 3621	1.45	5.59	11.20	0.87	1.44	3.64	3.29	1.24
N 4736	1.34	1.30	20.50	5.76	1.30	1.29	8.74	4.11
N 5055	5.09	4.11	5.09	4.96	15.74	3.47	15.80	13.50
N 6946	1.21	1.14	1.29	1.25	4.42	2.49	4.42	2.47
N 7331	8.63	7.06	8.65	8.21	29.60	15.6	29.60	26.00
N 7793	4.07	7.88	10.50	4.48	5.01	4.30	5.52	5.31
N 925	3.68	1.19	3.74	1.35	5.64	2.54	5.72	2.30

Results

Summarizing:

- Fits for $n = 1$ are poor, since most of the $\chi^2_{\rm red}$ are bigger than unity. However, the most important problem is the Rindler acceleration parameter does not converge to a single value. The computed parameter is in the interval $0.93^{+0.01}_{-0.44} < a < 9.57^{+0.06}_{-0.06}$ in Kroupa's model, to account for a difference of an order of magnitude, whereas $0.62^{+0.10}_{-0.10} < a < 7.79^{+0.06}_{-0.06}$ in diet-Salpeter's model, in a similar fashion as in the previous model.
- When the power-law parameter n is set free, the fits become better, and by comparing them Kroupa's did much better in 14 (out of 17) cases than diet-Salpeter's. However, the goodness-of-fit test does not render acceptable results since some galaxies present very high $\chi^2_{\rm red}$ values. The parameters spreads are large too: For Kroupa's model: $77.96^{+1.74}_{-1.75} < a < 32294.30^{+211.39}_{-212.01}$, accounting for two orders of magnitude in difference and $0.002 \sim n < 2.14^{+0.045}_{-0.044}$ that yields a three orders of magnitude difference. For diet-Salpeter parameters the spreads are similar.

Results

Summarizing:

- When the mass-to-light ratio is set free, the fits are better than previous models when n is a free parameter, but again the spread in parameters is high: $10 \sim a < 19022^{+153}_{-154}$, accounting for a three orders of magnitude difference and $0.002 \sim n < 1.52^{+0.009}_{-0.009}$, a difference of two orders of magnitude.
- Comparing these results with those of NFW and Burkert, the χ^2-test favors firstly Burkert and then NFW profiles over Rindler's modify gravity. The tendency is clearer for the standard Rindler ($n = 1$) that fits worst than both NFW's and Burkert's profiles for the Kroupa and diet-Salpeter stellar mass models. The generalized Rindler model ($n \neq 1$) for the diet-Salpeter stellar mass model results are slightly better than the Kroupa's model, but still the NFW profile fits better for 9 galaxies (out of 17) and Burkert profile achieves a better fit for 12 galaxies (out of 17) than the generalized Rindler model.

Conclusions

- We have tested the idea that a modification of the Newtonian potential stemming from a Rindler acceleration that modifies gravity, as

$$\phi_i(r) = -G\frac{M}{r} + a\,r^n \quad \Rightarrow \quad v_T^2(r) = \Upsilon_\star v_\star^2 + v_G^2 + a|\vec{r}|^n \tag{15}$$

- The overall conclusion is that although the Rindler modified gravity fits are achievable for the considered galaxies, in many cases they show high $\chi^2_{\rm red}$ values, and a high spread in the Rindler parameters $(a,\, n)$ that points for an inconsistent model. Furthermore, the standard dark matter profiles (NFW and Burkert) or the alternative BDM model do a better job to fittings of the rotation curves.