# The inflationary Origin of the Seeds of Cosmic Structure: The Need for Novel Physics and the Possible Gravitation-Quantum Theory Connection

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# THE GENERAL SETTING: QUANTUM THEORY , SPACE-TIME & QUANTUM GRAVITY

i) Bottom-Up Approach: Start with a proposal for a fundamental theory and work to connect with empirical world. Examples : String Theory, Causal Sets, LQG, etc.

ii)Top Down Approach: Consider existing well established theories, trustable in a limited regime, and push to "the boundary" of such regimes, looking for indications they need modifying. Examples : THE SME of Kostelecki et. al., QG phenomenology, etc.

We follow ii) : I.e. consider that, at the fundamental level, there is some deeper theory that ought to reduce to classical GR, QFT and Quantum Theory, in the respective regimes.

We have been influenced by R. Penrose arguments that in joining quantum theory and gravitation, we might have to modify both.

The regime we will consider here is that involving quantum aspects of cosmology, and will focus in the form that uncomfortable aspects of quantum theory take in that context: I am, of course, referring to the so called "measuring problem" ( MP).

Dealing with MP is essential if we want a framework applicable to closed systems, and, in particular the universe as a whole (or an isolated and non entangled region thereof).

The technical complications involved in considering seriously something like the whole universe often distract us from the conceptual issues. To avoid this let's focus on simple symmetry arguments.
Also convenient to consider in parallel a toy model where the issue takes a very simple form.
We focus on the problem of symmetry in cosmology, in particular:
In any proposal that has our very early universe being represented by an Homogenous and Isotropic quantum state, how do we account for the late time inhomogeneity and anisotropy ?
Examples: Hartle-Hawking Wave function of the Universe in WDW.
Inflation: The emergence of the seeds of structure from quantum fluctuations. This represents the ONLY CASE where General Relativity & Quantum Theory come together in explaining a situation for which we have EMPIRICAL INFORMATION.

## The observations

We see the CMB photons emitted at the (LSS) with a local temperature $T \approx 3000K^0$. subject to two redshifts: 1) That tied to the overall cosmological expansion leading to $T \approx 2.7K^0$. 2) That tied to the "exiting" from the local Newtonian potential well, tied to the local matter distribution (there are more complexities but this is enough for our purposes).

Thus $\frac{\delta T}{T_0}(\theta, \varphi) = \frac{1}{3}\psi(\eta_D, \vec{x}_D)$: characterizes the Newtonian Potential on the intersection of our past light cone with the LSS.

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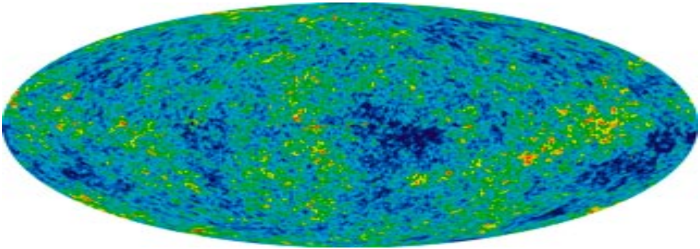


This map is characterized by an expansion in spherical harmonics: $\frac{\delta T}{T_0}(\theta,\varphi) = \sum_{lm} \alpha_{lm} Y_{lm}(\theta,\varphi)$.

Thus the coefficients

$$\alpha_{lm} = \frac{1}{3} \int d\Omega^2 \psi(\eta_D, \vec{x}_D) Y^*_{lm}(\theta,\varphi) \tag{1}$$

The determination of $\frac{\delta T}{T_0}(\theta,\varphi)$ provides the map from where one extracts the $\alpha_{lm}$.

Most studies focus on:

$$C_l = \frac{1}{2l+1} \sum_m |\alpha_{lm}|^2. \tag{2}$$

INFLATION is supposed to explain this and in fact everything seems to work fine. HOWEVER, the explanation relies on a choice of state (presumably as the result of the early phases of inflation) that is H & I to a very high degree ( i.e. deviations of order $e^{-N}, N = \text{No of e-folds} \sim 70$). How is the symmetry destroyed?

2) USUAL ANSWERS:

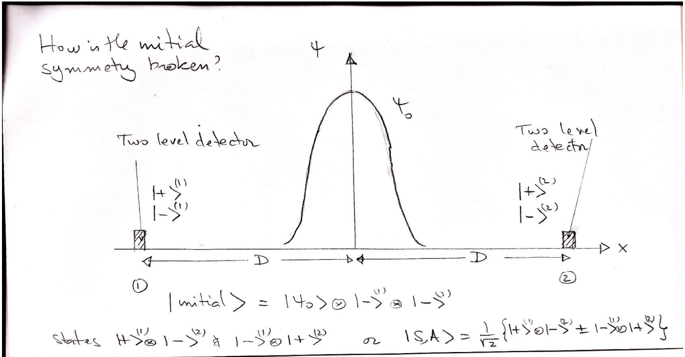
a) We make measurements, inducing the quantum state reduction .
We are here due to inhomogeneities.

b) Correlations?. Do not imply a breakdown of the symmetry before measurements ( think EPR-b).

c)Decoherence: environments + MANY WORLD Interpretation (MWI). Several issues ( Please ask at the end!).

d) Consistent Histories The answers depend on the way we ask the questions.

The best way to test any such proposal is to focus on a toy model:



3) If the known physics is unable to resolve the problem we must consider new physics

NOTE: This is the only situation where we find the combination :

Quantum Theory + General Relativity + Observations .

A satisfactory explanation to our question, must point to a physical process, taking place in time that can explain the emergence of the seeds of cosmic structure.

After all, “Emergence ” refers to : something that was not there initially being there at a later time .

We propose : adding to the inflationary paradigm some spontaneous dynamical collapse of the quantum state. (Inspired on ideas of Penrose/ Diosi).

Dynamical Collapse Theories : GRW, Pearle, Diosi, Penrose & recently Weinberg.

Example, CSL: It is defined by two equations:
i) A modified Schrödinger equation, whose solution is:

$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[ i\hat{H} + \frac{1}{4\lambda}[w(t') - 2\lambda\hat{A}]^2 \right]} |\psi, 0\rangle. \tag{3}$$

( $\hat{\mathcal{T}}$ is the time-ordering operator). $w(t)$ is a random classical function of time, of white noise type, whose probability is given by the second equation, ii) the Probability Rule:

$$PDw(t) \equiv {}_w\langle\psi, t|\psi, t\rangle_w \prod_{t_i=0}^{t} \frac{dw(t_i)}{\sqrt{2\pi\lambda/dt}}. \tag{4}$$

The processes $U$ and $R$ (corresponding to the observable $\hat{A}$) are unified. For non-relativistc QM the proposal assumes : $\hat{A} = \hat{\vec{X}}$.
Here $\lambda$ must be small enough not to conflict with tests of QM in the domain of subatomic physics, and big enough to result in rapid localization of "macroscopic objects". GRW suggested range: $\lambda \sim 10^{-16} sec^{-1}$. ( Exp. bounds suggest $\lambda^{(i)} = \lambda(m^{(i)}/m_N)^2$ ).

## 4) Our approach :

We need to adapt the approach to situations involving both Quantum Fields and Gravitation.

Dynamical reduction in the quantum state requiters the notion of " time" ( the collapse takes place in time). As QG has a problem with time, and its resolution generically involves passing to a sort of semiclassical regime. We make our analysis assuming we can rely on a semiclassical framework.
We consider that even if at the deepest levels gravitation must be quantum mechanical in nature at the meso/macro scales, it corresponds to an emergent phenomena, with traces of the quantum regime surviving in the form of an effective dynamical state reduction for matter fields.

Assume that in the inflationary regime one already has a good description of gravitation in terms of classical geometric notions, however , matter fields must still be considered using quantum theory . This seems reasonable as inflation is supposed to occur at GUT scales where $R << 1/l^2_{Planck}$.

The space-time is treated classically (in our case using a specific gauge and ignoring tensor perturbations):

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1-2\Psi)\delta_{ij}dx^i dx^j\right], \Psi(\eta, \vec{x}) \ll 1$$

The scale factor can be written as

$$a(\eta) = \frac{-1}{\eta H_I} \qquad (5)$$

with $\eta \in (-\mathcal{T}, \eta_0), \eta_0 < 0$.

The scalar field must be treated using QFT in CS .

The quantum state of the scalar field and the space-time metric satisfy Einstein's semiclassical eq.

$$G_{\mu\nu} = 8\pi G \langle \xi | \hat{T}_{\mu\nu} | \xi \rangle.$$

We will be concentrating on the modes other than the zero mode which is responsible for the overall inflationary expansion and which we treat classically as an effective approximation.

At the early stages of inflation, which we denote by $\eta = -\mathcal{T}$, the state of the scalar field perturbation is described by the Bunch-Davies vacuum, and the space-time is 100 % homogeneous and isotropic.

In fact, in the vacuum state the operators $\hat{\delta}\phi_k$ $\hat{\pi}_k$ are characterized by gaussian wave functions centered on $0$ with uncertainties $\Delta\delta\phi_k$ and $\Delta\pi_k$.
The collapse modifies the quantum state, and generically the expectation values of $\widehat{\delta\phi_k}(\eta)$ and $\hat{\pi}_k(\eta)$.

We must now specify the rules governing the collapse. This is the result of some unknown aspect of physics, which we will here encode into some effective collapse theory.

The approach is based on making an "educated guess", which can later be contrasted with observations. The collapse will be controlled mode by mode by some stochastic function.

Note: Our universe would correspond to one specific realization of these stochasticities (one for each $\vec{k}$).

The semi classical Einstein Equation we must focus on is:

$$-k^2\Psi(\eta,\vec{k}) = 4\pi G\phi_0'(\eta)\langle\hat{\delta\phi}'(\vec{k},\eta)\rangle = \frac{4\pi G\phi_0'(\eta)}{a}\langle\hat{\pi}(\vec{k},\eta)\rangle \tag{6}$$

( $\langle\hat{\pi}(\vec{k},\eta)\rangle \equiv \langle\psi,\eta|\hat{\pi}(\vec{k})|\psi,\eta\rangle$). As we said at the start of inflation( $\eta = -\mathcal{T}$) state is described by the Bunch-Davies vacuum, so $\langle\psi,-\mathcal{T}|\hat{\pi}(\vec{k})|\psi,-\mathcal{T}\rangle = 0$. however the effects of the collapse dynamics is to change this in a manner involving "randomness". The quantity of interest is:

$$\frac{\Delta T(\theta,\varphi)}{\bar{T}} = c\int d^3k e^{ik\cdot\vec{x}}\frac{1}{k^2}\langle\hat{\pi}(\vec{k},\eta)\rangle, \text{ where } c \equiv -\frac{4\pi G\phi_0'(\eta)}{3a}. \tag{7}$$

Here, $\vec{x}$ is a point on the intersection of our past light cone with the last scattering surface which corresponds to the $\theta,\varphi$ direction on the sky.Thus:

$$\alpha_{lm} = c\int d^2\Omega Y_{lm}^*(\theta,\varphi)\int d^3k e^{i\vec{k}\cdot\vec{x}}\frac{1}{k^2}\langle\hat{\pi}(\vec{k},\eta)\rangle. \tag{8}$$

There is no analogous to this expression in the standard approaches!

The eq. above shows that the quantity of interest can be thought of as a result of a "random walk" on the complex plane. One can't predict the end point of such "walk" but can focus on the magnitude of the total displacement:

$$|\alpha_{lm}|^2 = (4\pi c)^2 \int d^3k d^3k' j_l(kR_D) j_l(k'R_D) Y_{lm}(\hat{k}) Y^*_{lm}(\hat{k}') \tag{9}$$

$$\frac{1}{k^2 k'^2} \langle \hat{\pi}(\vec{k},\eta)\rangle \langle \hat{\pi}(\vec{k}',\eta)\rangle^*. \tag{10}$$

( We need the product of expectation values and not the expectation value of the product !!) and estimate the most likely value of such quantity by using an ensemble average. The ensemble here is just an auxiliary computational tool. Computing this at "late times":

$$\overline{(\langle \hat{\pi}(\mathbf{k},\eta)\rangle \langle \hat{\pi}(\mathbf{k}',\eta)\rangle^*)} = f(k)\delta(\mathbf{k}-\mathbf{k}').$$

Then,

$$\overline{|\alpha_{lm}|^2} = (4\pi c)^2 \int_0^\infty dk j_l(kR_D)^2 \frac{1}{k^2} f(k). \tag{11}$$

Now we need to use the theory controlling the Collapse

Here I 'll describe the results of using a version of CSL ( *PRD* , **87**, 104024 (2013). arXiv:1211.3463[gr-qc]). One needs to chose the operator $\hat{A}$ driving the collapse and the parameter $\lambda$.

We work with a rescaled field $y(\eta, \vec{x}) \equiv a\delta\phi(\eta, \vec{x})$ and its momentum conjugate $\pi_y(\eta, \vec{x}) = a\delta\phi'(\eta, \vec{x})$ .

For simplicity, put everything in a Box of size $L$ ( to be removed at the end), and focus on a single mode $\vec{k}$, so we write:

$$X \equiv (2\pi/L)^{3/2} y(\eta, \vec{k}), \qquad P \equiv (2\pi/L)^{3/2} \pi_y(\eta, \vec{k}). \tag{12}$$

As we saw, in order to compare with the observations, we need to evaluate the ensemble average $\overline{\langle \hat{P} \rangle^2}$, and determine under what circumstances, if any, this is $\sim k$.

Note: We must consider $\overline{\langle \hat{P} \rangle^2}$ and NOT $\overline{\langle \hat{P}^2 \rangle}$ !!

$\hat{P}$ as Generator of Collapse. Setting $\hat{A} = \hat{P}$ we obtain:

$$\overline{\langle \hat{P} \rangle^2} = \frac{\lambda k^2 \mathcal{T}}{2} + \frac{k}{2} - \frac{k}{\sqrt{2}\sqrt{1+\sqrt{1+4\lambda^2}}}. \qquad (13)$$

Note that if we set $\lambda = 0$ (turn off CSL), we have the standard quantum mechanics result $\overline{\langle \hat{P} \rangle^2} = 0$ since $\langle \hat{P} \rangle = 0$.

We see that agreement with the observed scale-invariant spectrum can be achieved if we assume the first term is dominant and we set

$$\lambda = \tilde{\lambda}/k, \qquad (14)$$

with $\tilde{\lambda} = \mathit{constant}$. We note that this replaces the dimensionless collapse rate parameter $\lambda$ by the parameter $\tilde{\lambda}$ with dimension $\text{time}^{-1}$ as in the original CSL.

In that case we obtain:

$$\overline{\langle\hat{P}\rangle^2} = \frac{\tilde{\lambda}k\mathcal{T}}{2} + \frac{k}{2} - \frac{k}{\sqrt{2}\sqrt{1+\sqrt{1+4(\tilde{\lambda}/k)^2}}}. \qquad (15)$$

The dominant term has the "right" behavior but there are very specific corrections.

Analogously, we consider: $\hat{X}$ as Generator of Collapse and obtain analogous results ( now the requirement is $\lambda = k\tilde{\lambda}$).
Comparisons with observations, using GUT scale inflation potential value and slow-roll parameter (order a few percent), we estimate $\lambda \sim 10^{-5}MpC^{-1} \approx 10^{-19}sec^{-1}$.

Not very different from GRW suggestion .

# Collapse on Field Operators

We would like to understand how the collapse looks when described in terms of the space-time field operators. In one case we can start by defining

$$\tilde{y}(\vec{x}) \equiv \frac{1}{(2\pi)^{3/2}} \int d^3k e^{i\vec{k}\cdot\vec{x}} k^{1/2} y(\vec{k}) = (-\nabla^2)^{1/4} \hat{y}(\vec{x}), \qquad (16)$$

The state vector evolution given by

$$|\psi, t\rangle = \mathcal{T} e^{-i\int_{-\mathcal{T}}^{\eta} d\eta' \hat{H} - \frac{1}{4\tilde{\lambda}} \int_{-\mathcal{T}}^{\eta} d\eta' \int d^3x [w(\vec{x},\eta') - 2\tilde{\lambda}\tilde{y}(\vec{x})]^2} |\psi, -\mathcal{T}\rangle. \qquad (17)$$

This is just the standard CSL state-vector evolution, where the collapse-generating operators (toward whose joint eigenstates collapse tends) are $\tilde{y}(\vec{x})$ for all $\vec{x}$.

Similarly, in the other case,

$$|\psi,\eta\rangle = \mathcal{T} e^{-i\int_{-\mathcal{T}}^{\eta} d\eta' \hat{H} - \frac{1}{4\tilde{\lambda}}\int_{-\mathcal{T}}^{\eta} d\eta' \int d^3x [w(\vec{x},\eta') - 2\tilde{\lambda}\tilde{\pi}(\vec{x})]^2} |\psi, -\mathcal{T}\rangle. \quad (18)$$

where $\tilde{\pi}(\vec{x}) \equiv (-\nabla^2)^{-1/4}\hat{\pi}(\vec{x})$.
Again, the standard CSL state-vector evolution, where the collapse-generating operators are $\tilde{\pi}(\vec{x})$ for all $\vec{x}$.

What are the fundamental reasons determining the appearance of the operators $(-\nabla^2)^{-1/4}\hat{\pi}(\vec{x})$ (or $(-\nabla^2)^{1/4}\hat{y}(\vec{x})$)?

A satisfactory answer will have to wait for a general theory expressing, in all situations, from particle physics, to cosmology, the exact form of the CSL-type of modification to the evolution of quantum states.

## 6) OTHER STUDIES & PREDICTIONS.

It is worth noting that in combining QT and gravitational situations we have found ( besides some severe mathematical difficulties) also some difficulties that seemed not to have counterparts in other cases:

1) The problem of Time in Canonical Quantum Gravity

2) The information loss paradox in BH evaporation processes.

Could it be that the resolution of these might come from following the path suggested by Penrose? Some very preliminary analysis suggests a positive answer ( *Found. of Phys.* in press Arxiv: gr-qc 1309.1730).

i) No tensorial modes (at 1-st order pert theory, semiclassical)

ii) Approach could offer a solution to the ”Fine Tuning” problem for the inflationary Potential. ( CQG, 27, 225017, (2010)).

iii) Multiple Collapses . More information about post collapse states. Limits on number of collapses per mode (CQG, 28, 155010, (2011)).

iv) Novel options for the analysis of No-Gaussianities (*Sigma* **8**, 024, (2012) & *PRD* **88**, 023526 (2013). e-Print: arXiv:1107.3054).

v) Development of the SSC formalism that incorporates dynamical collapses in the semi-classical GR setting (*JCAP*. **045**, 1207, (2012)).

Decoherence: environments + MANY WORLD Interpretation (MWI). This as well as the other proposals requires a long discussion

but the central points are:

i) Requires identification of the D.O.F that act as "environment". Implies using our experimental limitations as part of the argument.
ii) Decoherence does not imply the situation corresponds to one of the elements on the decohereing ( diagonal) density matrix. Seems to require MWI.
iii) However, a close examination of MWI indicates the reliance on some brain whose states of consciousness determine the BASIS characterizing the world splitting.

For more clarity focus on the toy model !