Primordial fluctuations from deformed quantum algebras

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December 9, 2013

Inflation as a probe of quantum gravity

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    - quantum gravity effects should be imprinted on distribution of galaxies in the sky

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- no calculable quantum gravity theories available $\Rightarrow$ consider phenomenological models that alter:
  - classical dynamics for large g (via semiclassical effective theories, modified dispersion relations, etc.), or
  - quantum dynamics for large g (e.g. deformed quantum algebras)

Deformed quantum algebras

- assume that Heisenberg algebra of quantum operators gets modified at high energy:

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- want to recover ordinary physics for small momenta $\hat{p}$ and have invariance under $(\hat{q},\hat{p}) \mapsto (-\hat{q},-\hat{p})$:

$$f(0) = 1 \qquad f(x) = f(-x)$$

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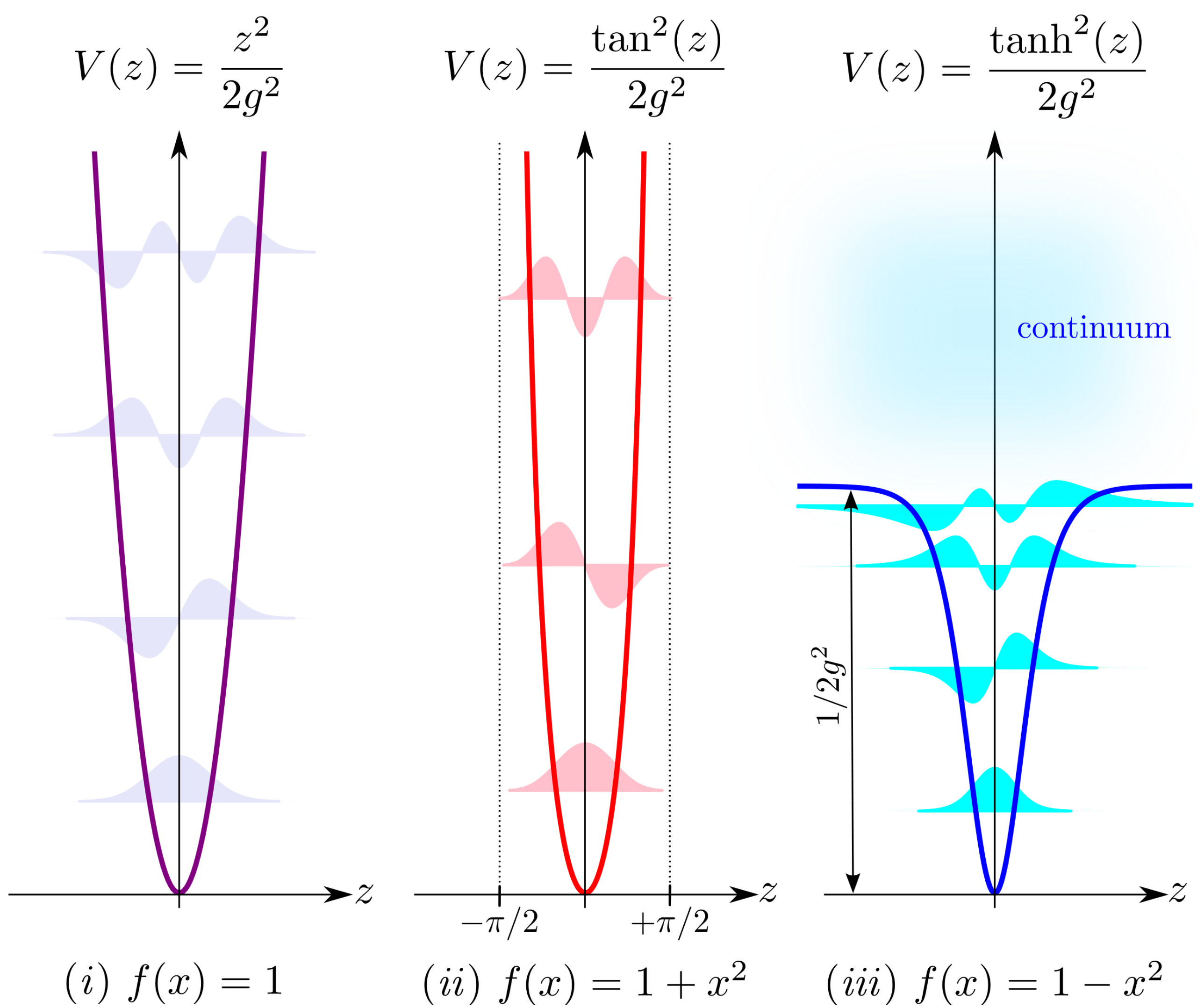
- number of interesting effects:
  - modified propagator
  - modified dispersion relations
  - field energy no longer quantized in units of $\hbar\omega$, energy levels follow from solution of a 1D Schrödinger bound state problem



($g = k/M_\star$ in this case)

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  - $\epsilon = \dfrac{\text{inflationary Hubble parameter}}{\text{exotic physics scale}} = \dfrac{H}{M_\star}$
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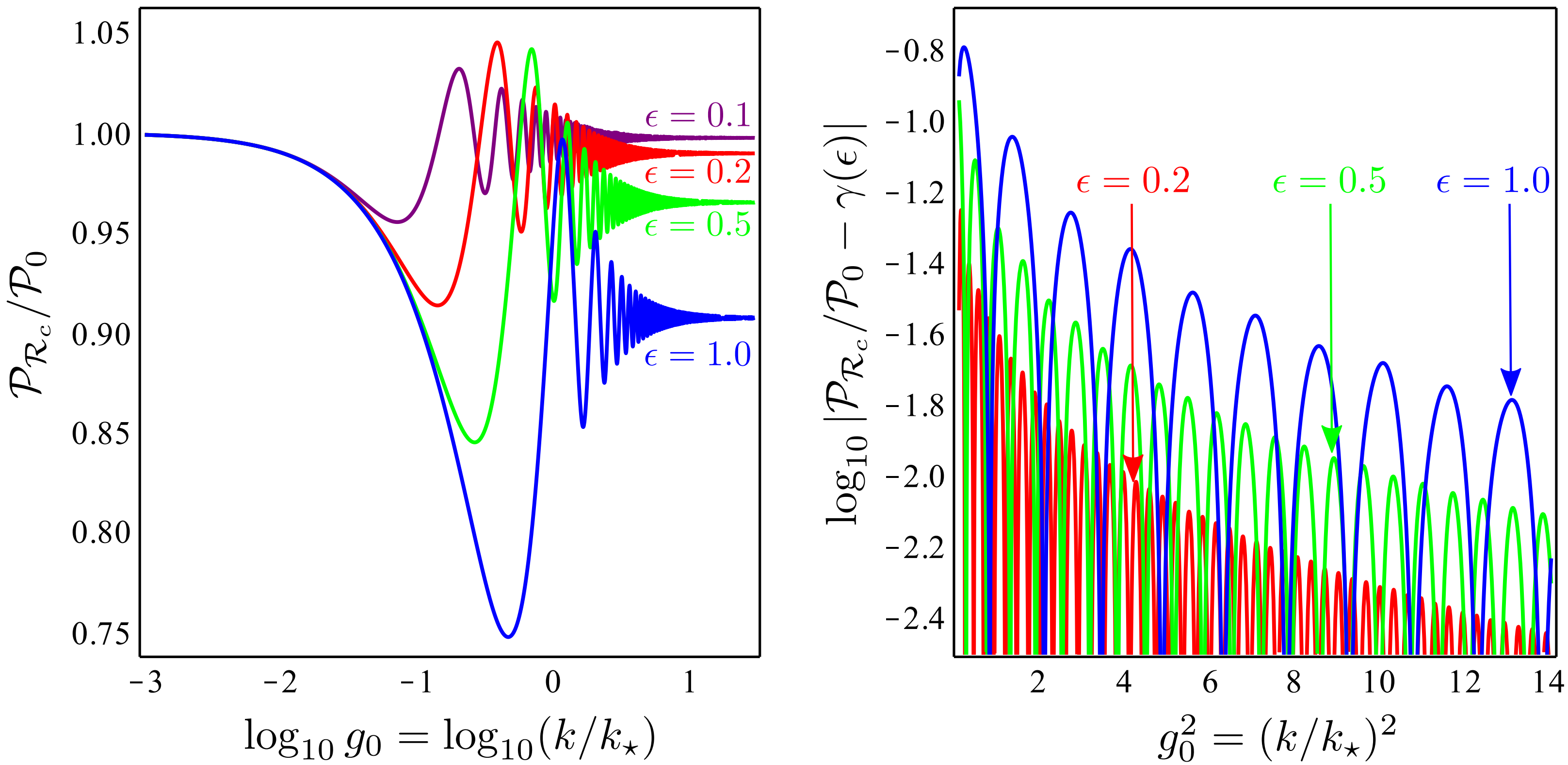
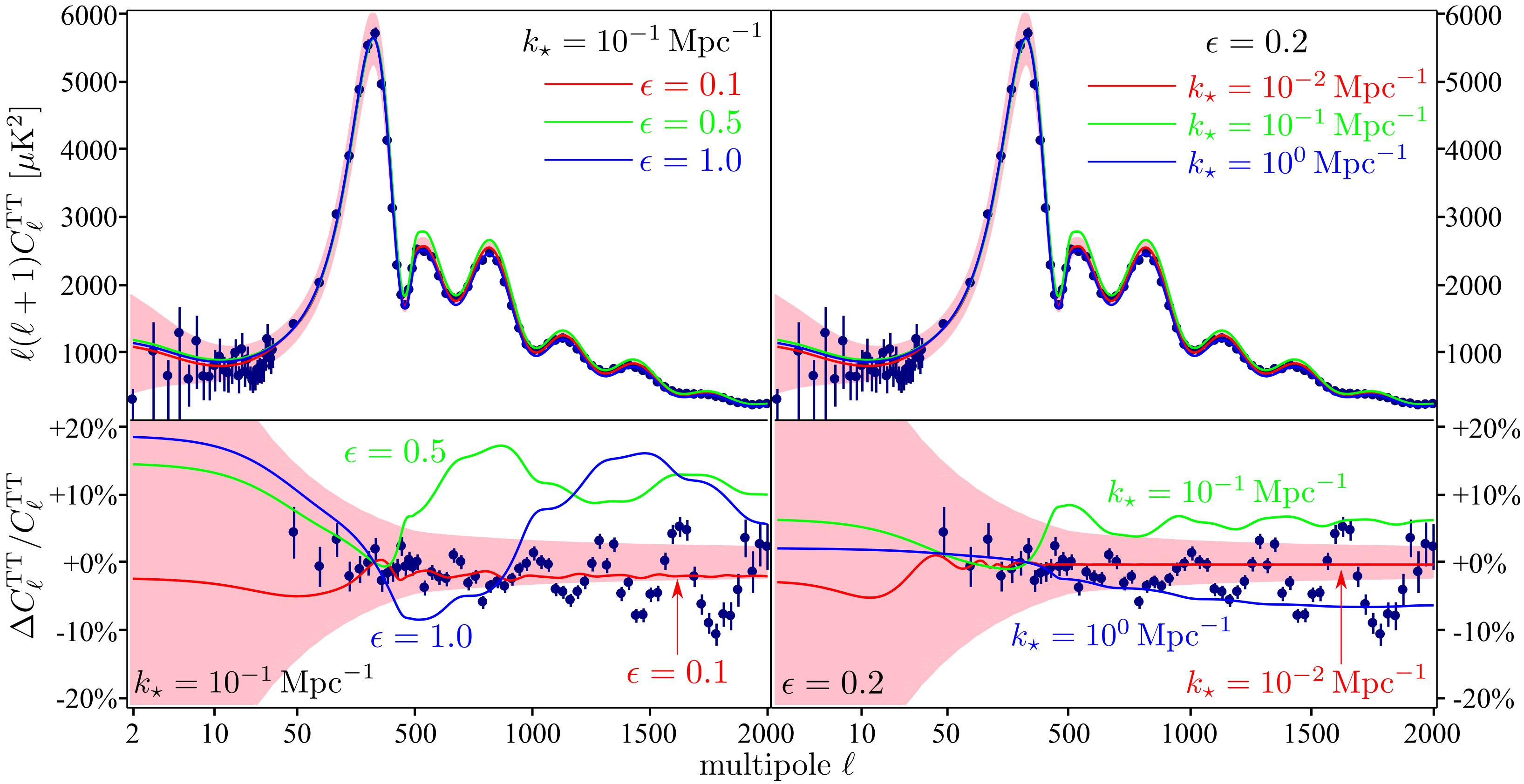
$$k_\star \sim \frac{6 \times 10^{-6}}{\text{Mpc}} \epsilon^{-1} \left(\frac{E_{\text{inf}}}{10^{16}\,\text{GeV}} \right) \left(\frac{e^{65}}{e^N} \right) \left(\frac{100}{\mathcal{G}} \right)^{1/12}$$

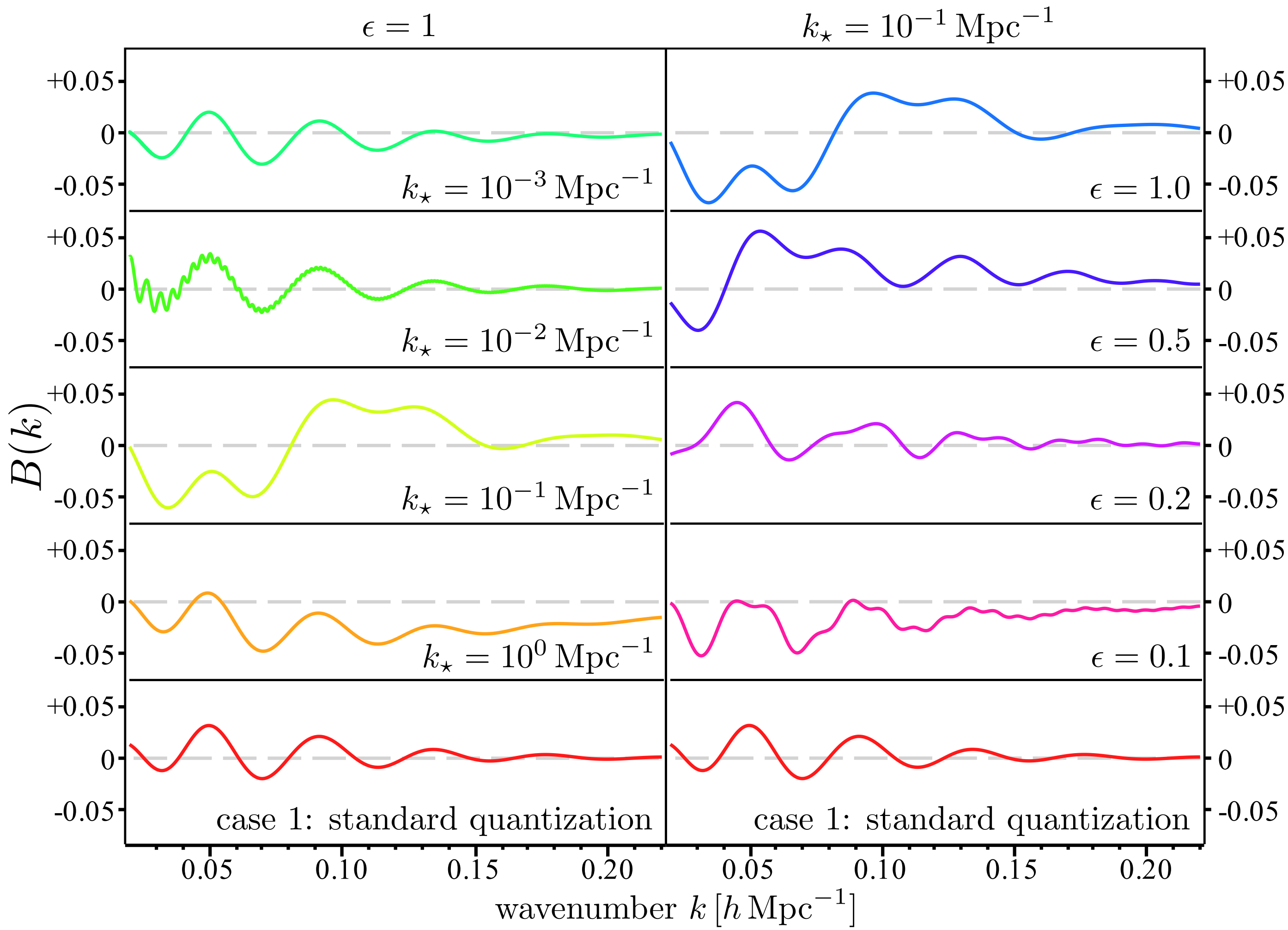


- comoving curvature perturbation results for $f(x) = 1 + x^2$
- $\mathcal{P}_0$ is ordinary slow-roll result—recovered for small k
- DQA effects $\propto \epsilon^2 = H^2/M_\star^2$

Primordial fluctuations



- CMB power spectrum results for $f(x) = 1 + x^2$
- high multipoles can distinguish between various values of $\epsilon = H/M_\star$



- Baryon acoustic oscillations (BAOs) for $f(x) = 1 + x^2$

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- effects were $\mathcal{O}(H^2/M_\star^2)$ (for a specific case)
- may be possible to see effects in CMB or BAOs if $H/M_\star$ not too small
- **still to do:** parameter estimation/limits on ϵ and $k_\star$